GTOC7 Solution Description

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The methods we used

Basically, we utilized the greedy algorithm to construct asteroid tours for three probes, companied with a whole search of the launch time of the mother ship to determine the best starting asteroids. And also there are some tricks when designing the returning rendezvous trajectory for all probes and the mother ship. Local optimal solution for the greedy step is carefully chosen so that the algorithm can approximate the global optimal solution as much as possible.

A simple description of our solution is given in this paragraph. First, the whole asteroid database is analyzed and divided into four groups: one group for the mother ship to rendezvous departing from the earth, and other three groups for each probes to explore. Second, the mother ship is launched from the earth with provided excessive velocity, and arrived at an asteroid belonging to the previous extracted group. Third, with at least one probe stay on this landing asteroid for at least 30 days, all three probes start to fly a tour in their private asteroid group, until they have just enough propellant to accomplish the return rendezvous. Fourth, three sets of N possibly feasible asteroids for probes are detected, and their intersection set (if empty, just enlarge N) are examined to decide which can be arrived by the mother ship from the original landing asteroid with left propellant. Then, finally, transfers to the chosen asteroid are constructed, and they all rendezvous at this asteroid, still with at least one probe stay on it for at least 30 days.

There are some highlights:

1. The latter three groups for probes have no intersections. They have semimajor axis a between 2.21 and 2.4 AU. , eccentricity e is no greater than 0.15, inclination i is no greater than 5, then they are equally divided by the longitude of ascending node Ω. Since i is small, Ω is not that much significant. In this way, the group properties are as uniform as possible.
2. All probes start from the same asteroid. Although the mother ship could bring one probe to a new asteroid, but the propellant is limited and we no benefit from it, especially to our return and rendezvous strategy.
3. In each inter-asteroid phase for one probe, we designed 8 ways to pick out a promising asteroid, the most powerful rule is: the asteroid gives the least total error of both position and speed at starting epoch t_1 and arriving epoch t_2. The required initial guess of flight time is 200 days, determined through a trial-and-error process.
4. Sometimes, the most promising asteroid does not give best (the ‘best’ will be explained later)

transfer, so for each step of the tour, we calculate transfers to three most promising asteroids, and choose the best one. (logic and details can be found in the flow chart at the end of the file) This is one of the most important greedy strategies.

5. 'Best' means propellant cost d*m* is least, and d*m*/d*t* is no smaller than 0.55. This is an important indicator. Because total mass 1200 kg over total time 365.25*6 days is approximately 0.55, if the index is less than 0.55, it means that the probe has wasted time to perform a transfer, and if it is greater, it means propellant is not most efficiently used. So the best result should be just around 0.55, which means the propellant is used up just when it rendezvous with the mother ship again. In our solution, this is exactly the situation of Probe 1 and 2, but Probe 3 leaves much propellant.
6. The final rendezvouses occur at an unvisited asteroid, detected a series of searches and judgments (as shown in flow charts at the end). At the rendezvous the probe and the mother ship usually has a phase angle, which causes great energy for a rendezvous, and it inevitably requires an optimization. Also the mother ship is too heavy to perform several maneuvers to retrieve all probes with limited propellant left. So we guided the mother ship to a new asteroid, and guarantee that this asteroid is achievable by all probes. Actually the procedure is reversed. We detecting which asteroids are achieved by the mother ship, from many asteroids that achievable for three probes (still refer to flow charts).
7. This problem is a large scale discrete optimization problem, or discrete planning, so it is nearly impossible to guarantee a global optimal solution. In this situation, however, human intelligence and feelings can quickly point out where could be possibly further improved. Actually, our Probe 3 visited one more asteroid which is excluded by the algorithm but picked out manually. We did this because we found much propellant was left by Probe 3 and its final transfer trajectory cost very long time but little fuel, which indicates it was far from optimal.

Due to the limited time, there are some unfinished works, but very promising to give even better solution:

1. The low-thrust solver utilized here assumes constant force 0.3 N and fixed thrust directions at starting asteroid and arriving asteroid. It allows to perform a Bang-Bang control or an All-Along control (with one mid-way direction switch). However, there are no time-optimization or fuel-optimization. So the solution here can definitely be improved with more a powerful solver.
2. Probes start their tours no too long after released from the mother ship, although it could wait for as most as about 5 years. This is a determinate result of our strategy and algorithm. We believe this can be improved by an evolutionary optimization. However, it requires a fast probe tour solver, which we do not have.

At the end, we would like to express our appreciation for such a challenging problem. It greatly motivates us to do a lot of works which we had never believed can be done within four weeks. After all, we really enjoy it and we would like to show our great thanks to all organizers.

Summary

Launch data, MJD	60225
Launch v_inf, km/s	5.930981297637148
Dates of probe 1 release, MJD	60711.2787435682
Dates of probe 2 release, MJD	60711.2787435682
Dates of probe 3 release, MJD	60711.2787435682
Flight time of probe 1, Year	5.999350308027395
Flight time of probe 2, Year	5.893793213255322
Flight time of probe 3, Year	5.990947260961535
Dates of probe 1 final rendezvous with the mother ship, MJD	62902.5414435752
Dates of probe 2 final rendezvous with the mother ship, MJD	62863.9867147097
Dates of probe 3 final rendezvous with the mother ship, MJD	62899.4722306344
Numbers of asteroid visited by probe 1	5
Numbers of asteroid visited by probe 2	5
Numbers of asteroid visited by probe 3	8
Probe 1 final mass, kg	894.180078919633
Probe 2 final mass, kg	822.244043346177
Probe 3 final mass, kg	851.044767832048
mother ship final mass, kg	7265.307021683932
Primary performance indexes	18
Secondary performance indexes, kg	2567.468890097858

Mother Ship Tour:

Earth --> 11287 --> 2735

Probe 1 Tour (the brackets means it stays but less than 30 days):

11287 --> 3094 --> 976 --> 2774 --> 6761 --> 12834 --> (2735) --> Mother Ship

Probe 2 Tour:

11287 --> 9991 --> 11541 --> 12821 --> 11326 --> 7715 --> (2735) --> Mother Ship

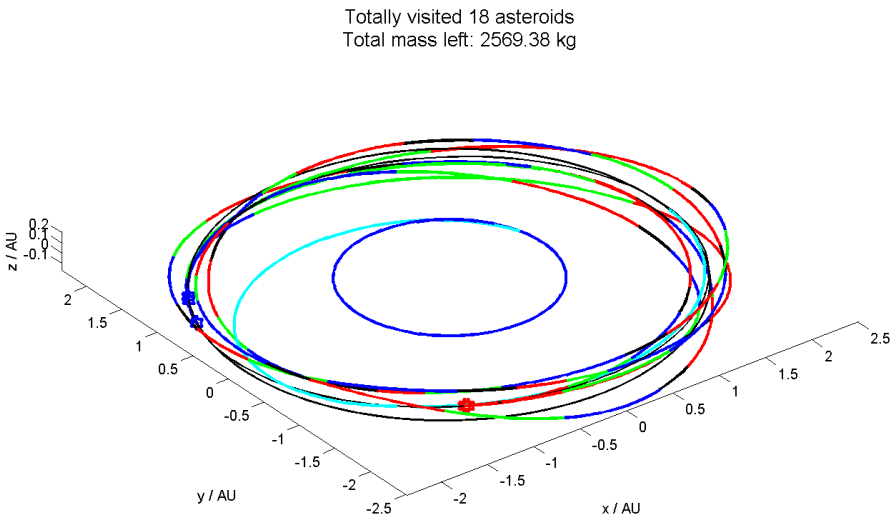
Probe 3 Tour:

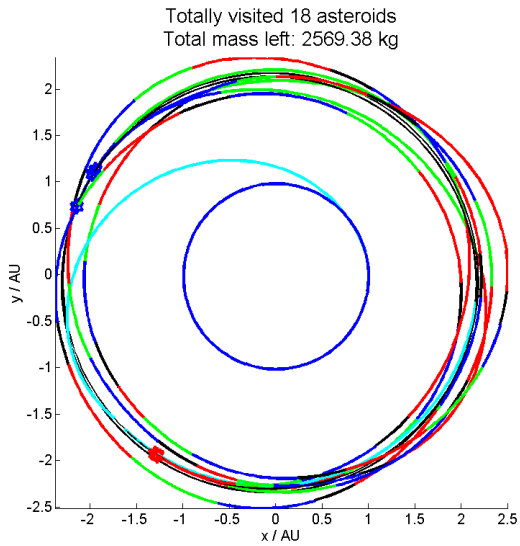
11287 --> 3526 --> 7617 --> 3086 --> 14489 --> 10234 --> 6745 --> 2735 --> Mother Ship

Asteroids gallery:

976, 2735, 2774, 3086, 3094,
3526, 6745, 6761, 7617, 7715,
9991, 10234, 11541, 11326, 11287,
12821, 12834, 14489

Visual representation





The above figure displays the tours of all three probes in two viewing angles.

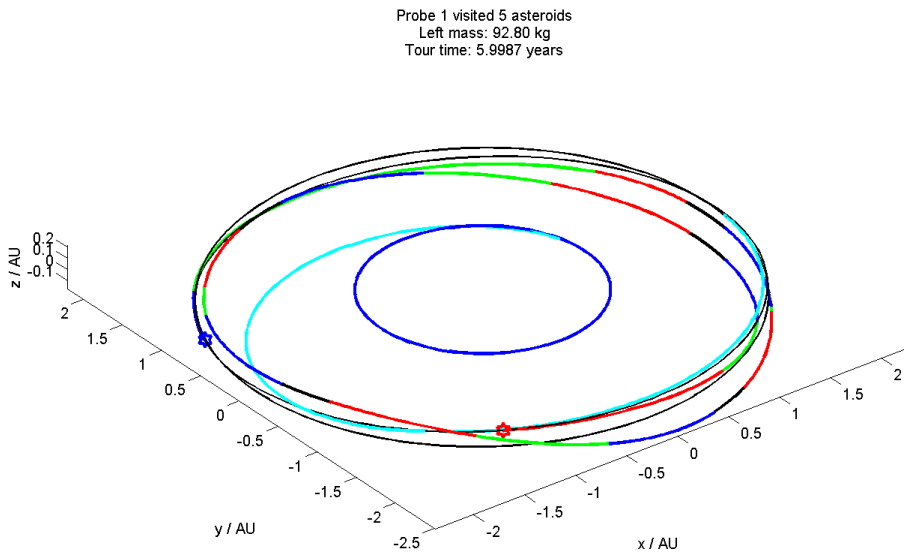
The **blue** circle in the center is the orbit of the earth.

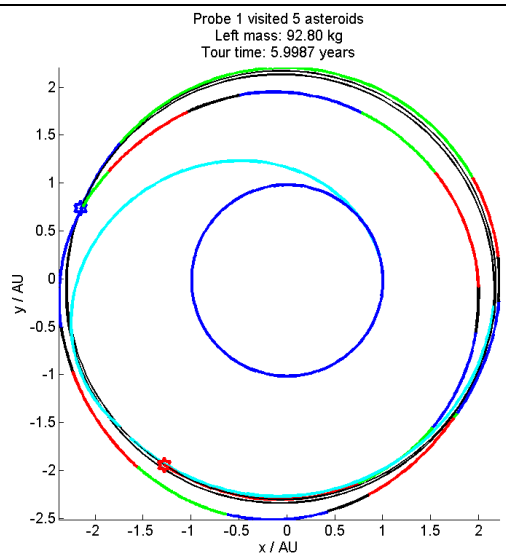
The two cyan long arcs are the two transfer of the mother ship: from the earth to an asteroid; then to a new asteroid.
The **black** arcs represents the probe and the mother ship is staying on an asteroid.
Each inter-asteroid low-thrust phase consists of three segments corresponding to a Bang-Bang control. They are colored by Red, Green, Blue as their orders, so that their order can be easily distinguished (simply remember **RGB** color):

- The **red** arcs are the departing segments with low-thrust along a constant direction.
- The **green** arcs are with no thrust
- The **blue** arcs represent the arriving trajectory with low-thrust along another constant direction.
- The **red** dots are starting positions of probes (all from asteroid 11287).
- The **blue** red dots are final position of probes (all on asteroid 2735).

Two black circles are the two asteroids the mother ship rendezvoused with. The graphic symbols are all the same below. On the right are the states and mass variation during the tours, where the rendezvous status of probes are not depicted for clarity.

Following figures are separate demonstration of each probes, all with the same notations.


Probe 1 visited 5 asteroids
Left mass: 92.80 kg
Tour time: 5.9987 years
z / AU
y / AU
x / AU


Probe 1 visited 5 asteroids
Left mass: 92.80 kg
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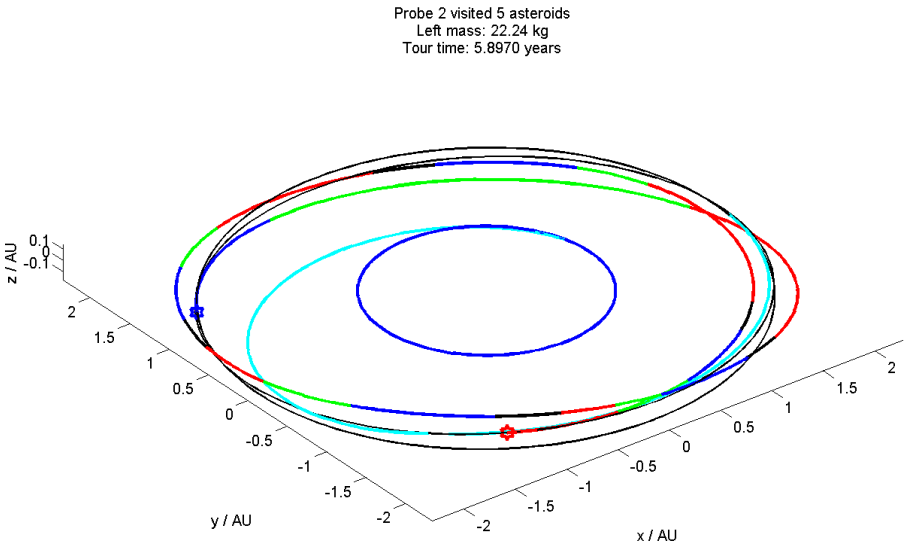
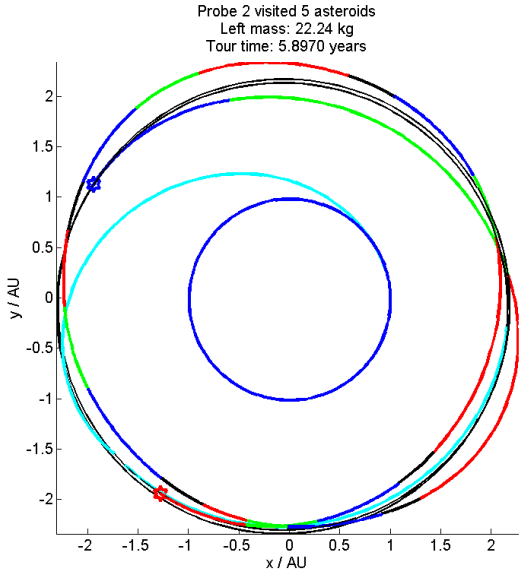
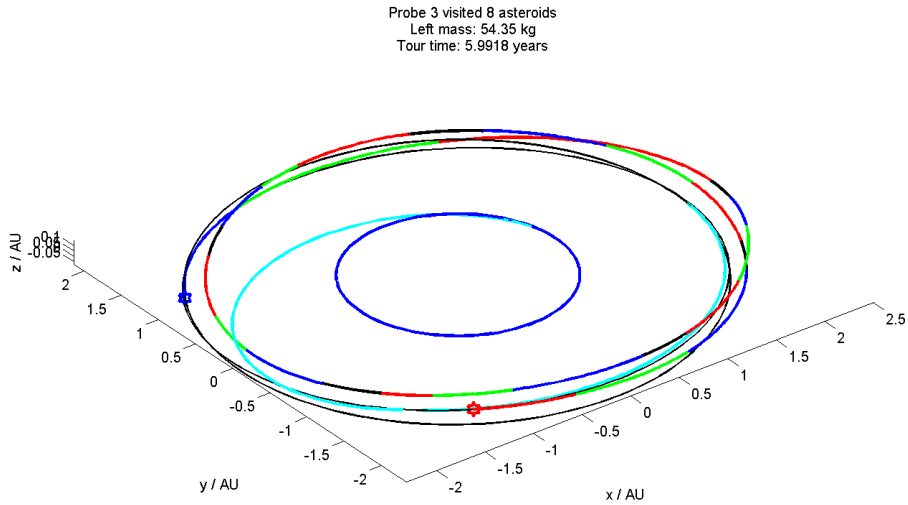
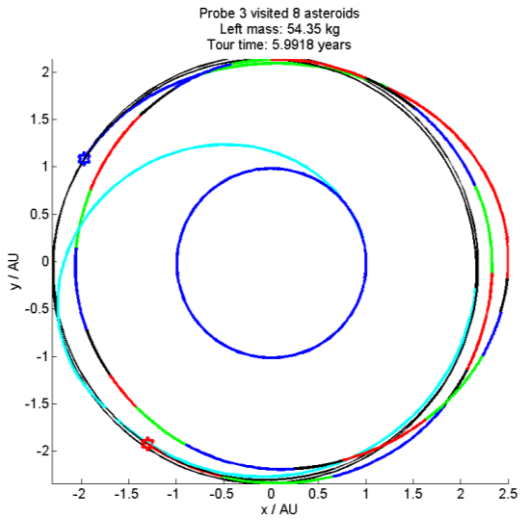
y / AU
x / AU

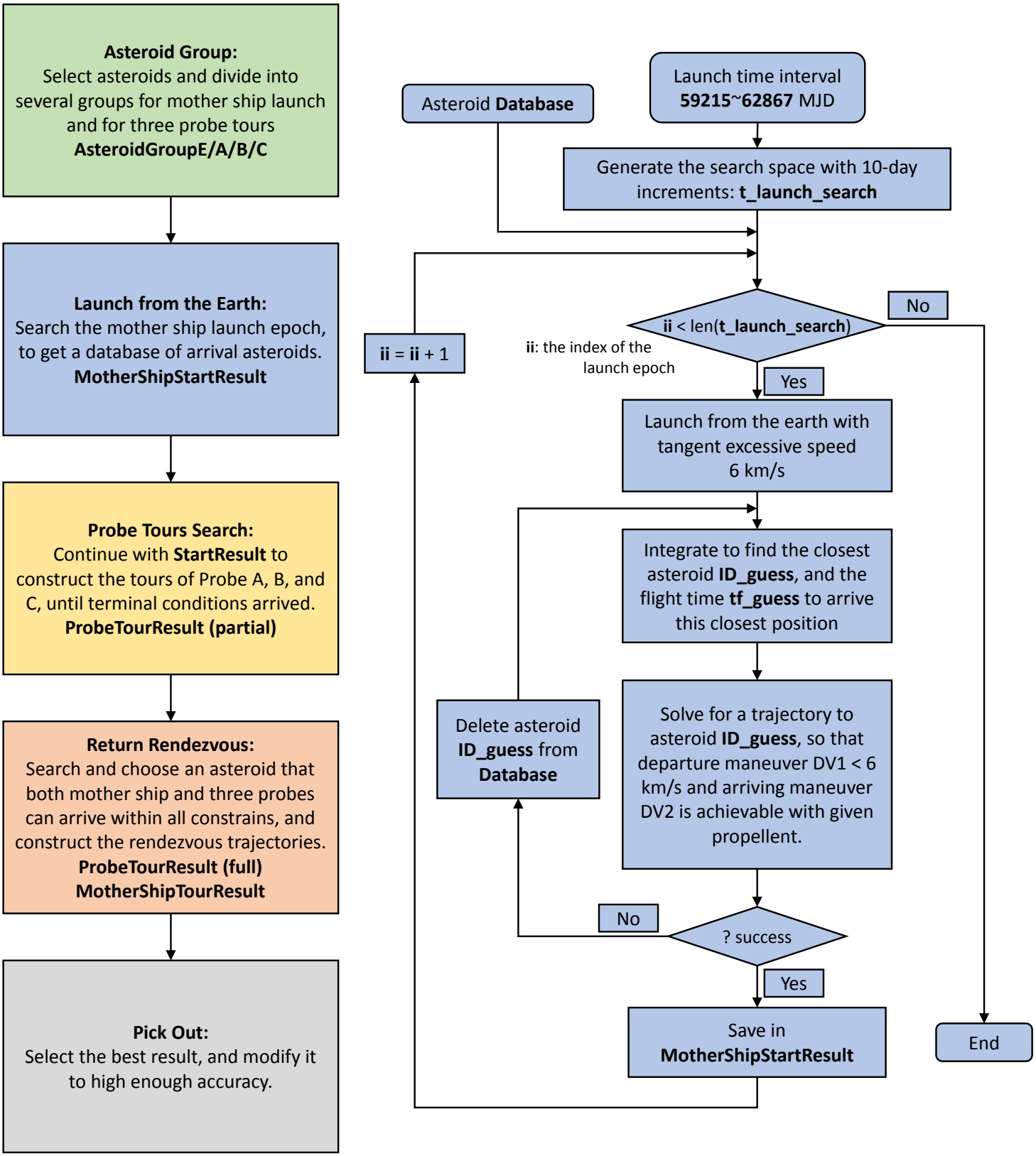

Probe 2 visited 5 asteroids
Left mass: 22.24 kg
Tour time: 5.8970 years
z / AU
y / AU
x / AU

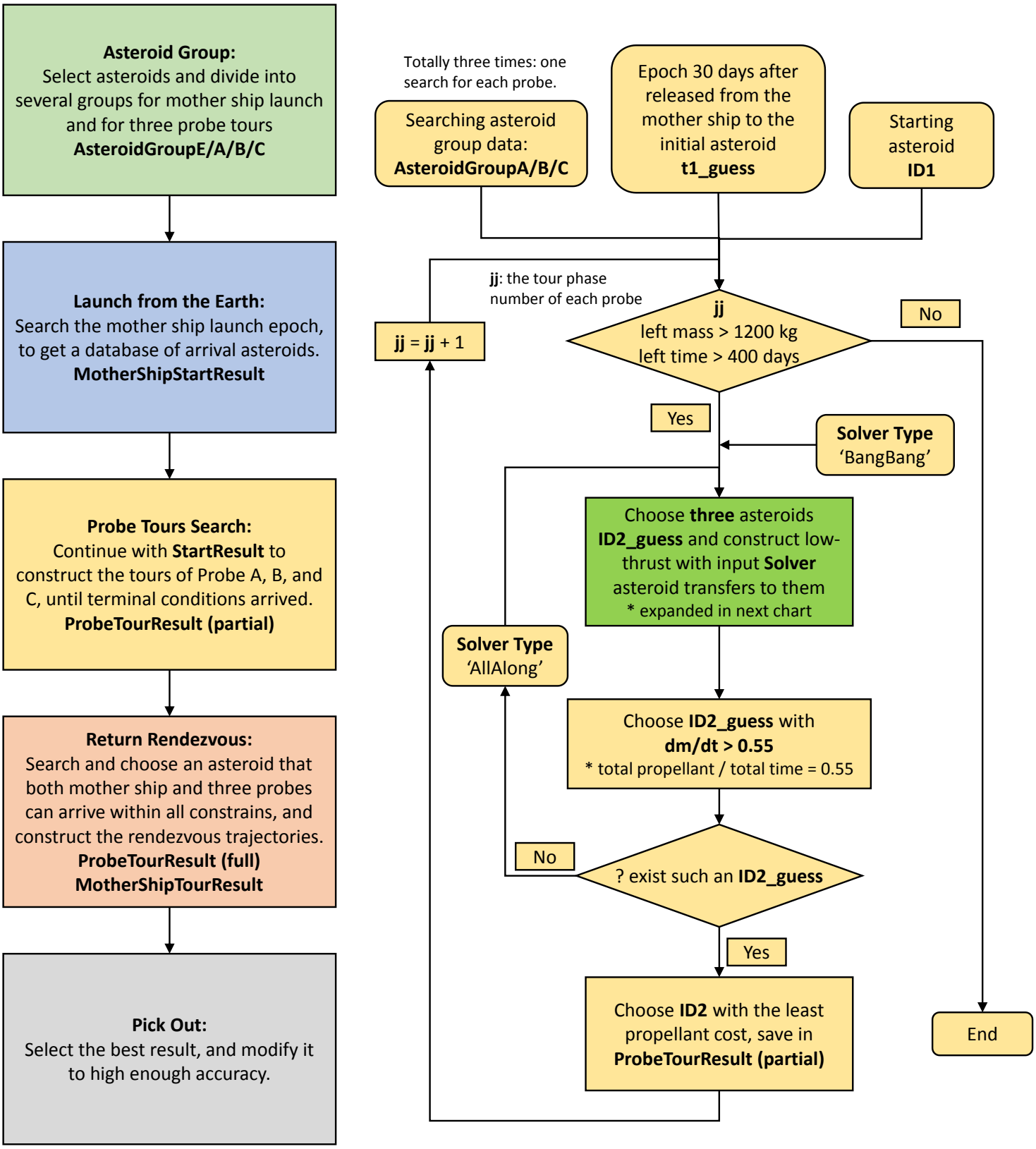

Probe 2 visited 5 asteroids
Left mass: 22.24 kg
Tour time: 5.8970 years
y / AU
x / AU


Probe 3 visited 8 asteroids
Left mass: 54.35 kg
Tour time: 5.9918 years
z / AU
y / AU
x / AU


Probe 3 visited 8 asteroids
Left mass: 54.35 kg
Tour time: 5.9918 years
y / AU
x / AU

Flow Chart of all the algorithm




Asteroid Group:
Select asteroids and divide into several groups for mother ship launch and for three probe tours
AsteroidGroupE/A/B/C
Launch from the Earth:
Search the mother ship launch epoch, to get a database of arrival asteroids.
MotherShipStartResult
Probe Tours Search:
Continue with StartResult to construct the tours of Probe A, B, and C, until terminal conditions arrived.
ProbeTourResult (partial)
Return Rendezvous:
Search and choose an asteroid that both mother ship and three probes can arrive within all constrains, and construct the rendezvous trajectories.
ProbeTourResult (full)
MotherShipTourResult
Pick Out:
Select the best result, and modify it to high enough accuracy.
Totally three times: one search for each probe.
Searching asteroid group data:
AsteroidGroupA/B/C
Epoch 30 days after released from the mother ship to the initial asteroid
t1_guess
Starting asteroid
ID1
jj: the tour phase number of each probe
jj = jj + 1
jj
left mass > 1200 kg
left time > 400 days
No
Yes
Solver Type
'BangBang'
Choose three asteroids ID2_guess and construct low-thrust with input Solver asteroid transfers to them
* expanded in next chart
Solver Type
'AllAlong'
Choose ID2_guess with
dm/dt > 0.55
* total propellant / total time = 0.55
No
? exist such an ID2_guess
Yes
Choose ID2 with the least propellant cost, save in
ProbeTourResult (partial)
End

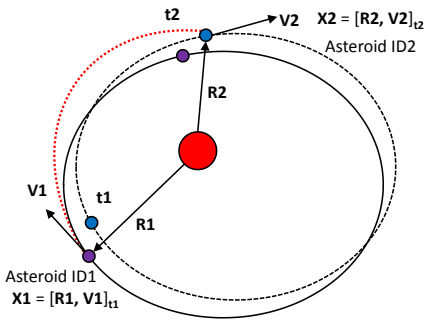
Asteroid Group:
Select asteroids and divide into several groups for mother ship launch and for three probe tours
AsteroidGroupE/A/B/C

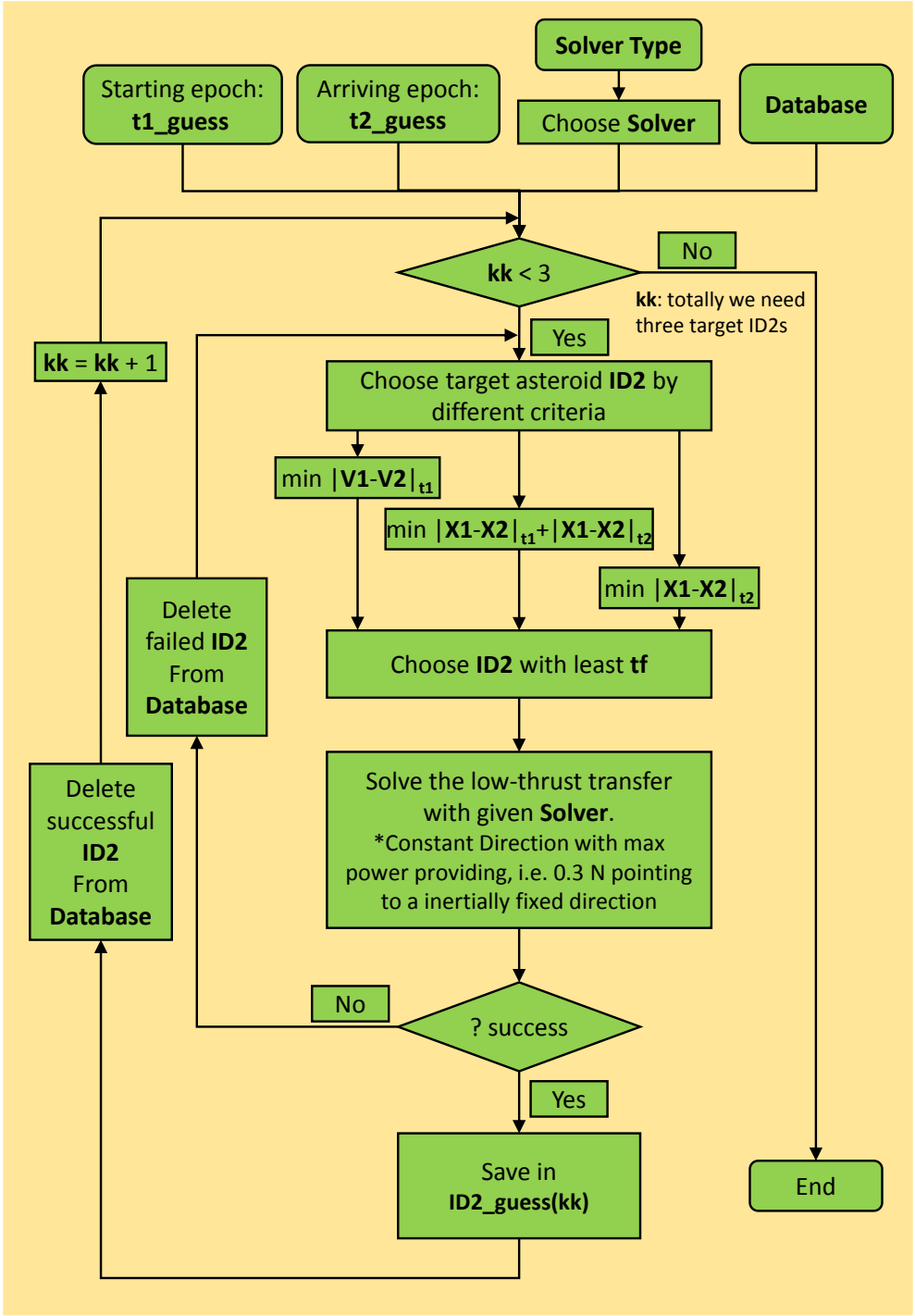
Launch from the Earth:
Search the mother ship launch epoch, to get a database of arrival asteroids.
MotherShipStartResult

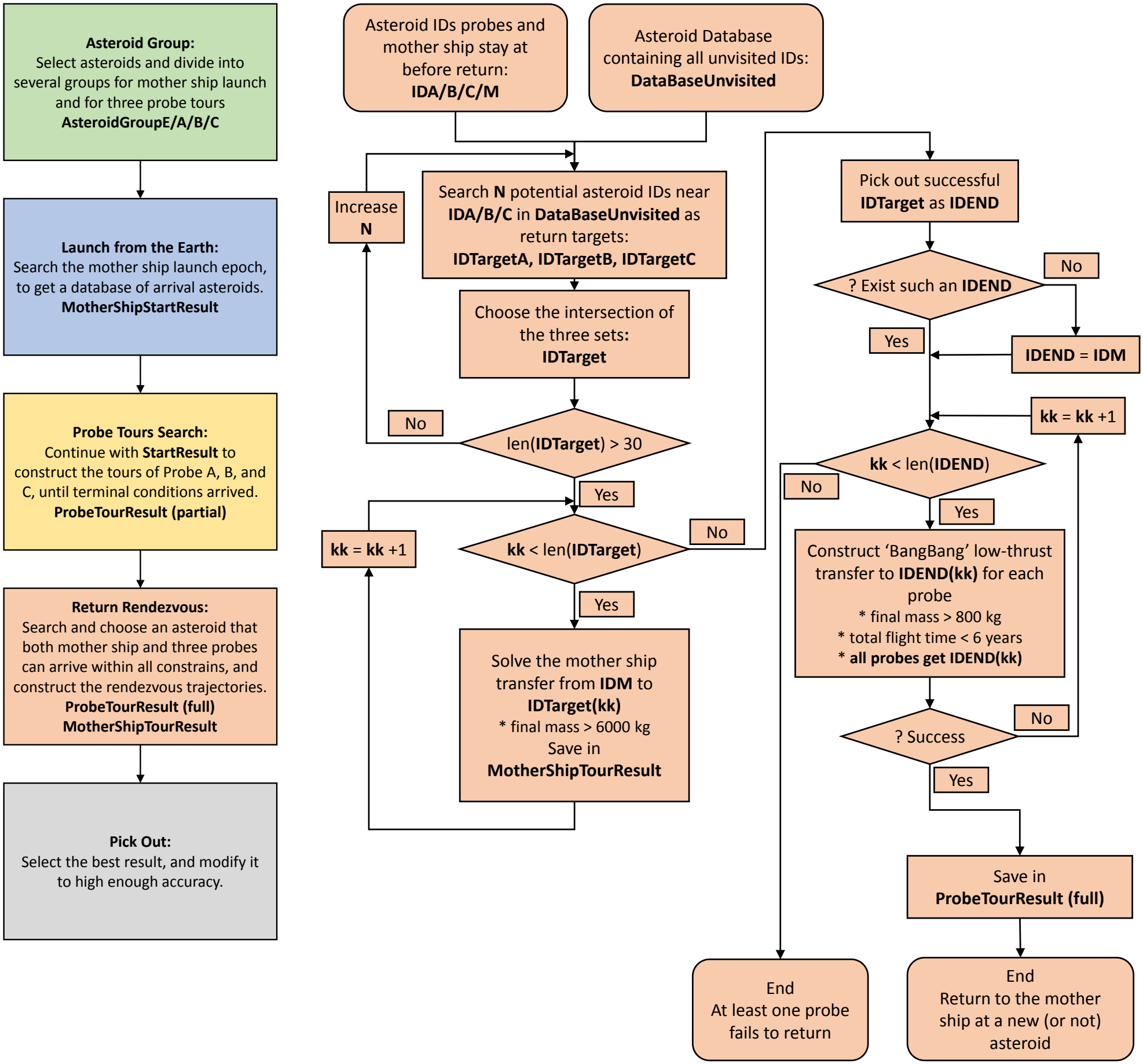
Probe Tours Search:
Continue with **StartResult** to construct the tours of Probe A, B, and C, until terminal conditions arrived.
ProbeTourResult (partial)

Return Rendezvous:
Search and choose an asteroid that both mother ship and three probes can arrive within all constrains, and construct the rendezvous trajectories.
ProbeTourResult (full)
MotherShipTourResult

Pick Out:
Select the best result, and modify it to high enough accuracy.



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AsteroidGroupE/A/B/C
Launch from the Earth:
Search the mother ship launch epoch, to get a database of arrival asteroids.
MotherShipStartResult
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Continue with StartResult to construct the tours of Probe A, B, and C, until terminal conditions arrived.
ProbeTourResult (partial)
Return Rendezvous:
Search and choose an asteroid that both mother ship and three probes can arrive within all constrains, and construct the rendezvous trajectories.
ProbeTourResult (full)
MotherShipTourResult
Pick Out:
Select the best result, and modify it to high enough accuracy.
Asteroid IDs probes and mother ship stay at before return:
IDA/B/C/M
Asteroid Database containing all unvisited IDs:
DataBaseUnvisited
Increase N
Search N potential asteroid IDs near IDA/B/C in DataBaseUnvisited as return targets:
IDTargetA, IDTargetB, IDTargetC
Choose the intersection of the three sets:
IDTarget
No
len(IDTarget) > 30
Yes
kk = kk +1
kk < len(IDTarget)
No
Yes
Solve the mother ship transfer from IDM to IDTarget(kk)
* final mass > 6000 kg
Save in
MotherShipTourResult
Pick out successful IDTarget as IDEND
? Exist such an IDEND
No
Yes
IDEND = IDM
kk = kk +1
kk < len(IDEND)
No
Yes
Construct 'BangBang' low-thrust transfer to IDEND(kk) for each probe
* final mass > 800 kg
* total flight time < 6 years
* all probes get IDEND(kk)
? Success
No
Yes
Save in
ProbeTourResult (full)
End
At least one probe fails to return
End
Return to the mother ship at a new (or not) asteroid