GTOC9: Results and Methods of Team 13 (NPU)

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Abstract

This paper presents the methods proposed by team 13-NPU in the 9th Global Trajectory Optimization Competition (GTOC9). The debris removal sequence in single submission and the reasonable allocation of all debris in submissions are designed by branch and bound algorithm and image method. Then a three-impulse transfer method involving a deep space maneuver is used to achieve the transfer of spacecraft between any two debris. The best solution found has a performance index 878.998 MEUR with 13 submissions.

1 Introduction

In GTOC 9, the issue of debris pieces removal is investigated [1]. To avoid the threat to orbiting satellites, a set of 123 critical debris pieces in Sun-synchronous orbits need to be removed by a series of missions. During the transfers, spacecrafts are subject not only the central force of the Earth's gravity, but also perturbed by the main effects of an oblate Earth, i.e. J_2 perturbation. The position and velocity vectors of each orbiting debris pieces are computed by updating Keplerian elements using the mean motion, which $[\Omega, \omega]$ is perturbed by J_2 perturbation. The only manoeuver allowed to control the spacecrafts is impulse propulsion to change the velocity vector instantaneously. Due to the fuel restrictions, a single spacecraft cannot clean up all the distributed debris, so multiple missions are designed to remove all the debris.

The performance to be minimized is

$$J = \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \left[c_i + \alpha(m_{0i} - m_{dry})^2\right] \quad (1)$$

where C_i is the cost charged by the contracted launcher supplier for the i-th mission and it is composed of a base cost c_i and a term $\alpha(m_{0i} - m_{dry})^2$ favouring a lighter spacecraft, m_{0i} and m_{dry} denote the initial spacecraft mass and dry mass of the i-th mission. Each spacecraft initial mass m_{0i} is the sum of dry mass, propellant mass, and the weight of N de-orbit packages: $m_{0i} = m_{dry} + m_p + N m_{de}$, where $m_{dry} = 2000$ kg, $m_p \leq 5000$ kg, $m_{de} = 30$ kg. The base cost c_i of each mission depends on how early the submission submitted to the authorities, and it in-

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creases linearly during the competition, varies from 45 MEUR to 55 MEUR.

The process of launching spacecraft from the ground to the first debris of any submission is not considered, only the transfer trajectories from one debris to another debris are calculated. After spacecraft arrived target debris, there is a waiting time constraint of 5 days to release the debris removal device before leaving the debris to the next debris. The time between any two successive debris rendezvous within the same submission must not exceed 30 days, which means the flight time of single transfer trajectory varies from 0 to 25 days. It is forbidden to operate different submissions in parallel, a time of at least 30 days must be accounted for between any two submissions. To prevent spacecrafts from colliding with the earth, the orbital periapsis r_p cannot be smaller than 6600 km.

The spacecraft is perturbed by J_2 perturbation during transfers, and the dynamics equation in the geocentric inertial coordinate is described as

$$\ddot{x} = -\frac{\mu x}{r^3}[1 + \frac{3}{2}J_2\frac{r_{eq}^2}{r^2}(1 - 5\frac{z^2}{r^2})] \quad (2)$$

$$\ddot{y} = -\frac{\mu y}{r^3}[1 + \frac{3}{2}J_2\frac{r_{eq}^2}{r^2}(1 - 5\frac{z^2}{r^2})] \quad (3)$$

$$\ddot{z} = -\frac{\mu z}{r^3}[1 + \frac{3}{2}J_2\frac{r_{eq}^2}{r^2}(3 - 5\frac{z^2}{r^2})] \quad (4)$$

where r_{eq} denoted the Earth average radius.

The position and velocity vectors of debris are computed by the classical orbital elements $[a, e, i, \Omega, \omega, M]$. The elements osculate due to the J_2 perturbation, and only the long-term influence of the ascending node Ω, the argument of perigee ω and the mean anomaly M are computed as follows

$$\dot{\Omega} = -\frac{3}{2}J_2\frac{r_{eq}^2}{p^2}\sqrt{\frac{\mu}{a^3}}\cos i \quad (5)$$

$$\dot{\omega} = \frac{3}{4}J_2\frac{r_{eq}^2}{p^2}\sqrt{\frac{\mu}{a^3}}(5\cos^2 i - 1) \quad (6)$$

$$\dot{M} = \sqrt{\frac{\mu}{a^3}} \quad (7)$$

where $p = a(1-e^2)$ denotes the semilatus rectum.

The only allowed manoeuver is instantaneous changes of the spacecraft velocity vector. The fuel consumption of the manoeuver ΔV is calculated using Tsiolkovsky equation

$$m_f = m_i \exp(-\frac{\Delta V}{I_{sp}g_0}) \quad (8)$$

where I_{sp} is the specific impulse, g_0 is the standard acceleration of gravity at sea level, m_i is the mass before the manoeuver and m_f is the mass after the manoeuver.

2 Methods

There are two main difficulties in solving the competition: how to allocate the debris to different submissions rationally to minimize the number of submissions, and for each submission, how to optimize the debris removal sequence to minimize the total manoeuver magnitude, so as to reduce the initial spacecraft mass.

Before the optimization of debris removal sequence, it is necessary to select a performance index about manoeuver of transfers between any two debris which can be calculated quickly. The transfer of the spacecraft between any two debris is considered as a lambert problem, and the double-pulse lambert transfer is the most common method. After the initial position and final position fixed, and the transfer time determined, the velocity increment required for the transfer can be calculated by lambert method. However, the transfer time is free before the debris removal sequence determined, and it takes times to optimize the transfer time to get a suitable transfer trajectory, so the lambert method is not suitable to measure the velocity increment when determining the debris removal sequence.

According to the knowledge of orbital mechanics, it takes much larger ΔV to change the spacecraft orbital plane than change the shape in the plane, so it is reasonable to make the debris with close orbital plane in the same submission. In the six classical orbital elements, the semi-major axis a and the eccentricity e describe the shape in orbital plane, the inclination i and the ascending node Ω describe the orbital plane, the argument of perigee ω and the mean anomaly M describe the spacecraft position in the orbit. All of the given 123 debris are located in Sun-synchronous with the constant inclination $i \in [1.6976, 1.7640]$ rad and the osculating ascending node $\Omega \in [0, 2\pi]$. Due to the J_2 perturbation, the average oscillation period of Ω is 364.7326 days, and the Ω changes about 8 cycles in 8 years mission time, so different rendezvous time have great influence on the angle between two debris orbital planes. According to the spherical trigonometry, the angle θ between any two orbital plane can be calculated as

$$\cos\theta = \cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos(\Omega_2 - \Omega_1) \tag{9}$$

But in the competition, we use a simpler formula as

$$\Delta\alpha = |i_2 - i_1| + |\Omega_2 - \Omega_1| \tag{10}$$

The ΔV required to accomplish the transfer between the two debris is assumed to be positively correlated to $\Delta\alpha$. The performance when determine the debris removal sequence is to minimize $\sum\limits_{j=1}^{m} \Delta\alpha_j$.

First, the debris removal sequence of each submission is designed by branch and bound algorithm. In order to minimize the number of submissions, a single submission should remove debris as many as possible. The transfer time between two debris varies 0$\sim$25 days, and the $\Delta\Omega$ also varies with time. Therefore both the order of debris and the time of debris rendezvous should be optimized in the sequence. The minimum $\Delta\Omega$ within allowable transfer time range between any two debris is computed by the different rate $\dot{\Omega}$, then the rendezvous time achieved. Two alternative debris are select as the next target debris, and up to 2^{13} sequence are selected in single mission, i.e. in the branch and bound algorithm, each branch has two branches, and the number of reserved sequence is up to 2^{13}, then the rest branches are deleted. It should be noted that in all submissions, only a few of the submissions are able to clean up more than 13 debris, and the debris numbers of others submissions are less than 13, then the sequence numbers will be reduced. The branch and bound algorithm is shown in Fig. 1, where each node in the sequence represents a alternate debris and the rendezvous time.

The branch and bound algorithm can only optimize the debris sequence in single submission, but cannot be used to achieve the solution of minimum number of submissions. For the global optimization problem, the approach we adopted is modifying the starting and ending time of each submissions and the associated debris to obtain the solution with minimum submission number. Since the $\Delta\Omega$ between debris varies greatly over time, the intersections of the element Ω of any two debris are found and plotted as Fig. 2, where the x-axis is

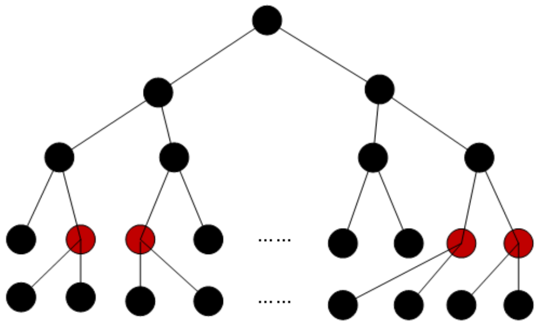

Figure 1: The branch and bound algorithm

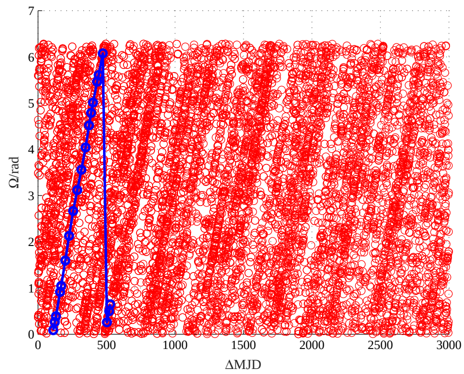

Figure 2: The intersections of the element Ω of any two debris

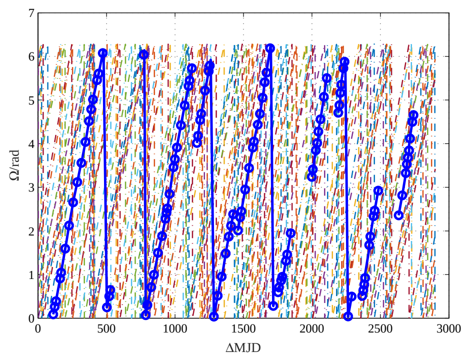

Figure 3: The cure of remaining debris Ω

the time from the 23467 MJD2000, the y-axis is the value of Ω, every red circle represents a moment that two debris have the same Ω. As showed in Fig. 2, the distribution of the intersection of Ω is asymmetrical, the time interval when many debris planes close to each other should be selected to be submission time, i.e. the section where the densest area of intersections should be selected. In Fig. 2, the intersections near the blue line is intensive, a sequence of 20 debris is selected along the blue line, and every blue circle represents a debris in the sequence and the rendezvous time. After several submissions determined, the intersections of the remaining debris are scattered and sparse, and the figure of the intersection becomes useless to select the sequences. Then the curve of remaining debris Ω over time is plotted to design the remaining submissions, as showed in Fig. 3.

Then the transfer trajectory is designed for spacecraft to leave last debris and rendezvous next debris. At the rendezvous moment, the phase angle of spacecraft should be the same as the target debris. Only the i and Ω constraints about orbital plane are accounted, and the ω and M constraints about phase angle in the orbit are neglected. If two-impulse transfer is used directly to accomplish the rendezvous at the rendezvous time in the sequence, a remarkable ΔV is needed to eliminate the effect of phase difference between spacecraft and target debris. The interval of two rendezvous time in sequence varies 5∼30 days. Once de-orbit package delivered, a small impulse is used to leave the debris and change the semi-major axis , thus the rate of the M in Eq.(1.5) is also changed. After many revolutions, spacecraft reaches the next debris at the rendezvous moment, then two-impulse lambert is used to accomplish the rendezvous.

The two-impulse lambert transfer is computed without J_2 perturbation, and the correction is required to take J_2 perturbation into account. Besides, the first impulse applied to adjust the phase angle may not take spacecraft reach the optimal position at the rendezvous time, so the first impulse is also modified. In the case of initial states and final states fixed, there are 7 optimization variables, including the first impulse $[\Delta v_{x1}, \Delta v_{y1}, \Delta v_{z1}]$, the second impulse $[\Delta v_{x2}, \Delta v_{y2}, \Delta v_{z2}]$ and the time for second impulse t_2, the terminal constraints are the same position of spacecraft with next debris, and the minimum performance index is the sum of three velocity increments: $\sum_{i=1}^{3} \Delta V_i$. The function fmincon in MATLAB is used for optimization. Take the transfer from debris 48 to debris 7 as example, three impulses are required: the first impulse to leave debris 48 and adjust the phase angle, the second and third impulse to accomplish the rendezvous. At the rendezvous time, the difference of phase angle between debris 48 and debris 7 is 1.1519 rad. By a small impulse of 6.3420 m/s leaving debris 48 after de-orbit package delivered, the spacecraft arrives at a location close to debris7 with similar phase angle, then the second and third impulse are used to accomplish the rendezvous. Figure 4. shows the transfer trajectory, the transfer time is 3.5067 days, and the total ΔV is 110.3 m/s.

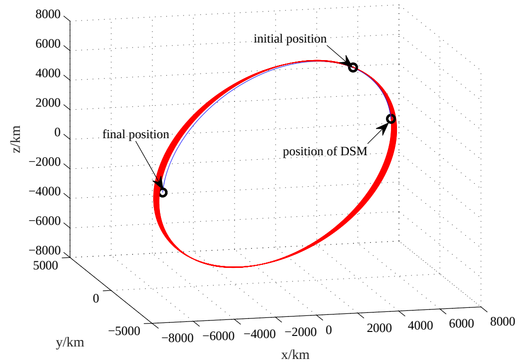

Figure 4: The transfer trajectory from debris 48 to debris 7

3 Results

By the methods introduced above, we got a feasible solution with 13 submissions. The performance index of the final result is $J = 878.998$ MEUR. Table 1 shows the detailed information of the result, including the removal debris number, ΔV, and the initial spacecraft mass of each submission. Figure 5 shows the Ω curves of spacecraft in the mission, which contains 13 curves, and each circle represents a debris removed in the moment.

Table 1: The final result with 13 submissions

Submissions	debris number	ΔV/(m/s)	m_{0i}/kg
1	20	2738	5468
2	4	2710	4708
3	19	3218	6254
4	14	3067	5718
5	14	2620	5009
6	7	2405	4456
7	9	3002	5396
8	3	733	2591
9	8	3156	5627
10	8	2300	4339
11	2	447	2353
12	9	3214	5701
13	6	2023	3911

4 Conclusions

In this paper, the methods that Team 13-NPU used for GTOC9 are introduced. The difference between any two debris orbital planes is used to approximate the impulsive velocity change, then the debris removal sequence of single submission is selected by branch and

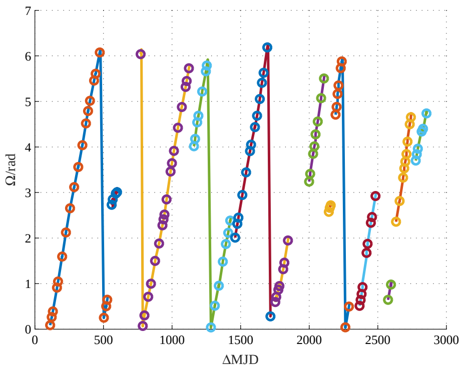

Figure 5: The Ω curve of spacecraft in the mission

bound algorithm, and the figures of intersections of Ω and the Ω curve over time are used to modify the starting and ending time of each submissions to make minimum number of submissions. A transfer method with 3 impulsive manoeuver is adopted to make spacecraft accomplish transfer between any two debris in the sequence: the first impulse to modify spacecraft phase angle, the second and third are lambert transfer. Then the 3-impulse transfers under J_2 perturbation are modified by the fmincon function in MATLAB. We ultimately got a solution with 13 submissions and the performance is 878.998MEUR.

References

[1] Dario Izzo. *Problem description for the 9th Global Trajectory Optimisation Competition*, May 2017.