GTOC 9, Multiple Space Debris Rendezvous Trajectory Design in the J2 environment

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Outline

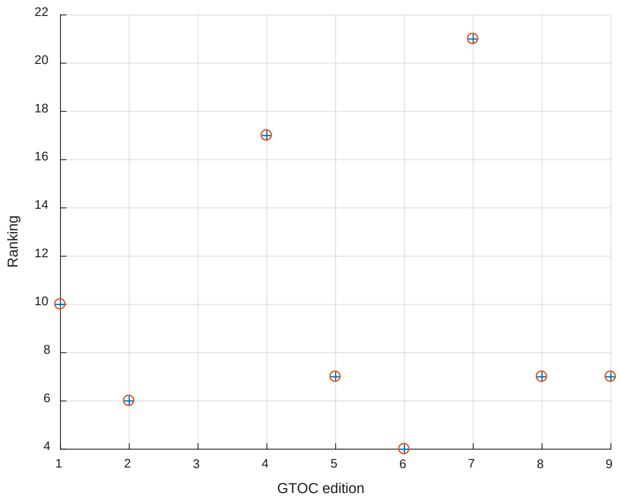
- DLRs history in GTOC
- Recap of the Problem Statement
- Overall strategy
- Combinatorial Problem
- Transfer Problem
- Results
- Conclusions

DLRs history in GTOC

- DLR has been part of GTOC from the beginning with an exception for the third GTOC

- The main motivation for us is to benchmark our own tools and code

- Excellent platform for networking in the space optimization domain (also with people from other domains)



Problem Statement

- Multiple Space Debris Rendezvous
- Scenario with n missions to collect a set of 123 debris pieces, distributed in the SSO region
- Cost function:
    - Base Cost per launch (increasing during competition time) plus used propellant mass square
- Launch Injection Parameters do not impact the cost function
- Five impulsive manoeuvres are allowed from debris2debris transfer
- Several constraints (max transfer time, min pericenter, mission time window)

$$J = \sum_{i=1}^{n} c_i + \alpha(m_{0_i} - m_{dry})^2$$

$$\dot{\Omega} = -\frac{3}{2} J_2 \left(\frac{r_{eq}}{p}\right)^2 n \cos i$$

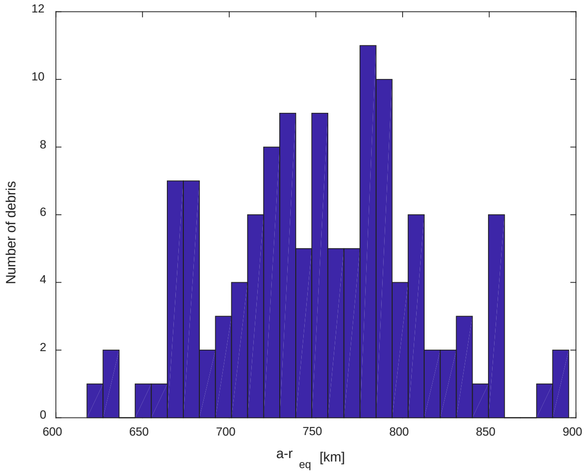
$$\dot{\omega} = \frac{3}{4} J_2 \left(\frac{r_{eq}}{p}\right)^2 n \left(5 \cos^2 i - 1\right)$$

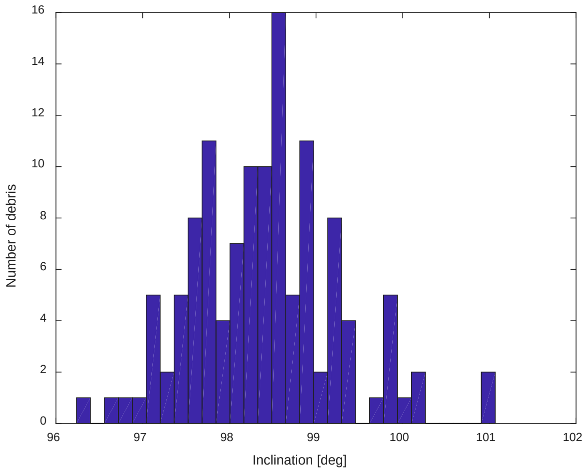
Overall Strategy

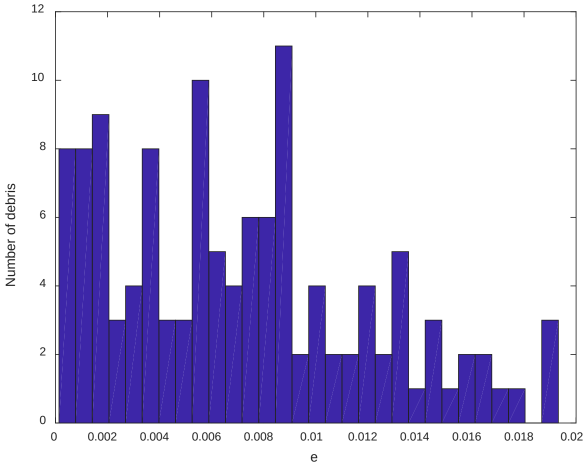
- The problem can be classified as a Time Dependent Traveling Salesman Problem (TDTSP)
- Nested Optimal Control Problem for the Transfer
- Combinatorial Permutation Space is huge
- On the Base Cost we don’t have any influence, besides working fast
- A fast transfer cost evaluation is needed for the combinatorial part

Overall Strategy

- How are the debris pieces spread out regarding eccentricity e, inclination i and semi major axis a ?







=> Nearly Circular, inclination ranges from 96° to 101° and the orbit height ranges from 600 km to 900 km

Overall Strategy

- If we only consider a change in inclination i and semi major axis a the problem can be treated as a TSP
- The cost in that case is the sum of the inclination change ΔV_{inc} plus the Hohmann transfer ΔV_{sma}

$$\Delta V_{inc} = 2V \sin((i_A - i_B) / 2)$$

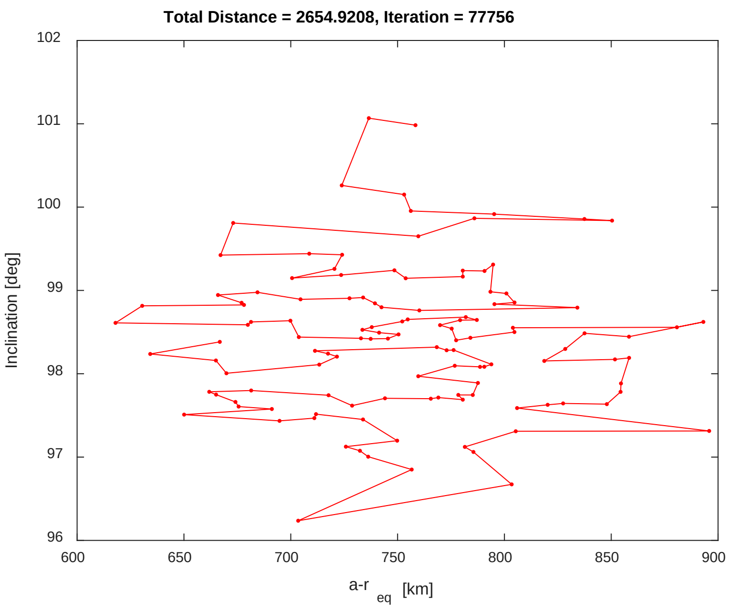
$$\Delta V_{sma} = \sqrt{\mu / r_1}(\sqrt{2k / (1+k)} - 1) + \sqrt{\mu / (r_1 k)}(1 - \sqrt{2 / (1+k)})$$

- Where *k* is the ratio between the two semi major axis a_A and a_B (assuming circular orbits)

$$\Delta V_{AB} = \Delta V_{inc} + \Delta V_{sma}$$

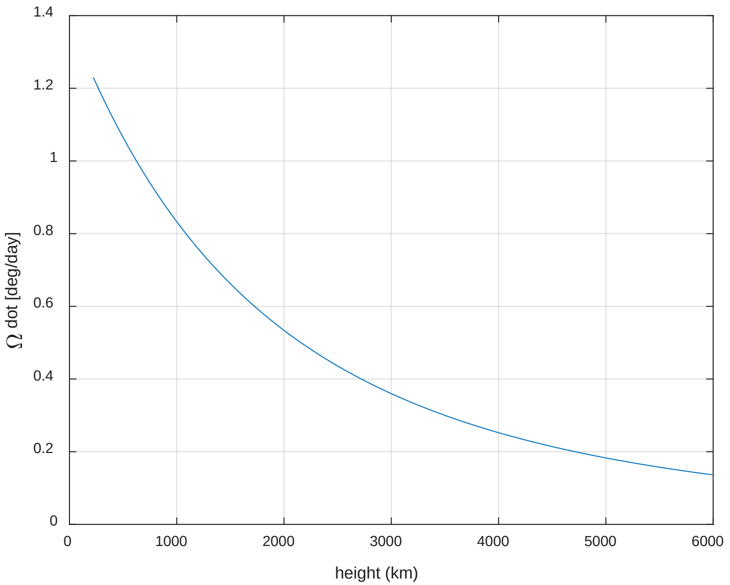
Overall Strategy, inc sma TSP

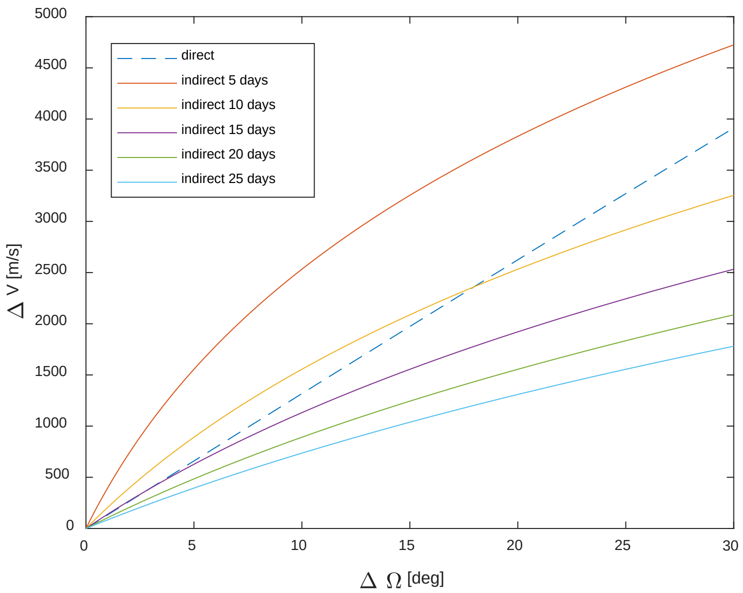
- With the above equations we can set up a cost matrix 123x123, which gives the distance in terms of velocity from debris A to debris B
- We used a genetic optimizer in *matlab* to find the shortest path between all 123 debris
- Population was set to 500 and 10^5 iterations were performed
- The total distance we get is around 2654m/s, so an average ΔV of 21.7m/s for one transfer



Overall Strategy, Change in Right Ascension

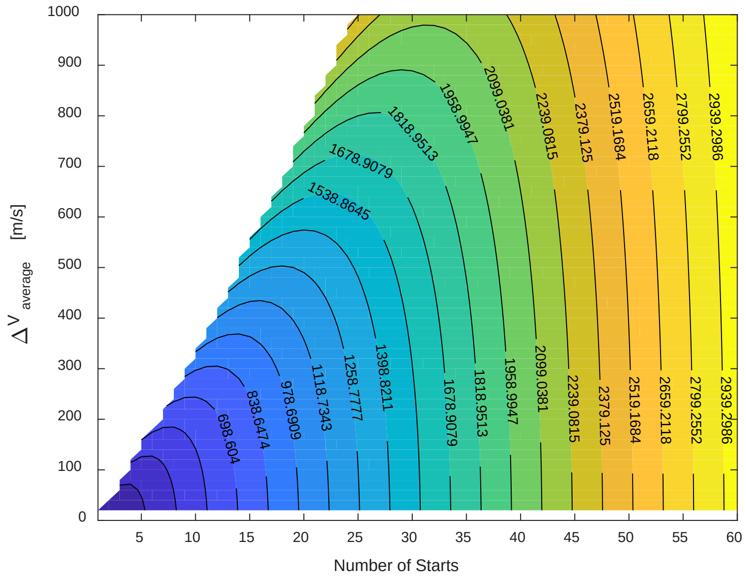
- The change of the Node Ω can be done in two ways:
  - A direct plane change
  - An indirect plane change via a change in semi major axis





Overall Strategy, How many missions?

- An important question to answer is: How many missions we need?
- Do we stack the launcher as full as possible?
- It can be seen that we should not use the maximum propulsion available
- If we analyze the J value for an average ΔV of 300m/sec:
    - 9 missions: 904.1MEUR
    - 12 missions: 827.6 MEUR

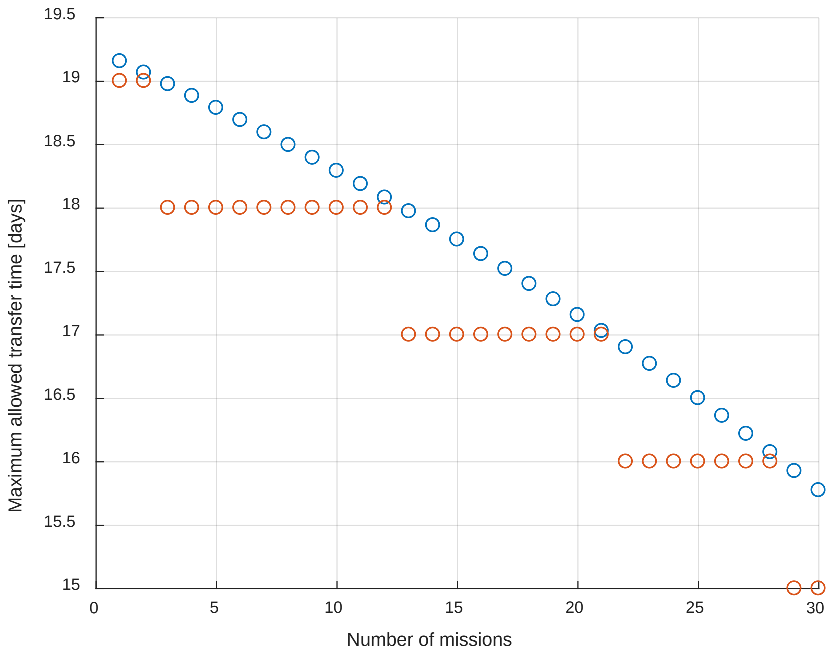


- So we have to reduce the total allowed ΔV per mission by 10% to 20% depanding on how many missions we may need

Combinatorial Problem

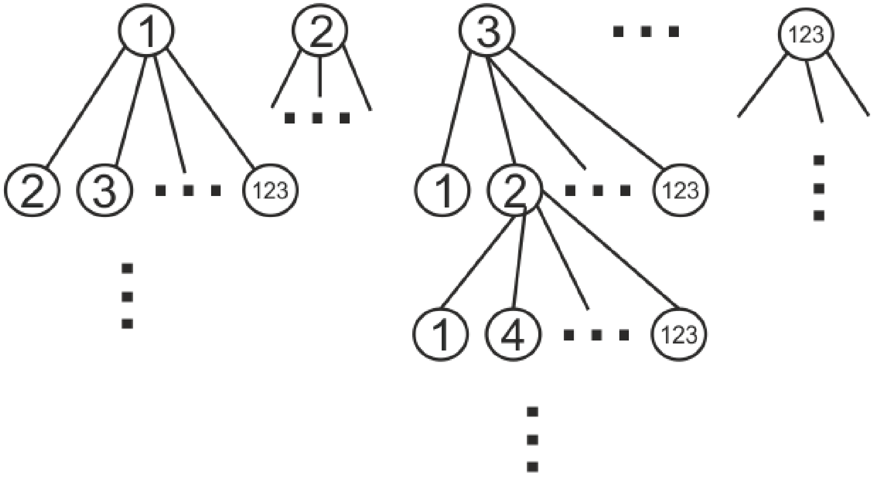
- If we use the allowed 25 days for the transfer we ending up with a total scenario time larger than the allowed 8 years

- In fact we can only use average transfer times between 15 and 19 days



Combinatorial Problem

- We used a graph approach which uses a certain beam width
- For the first mission we have 123 nodes or debris as an option to start from
- For each node the remaining transfers are calculated
- The Transfer Cost was precalculated and stored in a m-file for discret transfer times (14 days … 25 days)



Combinatorial Problem

- A pure Greedy search(depth first along the lowest ΔV) brought results in the range of $J = 2500\ MEUR$
- A full Breadth Search is not possible cause the permutations are to many
- Beam search is like a mixture between Greedy and Breadth search, instead of only looking on the best route, we take up to 50000 routes with us down the tree
- Beam search with a Beam Width between 50000 and 20000 delivered our committed solution

Combinatorial Problem

- After some levels in the graph we would get equal solutions in the tree
- e.g. the sequence 34-12-110-5 is equal to sequence 34-110-12-5
- We introduced a filter to only allow unique solutions to be passed to the next level
- That part is crucial, cause the goal is to get rid of the greedy effect (bad cookies stay over) and we can only improve if we have permutations

Combinatorial Problem

- The tree saves the beam after each level (or number of transfers performed)

- With that files we can enter the next mission

- The issue is when we use the file with a high number of transfers we have several drawbacks:
  - Fuel consumption is high, which may not result in an optimal J-value
  - The beamwidth gets low for the next mission, cause we only have a small beamwidth at that lvl

beamWidth=20000 NumTransfers=10
best_dv =1.7574km/s
beamWidth=20000 NumTransfers=11
best_dv =2.4213km/s
beamWidth=13850 NumTransfers=12
best_dv =2.695km/s
beamWidth=5114 NumTransfers=13
best_dv =2.9421km/s
beamWidth=753 NumTransfers=14
best_dv =4.1597km/s
beamWidth=37 NumTransfers=15
best_dv =4.625km/s
beamWidth=6 NumTransfers=16
best_dv =4.7616km/s
beamWidth=2 NumTransfers=17
best_dv =4.8884km/s
beamWidth=0 NumTransfers=18

Combinatorial Problem

MissionID	NumTransfer	Deb	TT	validBeams	NextBeamWidth		NodesLeft
1	20	21	17	42	4284		102
2	15	37	17	37	3182		86
3	13	51	17	207	14904		72
4	14	66	17	219	12483		57
5	12	79	17	1354	59576		44
6	6	86	17	1031	38147		37
7	7	94	20	321	9309		29
8	6	101	20	415	9130		22
9	5	107	20	1079	17264		16
10	4	112	20	230	2530		11
11	3	116	20	141	987		7
12	2	119	21	52	208		4
13	1	121	23	10	20		2
14	1	123	23				0

Transfer Part

- The debris to debris final transfer was implemented in matlab
- The sequence was re-optimized with respect to the transfer times, cause in the tree that was a fixed value
- The re-optimizer had as cost function the sum of all ΔV, the design variables were transfer time
- We also have to introduce an inequality constraint that the sum of the transfer times is not larger than the old sum of the fixed transfer times
- For the re-optimizer we are not using a look up table for the $\Delta \mathrm{V}$, we are calculating it during the *fmincon* call

Transfer Part

- For each transfer we know the following information:
    - departure epoch
    - arrival epoch
    - transfer time
    - estimated ΔV
- We used again matlab *fmincon* with an interior point method to tackle that problem
- The control parameters are the times between maneuvers and the 5 maneuvers itself in Cartesian form
- The cost function is quite easy in our case, it’s just the sum of all 5 ΔVs we applied
- The more demanding part is the constraint function

Transfer Part, Constraint Function

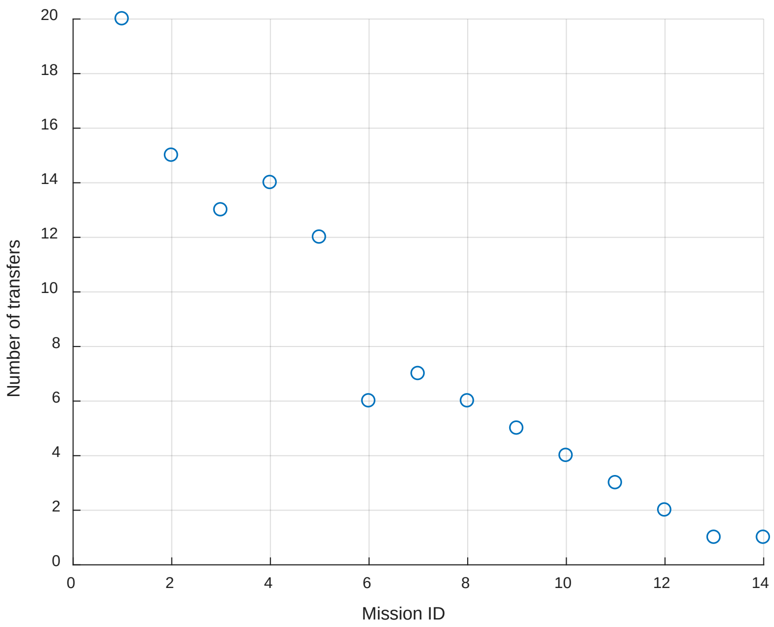
- In the constraint function we integrate the equation of motion between the maneuvers until we reach our final state
- Than the final state should equal the arrival debris state at that time (equality constraints)
- We have a global parameter in order to activate or deactivate the constraints, and we can choose between the Cartesian state vector, Keplerian elements, or a mixture, or a subset
- Another inequality constraint is that the sum off all transfer times between maneuvers is not larger than the transfer time we got out of the tree

Transfer Part, ode solver + *fmincon* issue

- The use of ode solvers in a *fmincon* call caused some issues with respect to stability
- When the error from the ode is to large and variable step size solvers are used the Jacobian and Hessian gets instable
- We implemented a RK8 in c++ with a step size of 50 sec to overcome that issue

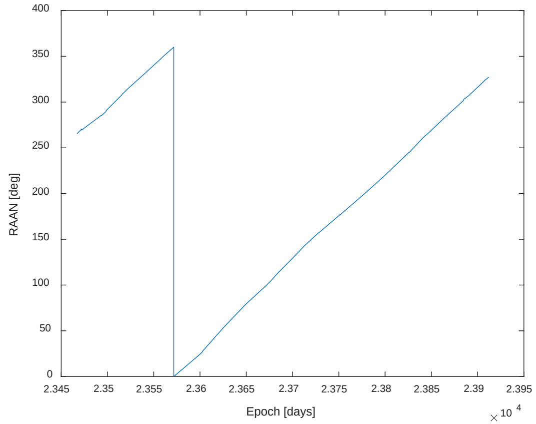
Results

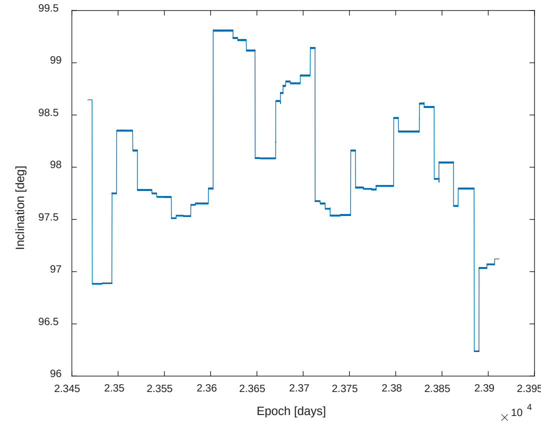
- Our submitted solution had a total of n =14 missions and a performance index J = 949.85MEUR

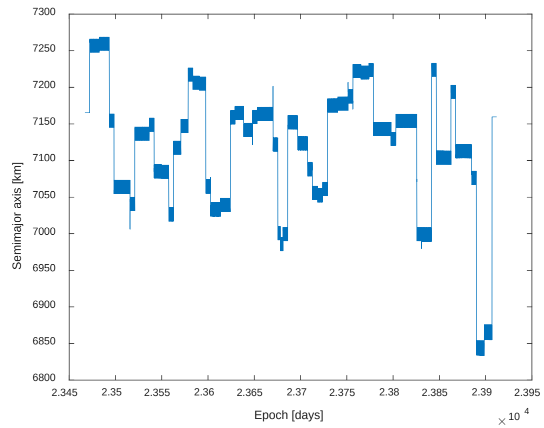


Results

- For the first mission the evolution of Ω, a and i are plotted over time







Conclusion

- For the combinatorial part genetic algorithms are suitable when the problem is time invariant
- But for time variant problems graph algorithms seem to be the better choice
- The Beam search algorithm brought reasonable results, but still suffers a bit from the greedy effect
- One option to improve that may be to select some feasible continuation beams randomly
- In the transfer problem we may use a multiple shooting method to get rid off our ode-integration issue