GTOCX: RESULTS AND METHODS OF TEAM 38 - TSINGHUA & XINGYI

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This paper summarizes our computing methods and results of solving GTOCX problem. The GTOCX problem is referred to settlement of one hundred thousand suitable galaxy star systems during 90 million years. Solving such a large-scale problem requires tremendous amount of calculation. To overcome this difficulty, our team proposed a series of optimization strategies based on genetic algorithms. First, we design a partitioning strategy to generate the initial set of settlement stars, which are settled by fast ships and mother ships. Then, a multiple phase optimization strategy based on genetic algorithm is proposed to generate substantial settlement stars. After that, in order to decrease the fuel consumption, a local optimization method is applied to exchange different transfers. Finally, fuel consumption of all the transfers are optimized by NPSOL and PSO. The final score of our team's results is 2070.53

INTRODUCTION

After ten thousand years from the present, technologies and knowledge have dramatically progressed. Humanity decides to venture into galaxy and settle other star systems. The GTOCX task focuses on the settlement of one hundred thousand galaxy star systems during 90 million years (Myr). The settlement process starts with 3 Mother Ships (MS) and 2 Fast Ships (FS), which are launched from our home star, Sol. Each MS can execute 3 maneuvers and settles up to 10 stars, while each FS maneuvers twice and only settle 1 star. Once a star has been settled for 2 Myr, up to 3 Settler Ships (SS) depart from it and settle 1 star severally. One merit function is defined as a product of a configuration term, which is related to settlement stars' number and spatial distribution, and a fuel term. The more stars are settled, the more uniformly settlement stars are distributed, or the less fuel is consumed, the better result is.

In the GTOCX problem, the number of alternative settle star system is huge. Due to this character, finding the optimization solution requires tremendous calculation. To simplify the problem, our team divide the original optimization problem into global sequence optimization and local transfer optimization. Note that the configuration term and fuel term are independent, the global search process mainly optimize the configuration part, while the optimization of fuel consumption will be done during the local search process. The global sequence optimization is conducted

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in two phases. First, we choose the initial settlement star systems based on the analysis of reachable domain of mother ships and fast ships. Then, taken the configuration part of merit function as objective function, the optimal settlement star sequence are obtained by genetic algorithms. In the local search process, the fuel consumption is first optimized through a settlement star exchange process. Then, fuel consumption of all transfer will be optimized together by NPSOLVE and PSO.

This paper is organized as follows. First we introduce our global search strategies, including approximate model for fuel consumption calculation, determination of initial settlement star systems and propagation strategy of settlement star systems. After that, the further local search methods are described in details, which insist of impulsive maneuvers optimization and settlement star replacement. Finally, the results are discussed and the conclusions are drawn in the last two sections.

GLOBAL SEARCH STRATEGY

This section describe our global search strategy for solving GTOCX problem. In global search process, the main optimization objective is to maximize the configuration part of merit function. First, we introduce the approximate model which is used in global search process to calculation fuel consumption. Then, the initial settlement star systems generation strategy will be described in details. Finally, the propagation strategy of settlement star systems is introduced.

Approximate Model for Fuel Consumption Calculation

The consumption calculation is approximated by a series of polynomial models during the global search process. The relationship between the motion period of stars and their radius is shown in Figure 1. It can be seen that the mission duration of 90 Myr is quite short compared to the period of most stars. Therefore, it is suitable to approximate the motion of stars using polynomial models.

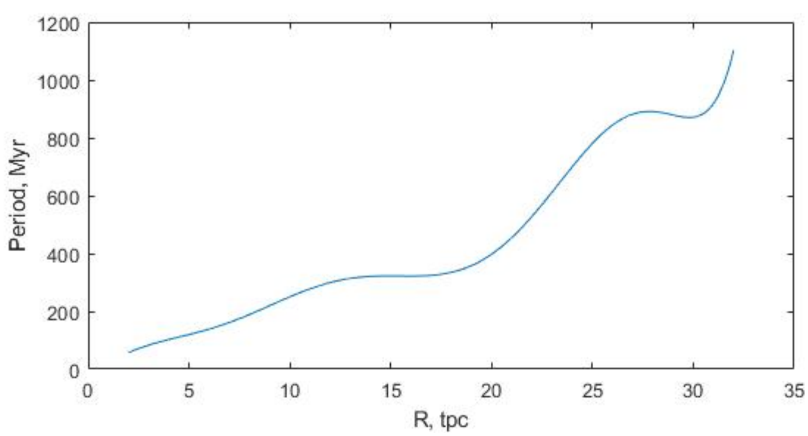

Figure 1. Star Period.

linear model

The simplest polynomial model is the linear model that is firstly proposed in [1] to approximate transfers between asteroids. Assume this scenario: a ship departs from a star s_1 to a star s_2. At the departure time, the position and velocity of star s_1 is r_1 and v_1, and the position and velocity of star s_2 is r_2 and v_2. In the linear model their motion equations are:

$$r_{s1} = r_1 + v_1 t$$
$$r_{s2} = r_2 + v_2 t \tag{1}$$

If the transfer time is Δt, then the velocity increment in the linear model is:

$$\Delta v_1 = \frac{(r_2 - r_1)}{\Delta t} + (v_2 - v_1)$$
$$\Delta v_2 = \frac{(r_1 - r_2)}{\Delta t} \tag{2}$$

Accuracy of the linear model should be checked before apply it. Results are shown in Figure 2. In the figure the x axis represents the radius of departure stars and y axis represents the relative error of accurate Δv and approximate Δv. The departure date is 0 Myr and the transfer time is 2 Myr, 6 Myr, and 10 Myr respectively. The departure stars are selected randomly, and for each departure star target stars are its 100 nearest stars (Euclidean distance). We can see from Figure 2 that the linear model is very accurate when the transfer time is small and the radius of the departure star is large.

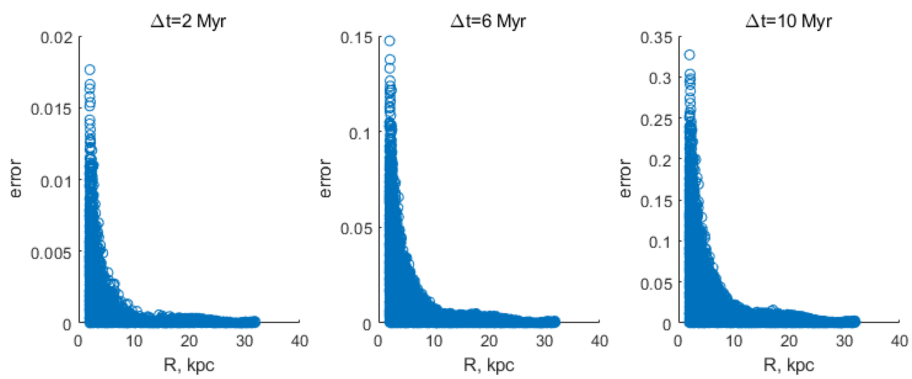

Figure 2. Accuracy analysis of the linear model.

Cubic model

The linear model assumes that stars move with constant speed in a straight line, and the cubic model assumes that stars move with acceleration and jounce. The motion equation of the cubic polynomial model is:

$$r = r_0 + v_0 t + \frac{1}{2} a_0 t^2 + \frac{1}{6} c_0 t^3$$
$$v = v_0 + a_0 t + \frac{1}{2} c_0 t^2 \tag{3}$$
$$a = a_0 + c_0 t$$

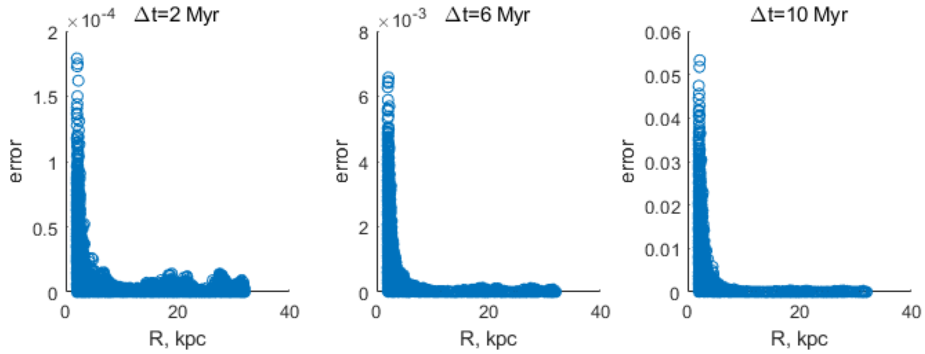

Figure 3. Accuracy analysis of the quart model.

Generation of Initial Settlement Star Systems

In the selection of initial settlement star systems, the first step is to generate the tabular databases of the initial settlement stars for MS and FS. Firstly, we aim to obtain the tabular database for FS. Considering that the initial settlement stars' number accounts for only a small fraction of all settlement stars, it is assumed that all of the fuel of FS is used. A minimum-time solution is search for the tabular database. To get the minimum-time solution for a target star (star's ID is N), the optimization parameters are the initial time t_0 when FS leaves Sol and the final time t_f when FS arrives the target star. The initial and final impulses are obtained by solving Lambert problem using shooting method. Then, the performance index is

$$f_{\mathrm{FS}}=\begin{cases} t_f\text{, if } \Delta V-\Delta V_{\max}\le 0 \\ t_f+p_f\left(\Delta V-\Delta V_{\max}\right)\text{, if } \Delta V-\Delta V_{\max}>0 \end{cases} \tag{4}$$

where ΔV is the impulse used for orbit transfer, $\Delta V_{\max}$ is maximum total ΔV, and $p_f = 10^5$ is a penalty function. Finally, the solutions for all target stars are concluded in the tabular database. The target stars which could be settled during 25~35Myr are shown in Figure 4.

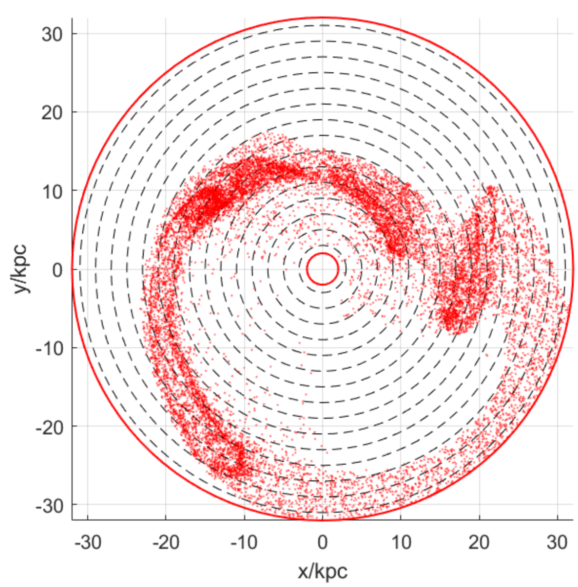

Figure 4. The positions of target stars at 90 Myr.

Secondly, the tabular database for MS is more complex to obtain because of the multi-impulse maneuver and multiple settlement targets. Using the aforementioned algorithm for searching minimum-time solutions of FS, the minimum-time solutions of MS could also be conducted by involving multi-impulse optimization and settling only one star. This is first part of the tabular database. Another part is a collection of 6-flyby MSs, which settle 6 stars by 3-impulse maneuver and settle 2 stars after an impulse. It should be noted that settling more than 2 stars after an impulse is possible. But these rare cases are not employed there for their complicated optimization. The MS is assumed to flyby 2 stars after the first impulse. As shown in Figure 5, the target stars is determined by traversal method. For target 1 and target 2, the shooting method is utilized to find the initial time t_0, the initial impulse $\boldsymbol{dv}_0$, the first flyby time t_1 and the second flyby time t_2. The shooting function is that the position of MS at t_1 equals to the position of target 1 and the position of MS at t_2 equals to the position of target 2. After the first impulse applied, the second impulse, the third and the forth settle stars are determined similarly. Finally, the 6-flyby MSs and target stars are obtained.

At last, the tabular databases are formulated. The initial settlement stars are further analyzed using the reachable domains of MS and FS.

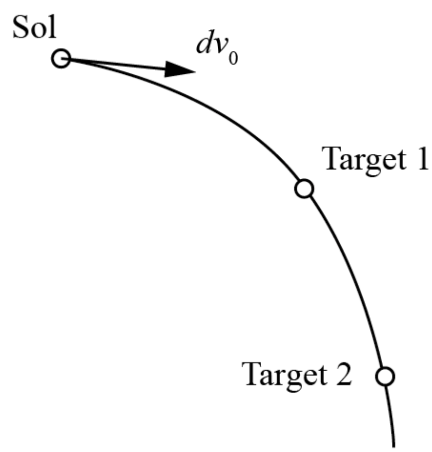

Figure 5. The 2-star flyby after the first impulse.

The reachable domain of FS, which is shown in Figure 4, could be obtained using above tabular database. For MS, the minimum-time optimization is time-consuming, and the reachable domain is obtained by randomly applying impulses and getting the position at 40 Myr. The results is shown in Figure 6.

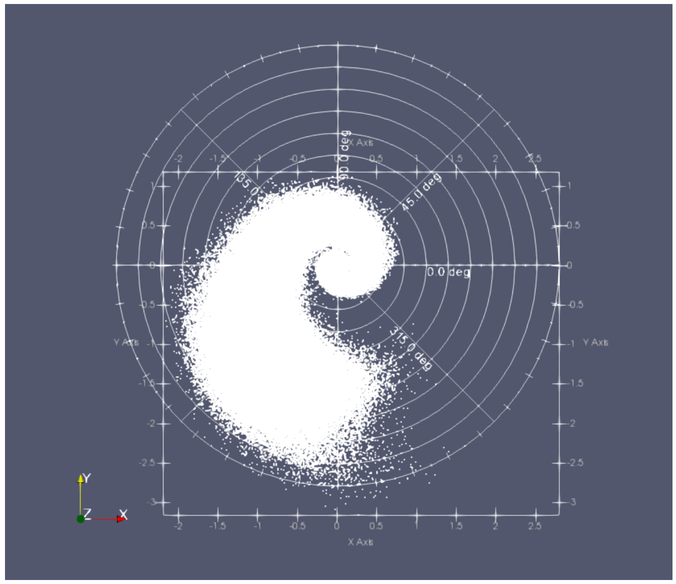

Figure 6. The reachable domain for MS at 40Myr.

Considering the uniform spatial distribution of the settled stars in the galaxy, the initial settlers from Sol is very important. According to the previous analysis of reachable domain of mother ships and fast ships, it can be conclude that fast ship are suitable for the settlement of reverse and outer stars and mother ships should be used to settle the inner stars for the advantage of multi-Pod and multi-flyby performance. As shown in Figure 4, given a specific flight time, if a fast ship choose a star with smaller orbital radius, its final phase angle θ_f can be bigger. As for the reachable domain of mother ships shown in Figure 6, depending on the different revolution speed of stars, mother ships flying to the inner galaxy can cover a wider range of phase angle, while mother ships flying to the outer galaxy spend more flight time and can only achieve lager radius stars' settlements. Thus, one mother ship is assigned flying to the inner galaxy to achieve phase angle coverage, while the other two motherships fly to the region out of Sol and disperse according to phase angle, and the fast ship are used to cover larger orbital radius and the remaining phase angle gaps.

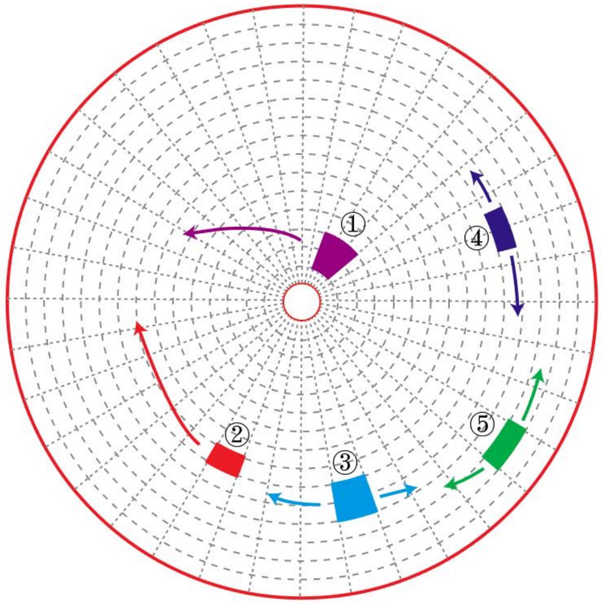

Figure 7. Distribution of initial settlement star sequence

According to the previous analysis, distribution of the selection of initial settlement star sequence is shown as Figure 7. Two mother ships are assigned to fly by and settle three stars respectively (zone 1 and zone 2), and one mother ship are assigned to settle a star with lager orbital radius exerting three maneuvers (zone 3). The settlement target of fast ships are selected for the remaining space of orbital and phase angle space (zone 4 and zone 5). The expected direction of developing settlements are shown with arrows in Figure 7. It is worth mentioning that the first settled star of mother ship 1 and the last settled star of mother ship 2 are restricted by zone 1 and zone 2, respectively.

Propagation Strategy of Settlement Star Systems

Given the initial settlement stars, further settlement stars are decided by generic algorithm according to the configuration part of merit function, which can be defined as following:

$$J = \frac{N}{1 + 10^{-4} \cdot N(E_r + E_\theta)} \tag{5}$$

Where N represent the number of settled stars, E_r, E_θ are refer to reference [2]. There are two phases in the decision of settlement star for each generation. One is the determination of unsettled star systems, the other is the detachment of settled star systems. In the propagation process, the further settlement stars are chosen in the current settlement star's reachable domain, which is show as Figure 8. Since the reachable domain of any settlement star is limited by remaining time, it need to be calculated dynamically. The star selection is performed by an integer coding genetic algorithm based on the current distribution of settled star. The coding mechanism is shown in Figure 9. Each individual is composed of a series of genes with the same number of settled stars. Each gene position represents a settled star, while the number in the gene position represents the serial number of unsettled star in the reachable domain of the current settled stars. Once the serial number is generated, the settlement star is correspondingly determined. The optimal solution can be obtained after a series of genetic manipulation including selection, crossing and mutation.

Figure 8. Propagation process of one settled star system.

Individual1	m_1	n_1	p_1	…	l_1	k_1
Individual2	m_2	n_1	p_1	…	l_1	k_1
…						
Individualn	m_n	n_n	p_n	…	l_n	k_n

Figure 9. Coding mechanism of propagation process.

The detachment process is a little different from the selection process, for that the number of settled star can only be decreased. All the settled star systems which do not launch settle ship are taken as candidates. Then, the selection of which settlers to remove are implemented by genetic algorithm according to the current distribution of settled star. The coding mechanism is shown in Figure 11. Each gene position corresponds to a star candidate. If the settled star system is deleted, the value of corresponding gene is 0, otherwise 1. The optimal solution can be obtained after a series of genetic manipulation including selection, crossing and mutation.

Figure 10. Detachment process of one settled star system.

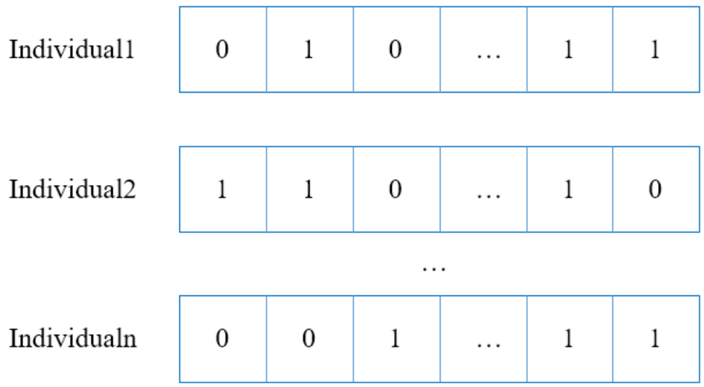

Figure 11. Coding mechanism of detachment process.

LOCAL OPTIMIZATION METHODS

This section describe our local optimization methods. First, a sequence adjustment process is implemented to reduce the fuel consumption and improve the configuration. Impulsive maneuvers are optimized to further reduce the fuel consumption.

Settlement Star Sequence Adjustment

First, star sequence adjustment is implemented. There are two types of star sequence adjustment, as illustrated in Figure 12. The first is exchange. Chose two transfers in the solution which are obtained by global optimization, and exchange the target stars and their rendezvous times. The second is replacement. Replace a star with a star that is not settled, and keep rendezvous time unchanged. There are two objectives of sequence adjustment. One is to reduce the fuel consumption, and the other is to improve the configuration. Because exchange does not change the configuration, altogether three adjustment processes are implemented: exchange that reduces the fuel consumption, replacement that reduces the fuel consumption, and replacement that improves the configuration.

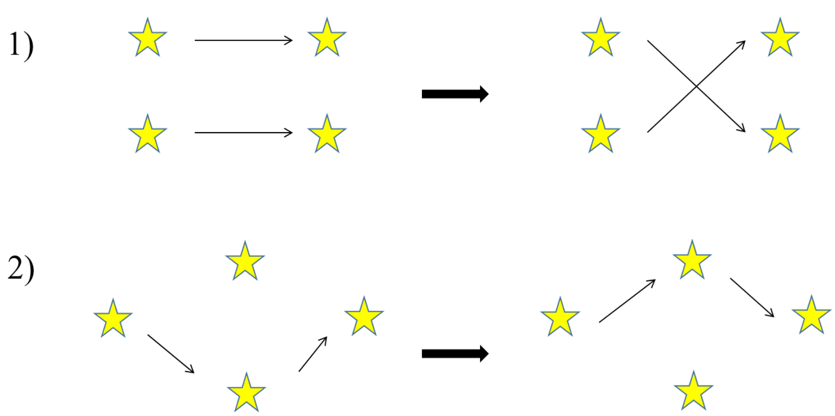

Figure 12 Settlement Star Sequence Adjustment

Impulsive Maneuvers Optimization by NPSOL and PSO

Given the sequences of settlement stars, nonlinear programming software NPSOL is utilized to improve the fuel term in merit function. The optimization variables are the initial time t_0 and final time t_f of all the transfers and the object function is the total Δv. As the number of impulses is unknown, so each transfer is assumed 2-impulses first and a Non-Kepler Lambert problem is solved. If the magnitude of impulse at t_0, named Δv_0, is beyond the limit of single impulse, 175 km/s, it is set 175 km/s and an impulse with a magnitude of Δv_0-175 km/s is inserted at t_0+1 Myr. Likewise, if the magnitude of impulse at t_f, named Δv_f, is beyond 175 km/s, it is set 175 km/s and an impulse with a magnitude of Δv_f-175 km/s is inserted at t_f-1 Myr. By this means, up to 4 impulses are considered for each transfer but the result is not optimal because the intermediate impulses are not optimized. Based on this result, the number of impulses for each transfer is determined and NPSOL is used again to optimize all the impulses. The optimization variables consist of all the epochs when an impulsive maneuver is employed as well as all the impulsive vector.

After the whole sequences are optimized by NPSOL, all the last two generations in the sequences are picked out and optimized by Particle Swarm Optimization (PSO). Assuming N generation of settlement stars have been developed in a sequence, there are three topological structures for the last two generations, ternary tree, binary tree and chain, see Figure 11.

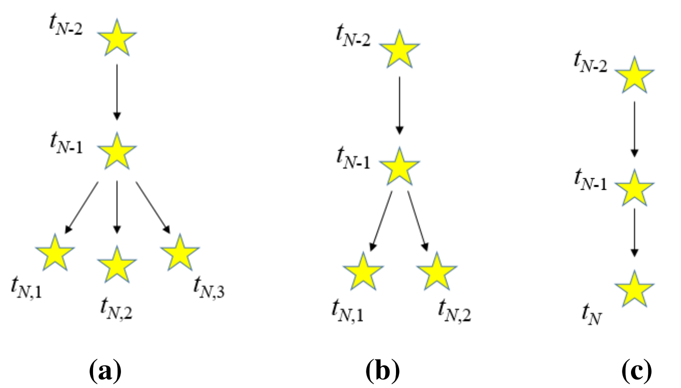

Figure 13 topological structures for the last two generations

(a) ternary tree, (b) binary tree, (c) chain

The variable t in Figure 13 represents the epoch when a settler ship rendezvous with a star. When optimizing the transfers among the last two generations, $t_{N\text{-}2}$ is fixed to avoid influencing former generations, $t_{N\text{-}1}$ is regarded as one of the optimization variables, and $t_{N,k}$, k=1,2,3, are set 90 Myr. For each transfer, 4 impulsive maneuvers are employed so the impulsive vectors of the first two maneuvers and the epochs of the two intermediate maneuvers are also chosen as optimization variables. Thus, there are 33, 25, 17 optimization variables for the three topological structures, respectively. Using PSO to optimize them separately, and comparing the fuel consumption with that of previous result, the better result is reserved.

RESULTS

Our final result consists of 2806 settlement stars with a score of 2070.53. The configuration of our result is show in Figure 14. The first MS voyages inwards while the second outwards, settling 4 and 6 stars, respectively. The third MS exerts all its energy to settle an outer star. The 2 FS voyage to specific districts we set to make the final spatial distribution as uniform as possible.

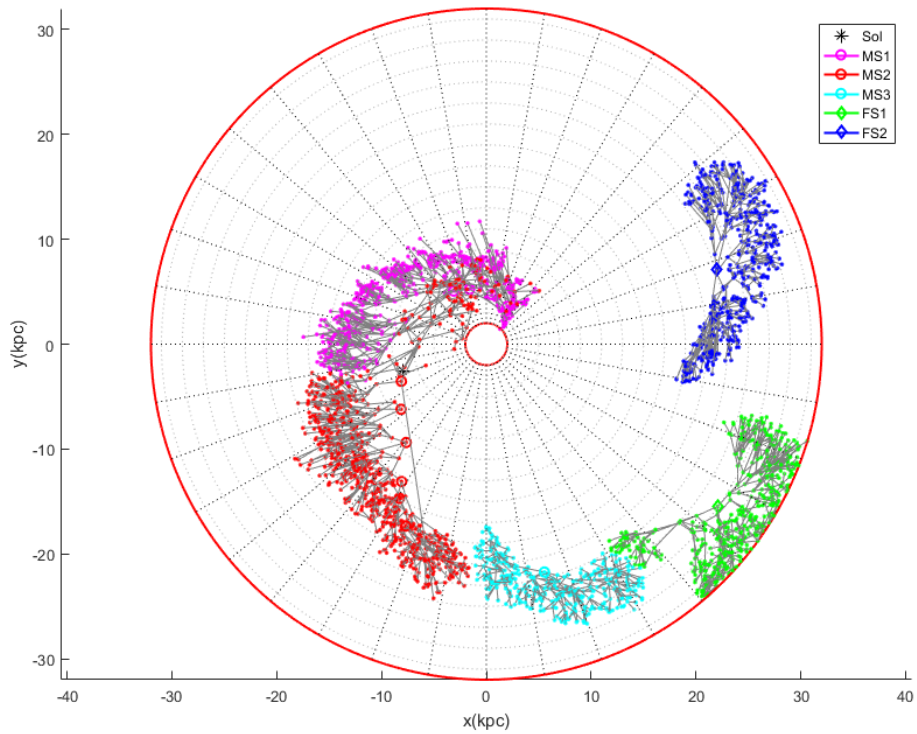

Figure 14. Distribution of settled star sysytems.

CONCLUSION

A multiple phase optimization strategy based on genetic algorithm is proposed in this paper. The genetic algorithm is illustrated to be effective in generating settlement star sequence. Furthermore, NPSOLVE and PSO are proved to be effective methods on optimizing the fuel consumption. Look forward to the next GTOC.

ACKNOWLEDGMENTS

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