

A Near Deterministic Method For Constructing the GTOC-11 Dyson Ring

12/18/2021

James Pezent¹, Jared Sikes², Carrie Sandel³, Ari Rubinsztein⁴, Rohan Sood⁵

Astrodynamics and Space Research Lab (ASRL)

The University of Alabama

1. PhD Student , Team Leader
2. PhD Student
3. PhD Student
4. PhD Candidate
5. Assistant Professor, Lab Director



Overall Strategy

- Planar Dyson ring
- Construct many independent “Tours” wrt. fixed bins and radius
 - 1 Mothership flyby sequence
 - Transfers of all asteroids in sequence to at least one station
 - Sequential station build order
 - Equal build time for each station
- Combine 10 Tours in post processing
 - Bin-pack to increase minimum mass
- Full fidelity almost all the time

Tools

➤ Software

➤ ASSET: Astrodynamics Software and Science Enabling Toolkit

- Custom Sparse Non-Linear Optimizer (PSIOPT & Jet)
- General Purpose Optimal Control
- Spacecraft Trajectory Design
- C++/Python
- Developed at UA - Open source at EOY 2022

```
=====
                        ASSET
=====
Astrodynamics Software and Science Enabling Toolkit
=====

Development funded by NASA under Grant No. 80NSSC19K1643
Authors:
James B. Pezent      jbpezent@crimson.ua.edu      Lead Developer
Jared D. Sikes       jdsikes1@crimson.ua.edu      Developer
William G. Ledbetter wgledbetter@crimson.ua.edu    Developer

Principle Investigators:
Rohan Sood           rsood@eng.ua.edu             PI
Kathleen Howell      howell@purdue.edu            Co-PI
Jeff Stuart          jeffrey.r.stuart@jpl.nasa.gov Co-PI
=====

Software Version      : 0.0.1
Python Version        : 3.8
System Core Count     : 8
System Thread Count   : 16
Vectorization Mode    : AVX2 - 256 bit - 4 doubles
Linear Solver         : Intel MKL Pardiso
Compiled With         : Clang 13.0.0
Compiled On/For       : Windows 10.0.19043
=====
```

➤ Hardware

➤ Two desktop computers

- i9 9900k (8 Core)
- i7 10700k (8 Core)

```
=====
                        PSI-OPT
=====
Parallel Sparse Interior-point Optimizer

Problem Statistics
Primal Variables      : 195
Equality Constraints   : 130
Inequality Constraints : 0

KKT-Matrix DIM (P+E+2*I) : 325
KKT-Matrix NNZs        : 1674
KKT-Matrix MNZs        : 1.584852%

Beginning: PSIOPT
Beginning: KKT-Matrix Analysis
LDLT Factor NNZs      : 3416
LDLT Factor FLOPs     : 0 MfLOPs
Analysis/Reorder Time : 3.893 ms
Finished: KKT-Matrix Analysis
Beginning: Optimization Algorithm

Iteration | Mu Val | Prim Obj | Bar Obj | KKT Inf | Bar Inf | ECons Inf | ICons Inf | AlphaP | AlphaD | LSI | PPS | HFI | HPert |
0 | 1.00e-03 | 1.268e+00 | 0.000e+00 | 4.214e-03 | 0.000e+00 | 0.000e+00 | 0.000e+00 | 1.00e+00 | 1.00e+00 | 0 | 0 | 0 | 0.0e+00 |
1 | 1.00e-03 | 7.616e-01 | 0.000e+00 | 1.000e-03 | 0.000e+00 | 2.4160e-11 | 0.000e+00 | 1.00e+00 | 1.00e+00 | 0 | 0 | 0 | 0.0e+00 |
2 | 1.00e-03 | 7.616e-01 | 0.000e+00 | 2.8860e-15 | 0.000e+00 | 0.8810e-10 | 0.000e+00 | 1.00e+00 | 1.00e+00 | 0 | 0 | 0 | 0.0e+00 |

Optimal Solution Found
Iterations : 3
Prim Obj : 7.61642723e-01
KKT Inf : 2.88607886e-15
Bar Inf : 0.00000000e+00
ECons Inf : 8.88178240e-10
ICons Inf : 0.00000000e+00

NLP Function Evaluation Time : 0.071 ms 0.324 ms/iter
KKT Matrix Factor/Solve Time : 0.905 ms 0.382 ms/iter
Console Print Time : 3.752 ms 1.251 ms/iter
Total Time (NLP+KKT+Print) : 5.329 ms 1.776 ms/iter

Finished: Optimization Algorithm
PSIOPT Total Time : 18.866 ms
Finished: PSIOPT
=====
```

```
=====
                        JET
=====

Beginning: Jet

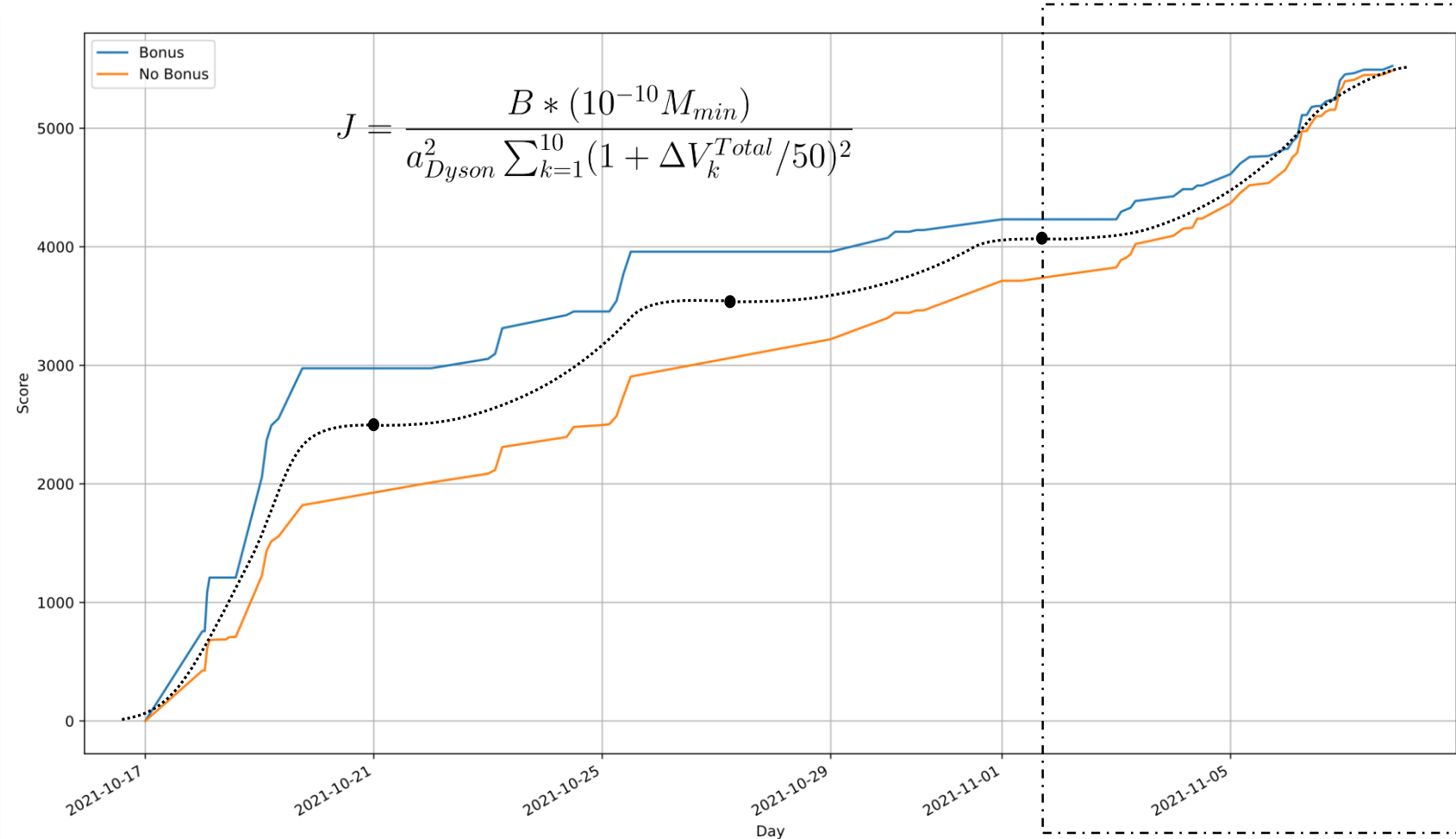
Remaining Time : 7.99 s
Elapsed Time : 0.86 s
Progress : 9.72 %

[#####.....]

Completed : 292/3004
Optimal : 292/3004
Acceptable : 0/3004
Not Converged : 0/3004
Diverged : 0/3004
```



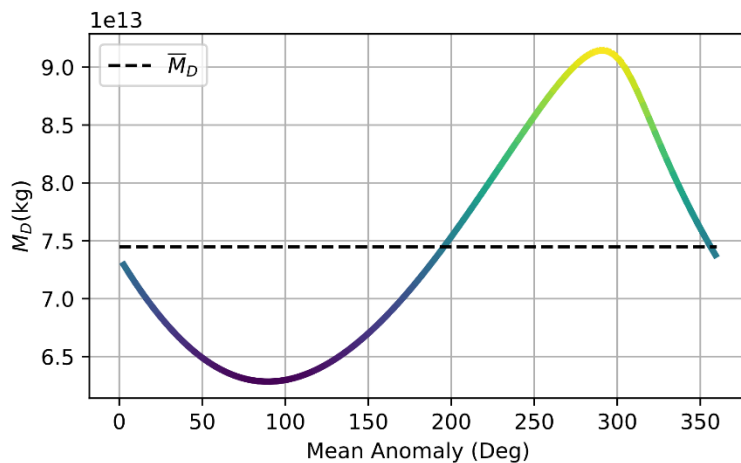
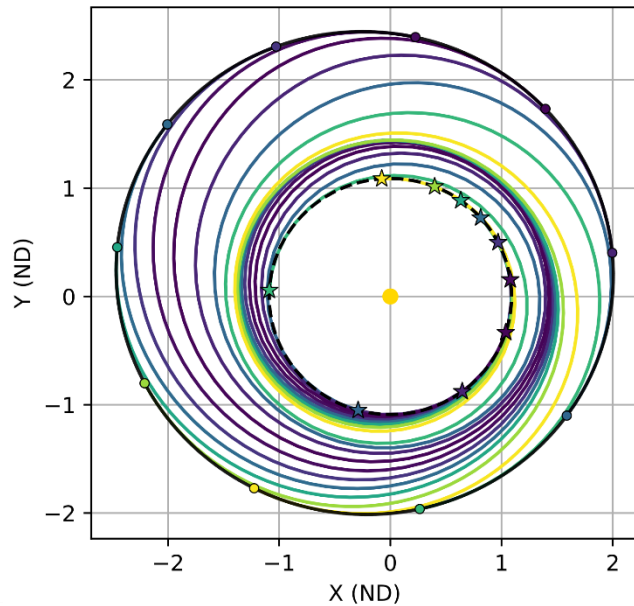
Development Cycle



- Overhauled search 4 times
 - Restart when scores went logistic
 - Obtain old score as quickly as possible
 - Methods became simpler over time

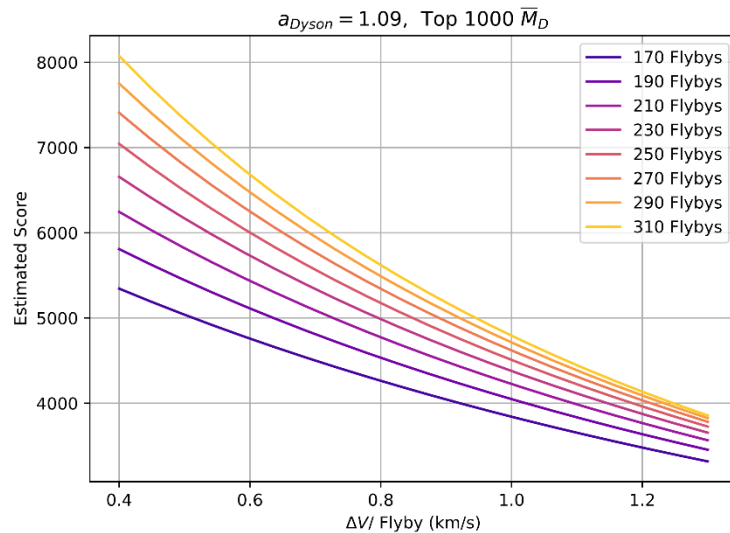
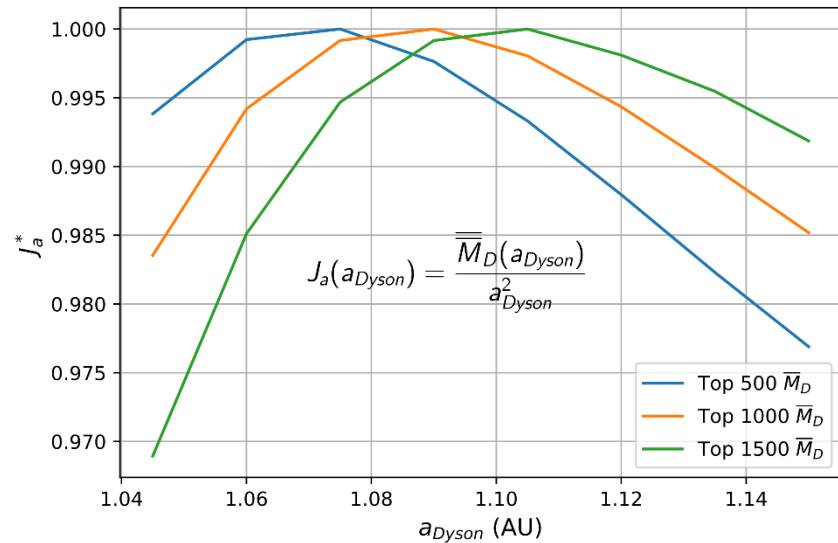
- Focus on final strategy today
 1. Pre-Analysis
 2. Tour-Sequence Construction
 3. Tour-Transfer Construction
 4. Final Solution Construction

Pre-Analysis



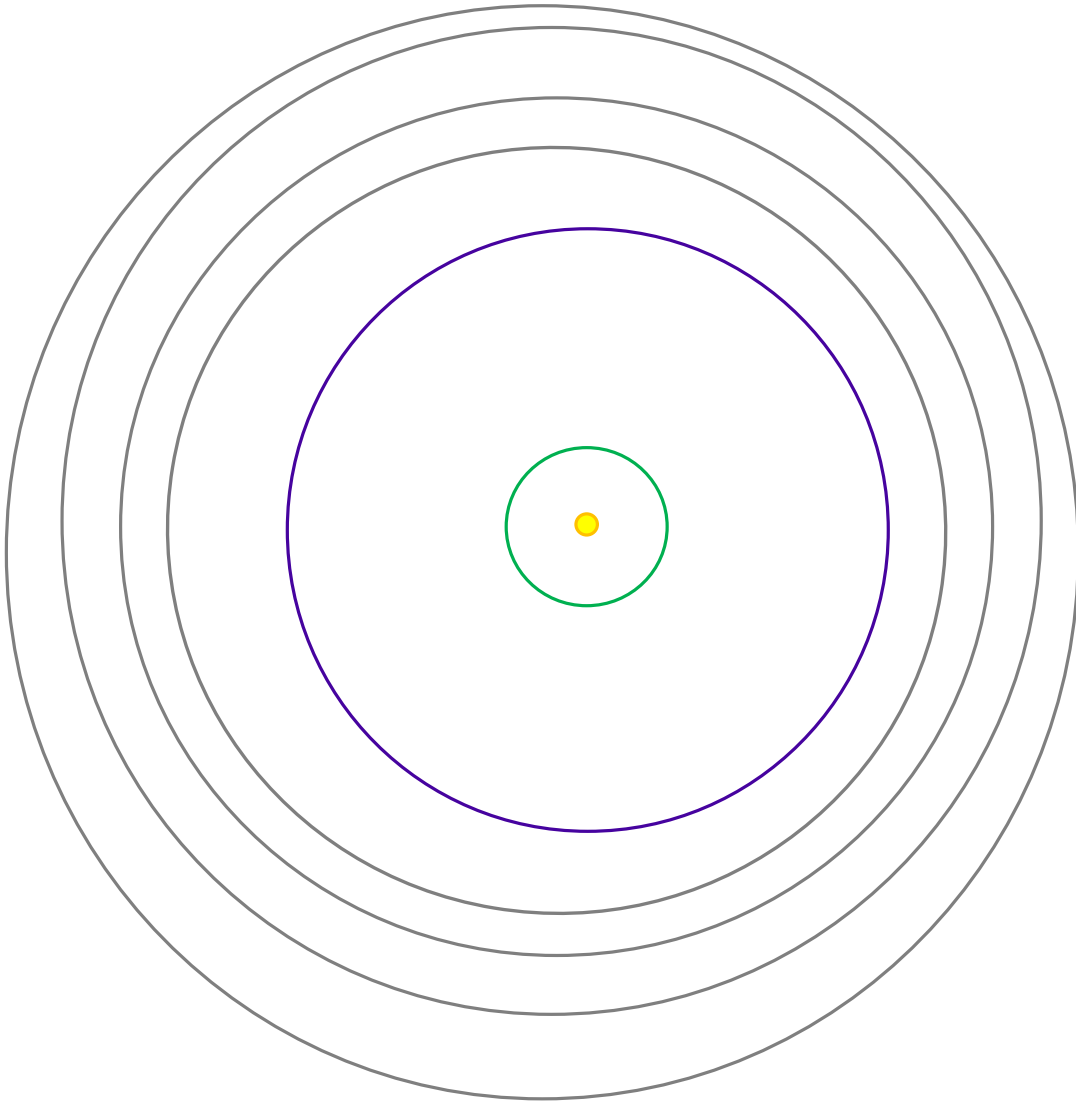
- Estimate best radius
- Estimate score for subsets of asteroids
- Compute mean mass delivered for highest mass asteroids
 - MEE Low-thrust model
 - Disregard phasing/rendezvous w/ station
 - Average over mean anomaly
- Repeat over range of station radii

Pre-Analysis Results



- Best target radius depends on asteroid set
 - Target top 1000 average mass delivered (\bar{M}_D)
 - Chose 1.09 AU
- Derive upper bound on tour TOF
 - 15.5 years
- Derive acceptable ΔV targets to meet a desired score
- Target ΔV /flyby of .75 km/s and 190 – 230 flybys
 - Chosen due to time constraints
 - Should score 5000 or better

Sequence Construction



➤ For each asteroid in high mass delivered set

➤ Find lowest ΔV Lambert solution from Earth

- Must depart within first year

➤ Converge full fidelity flyby

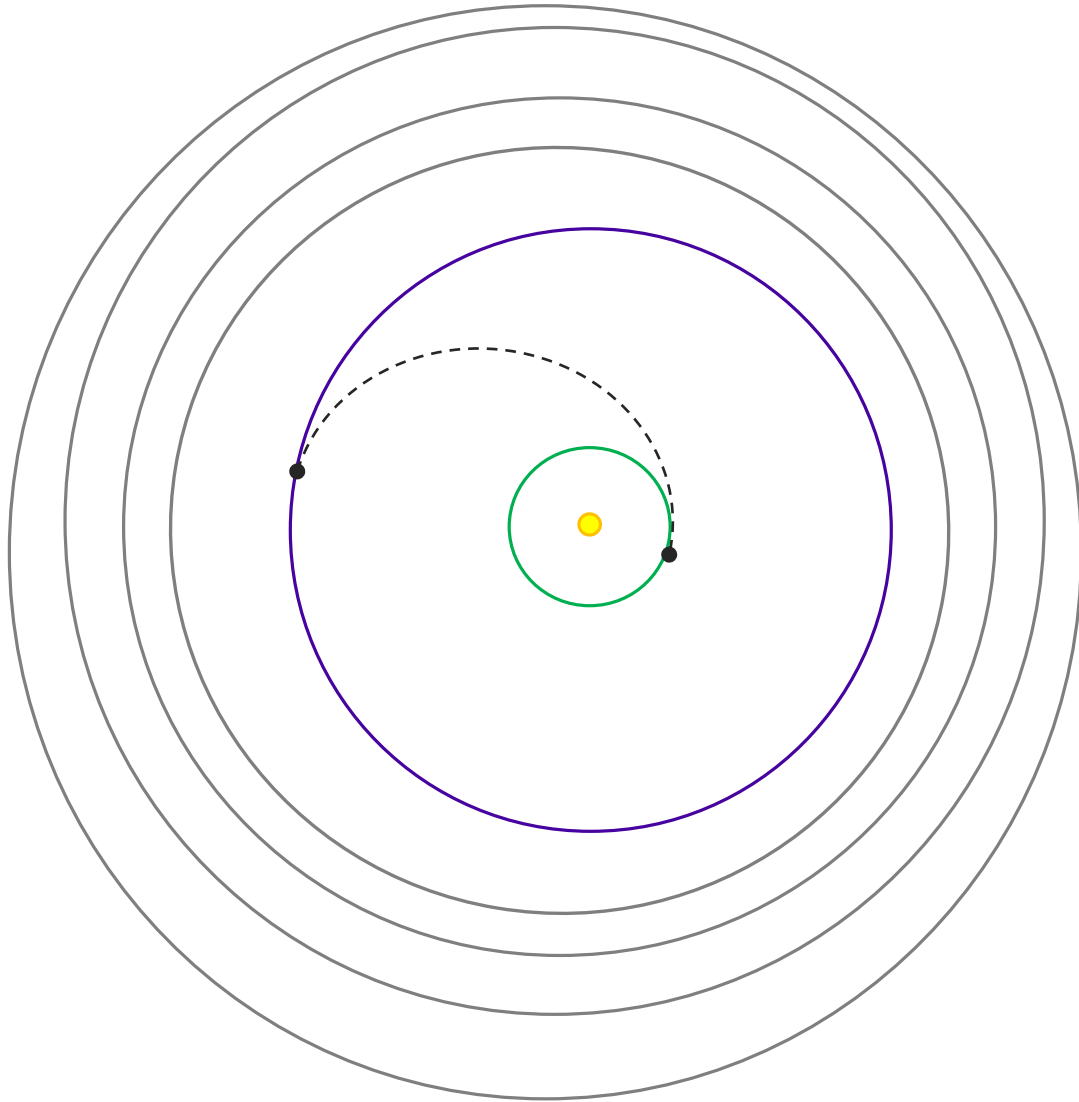
- Two ΔV s per leg

➤ Loop

- Estimate ΔV to all other asteroids in high mass set
 - Lambert solver
- Select minimum n Lambert ΔV asteroids
- Solve n full fidelity OCPs with these asteroids as next target
 - Two ΔV s per leg
 - Kepler propagator shooting scheme
 - Upper bound on tour TOF
- Lowest ΔV solution becomes new sequence

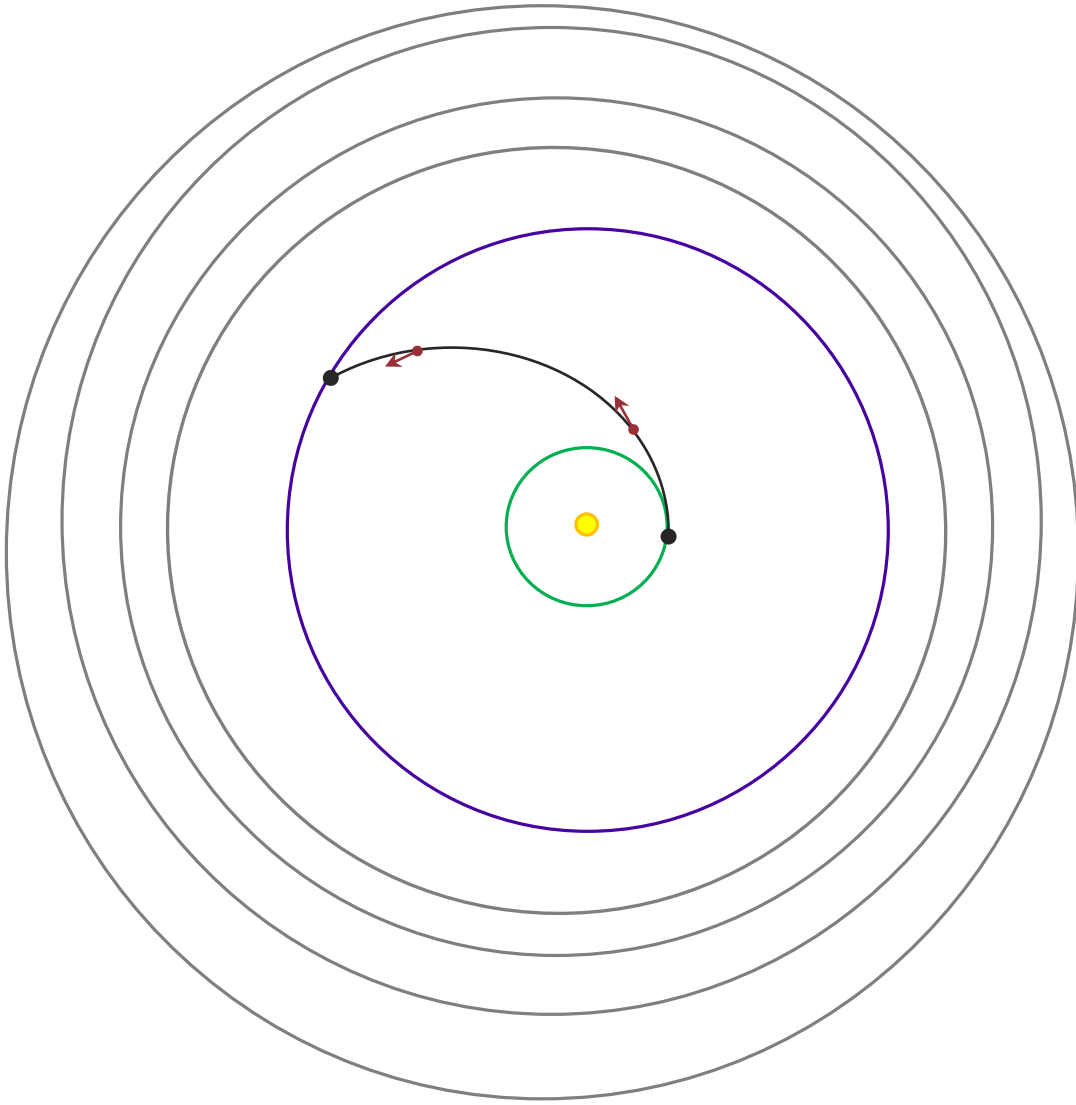
➤ Repeat until TOF limit reached

Sequence Construction



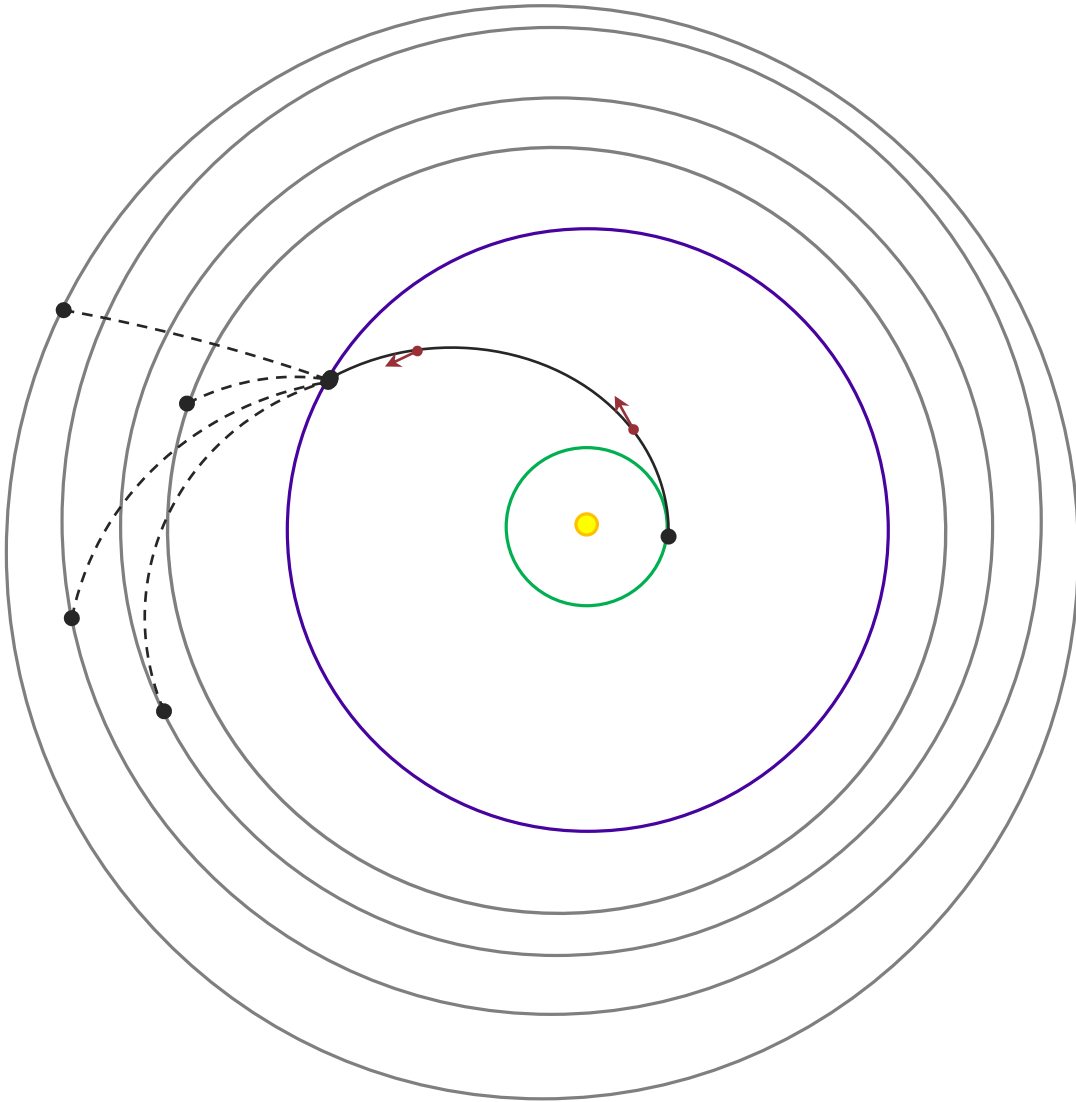
- For each asteroid in high mass delivered set
 - Find lowest ΔV Lambert solution from Earth
 - Must depart within first year
 - Converge full fidelity flyby
 - Two ΔV s per leg
 - Loop
 - Estimate ΔV to all other asteroids in high mass set
 - Lambert solver
 - Select minimum n Lambert ΔV asteroids
 - Solve n full fidelity OCPs with these asteroids as next target
 - Two ΔV s per leg
 - Kepler propagator shooting scheme
 - Upper bound on tour TOF
 - Lowest ΔV solution becomes new sequence
 - Repeat until TOF limit reached

Sequence Construction



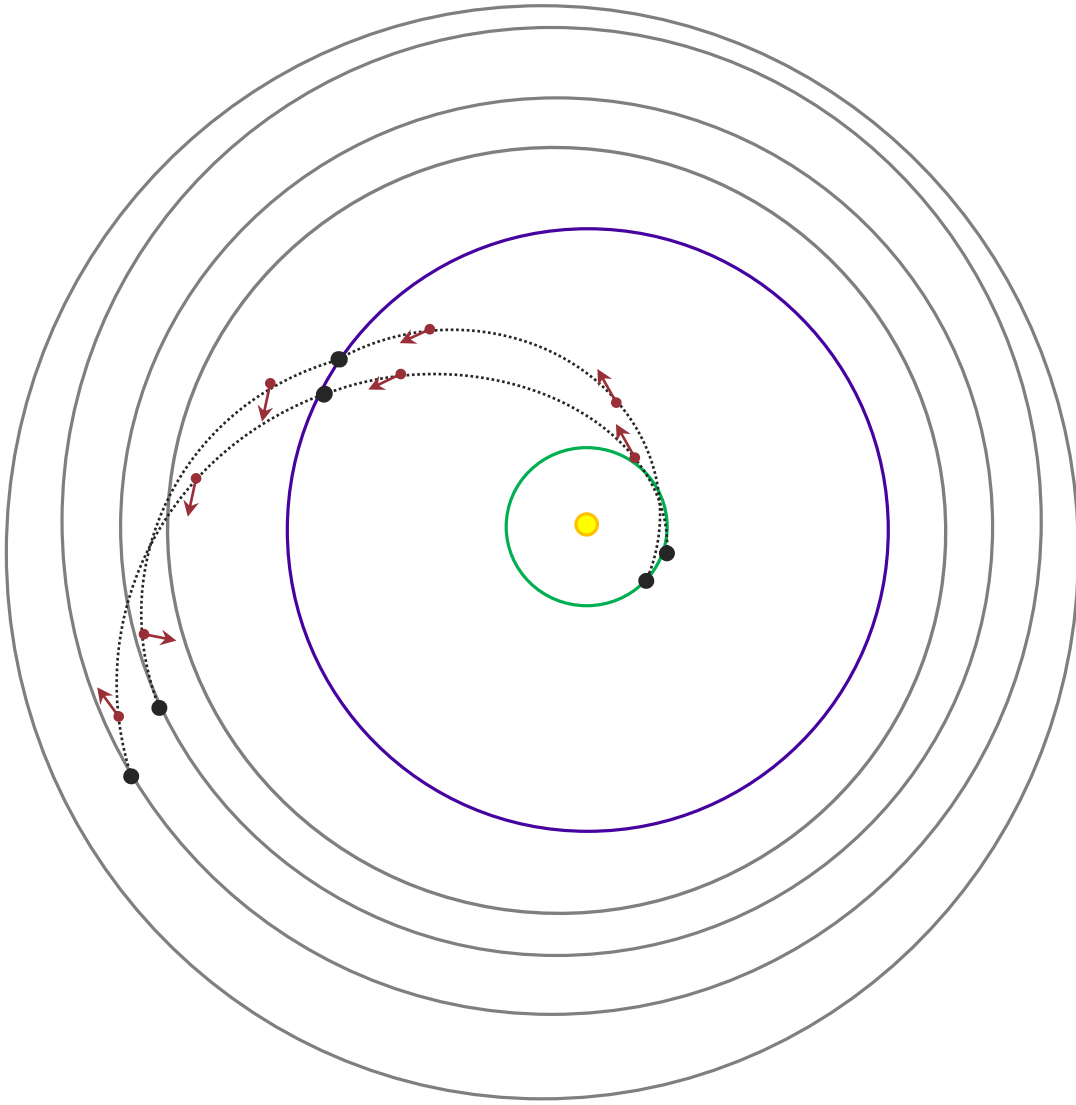
- For each asteroid in high mass delivered set
 - Find lowest ΔV Lambert solution from Earth
 - Must depart within first year
 - **Converge full fidelity flyby**
 - Two ΔV s per leg
 - Loop
 - Estimate ΔV to all other asteroids in high mass set
 - Lambert solver
 - Select minimum n Lambert ΔV asteroids
 - Solve n full fidelity OCPs with these asteroids as next target
 - Two ΔV s per leg
 - Kepler propagator shooting scheme
 - Upper bound on tour TOF
 - Lowest ΔV solution becomes new sequence
 - Repeat until TOF limit reached

Sequence Construction



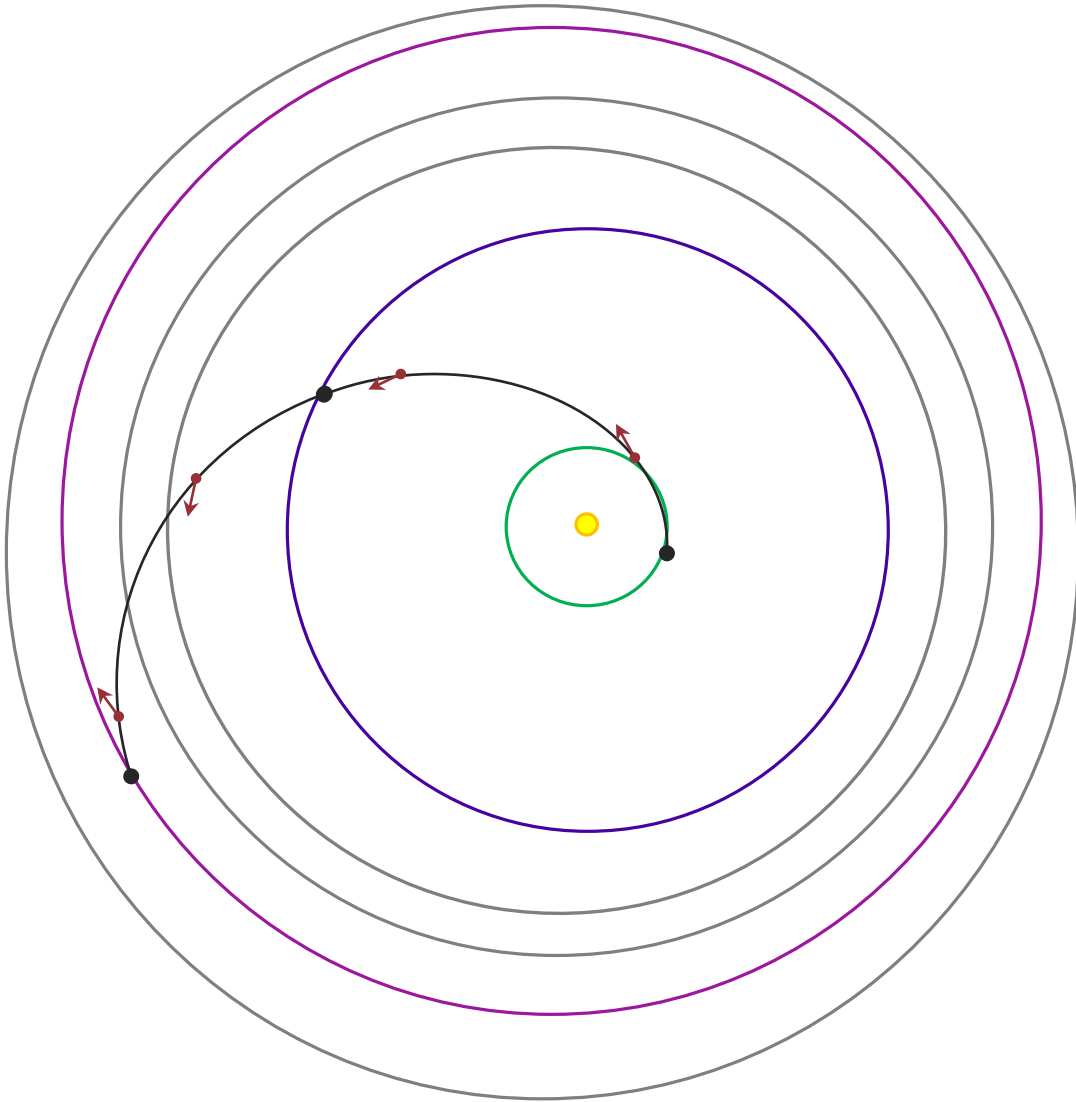
- For each asteroid in high mass delivered set
 - Find lowest ΔV Lambert solution from Earth
 - Must depart within first year
 - Converge full fidelity flyby
 - Two ΔV s per leg
- Loop
 - Estimate ΔV to all other asteroids in high mass set
 - Lambert solver
 - Select minimum n Lambert ΔV asteroids
 - Solve n full fidelity OCPs with these asteroids as next target
 - Two ΔV s per leg
 - Kepler propagator shooting scheme
 - Upper bound on tour TOF
 - Lowest ΔV solution becomes new sequence
- Repeat until TOF limit reached

Sequence Construction



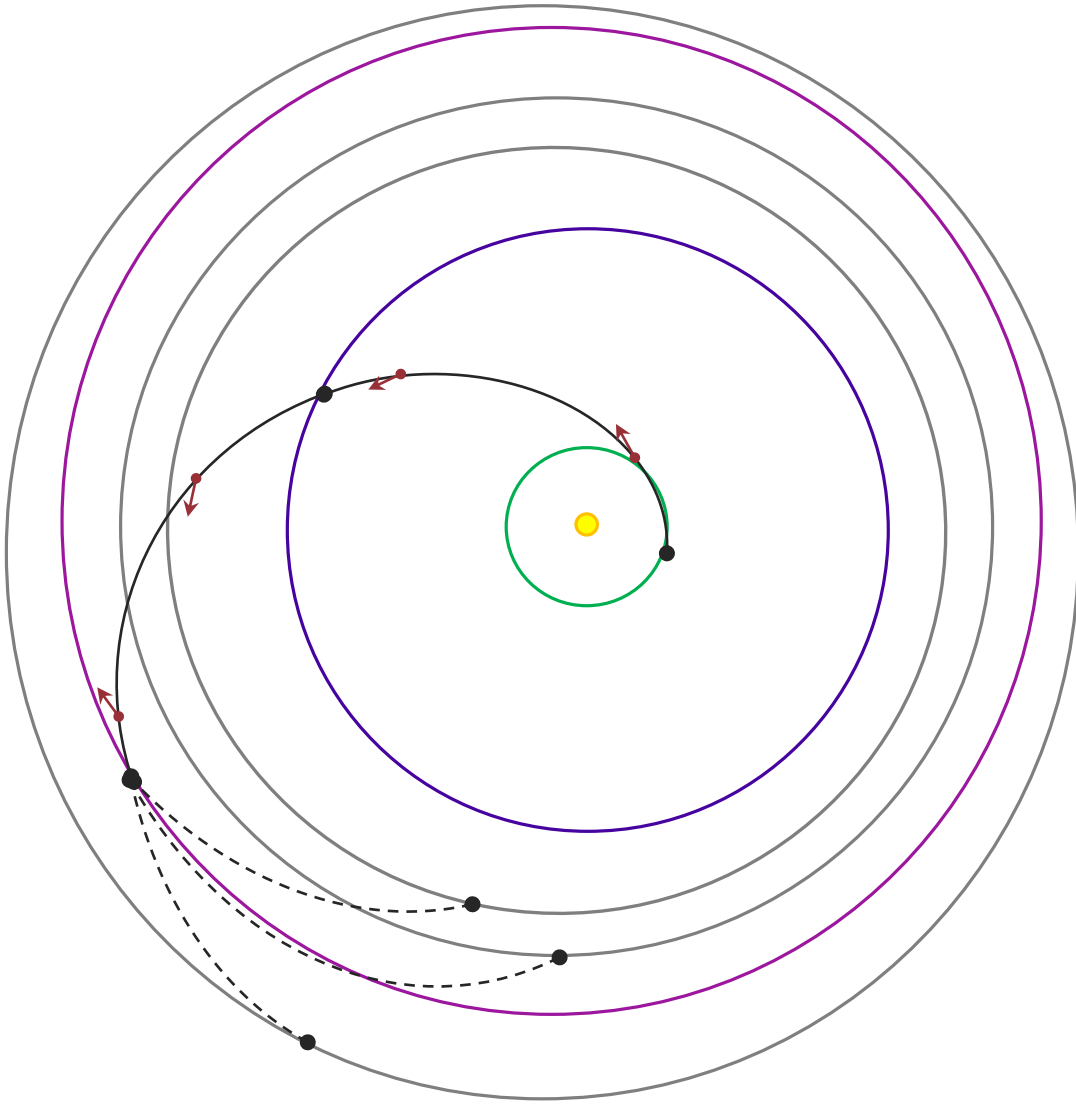
- For each asteroid in high mass delivered set
 - Find lowest ΔV Lambert solution from Earth
 - Must depart within first year
 - Converge full fidelity flyby
 - Two ΔV s per leg
- Loop
 - Estimate ΔV to all other asteroids in high mass set
 - Lambert solver
 - Select minimum n Lambert ΔV asteroids
 - Solve n full fidelity OCPs with these asteroids as next target
 - Two ΔV s per leg
 - Kepler propagator shooting scheme
 - Upper bound on tour TOF
 - Lowest ΔV solution becomes new sequence
- Repeat until TOF limit reached

Sequence Construction



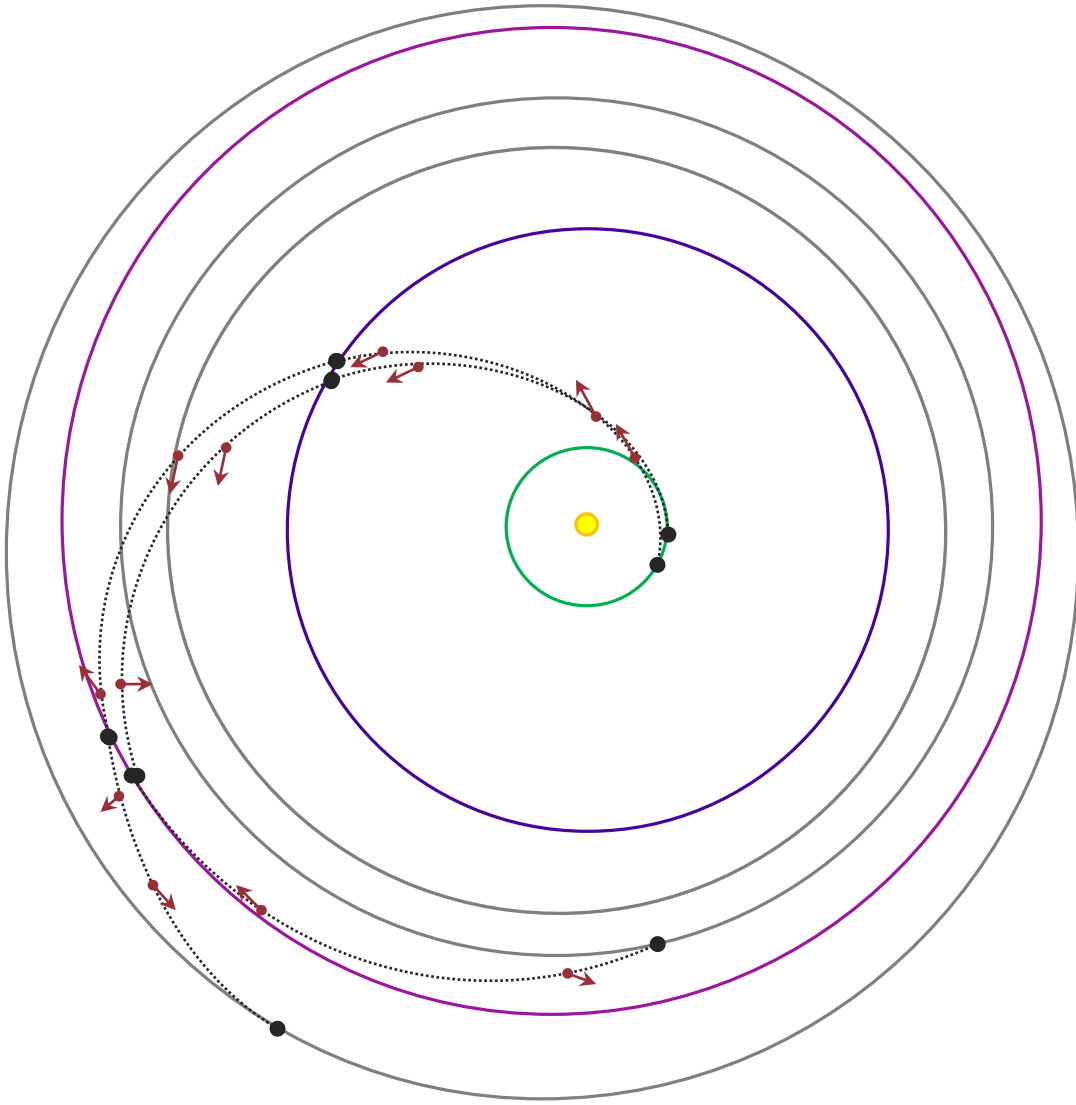
- For each asteroid in high mass delivered set
 - Find lowest ΔV Lambert solution from Earth
 - Must depart within first year
 - Converge full fidelity flyby
 - Two ΔV s per leg
- Loop
 - Estimate ΔV to all other asteroids in high mass set
 - Lambert solver
 - Select minimum n Lambert ΔV asteroids
 - Solve n full fidelity OCPs with these asteroids as next target
 - Two ΔV s per leg
 - Kepler propagator shooting scheme
 - Upper bound on tour TOF
 - **Lowest ΔV solution becomes new sequence**
- Repeat until TOF limit reached

Sequence Construction



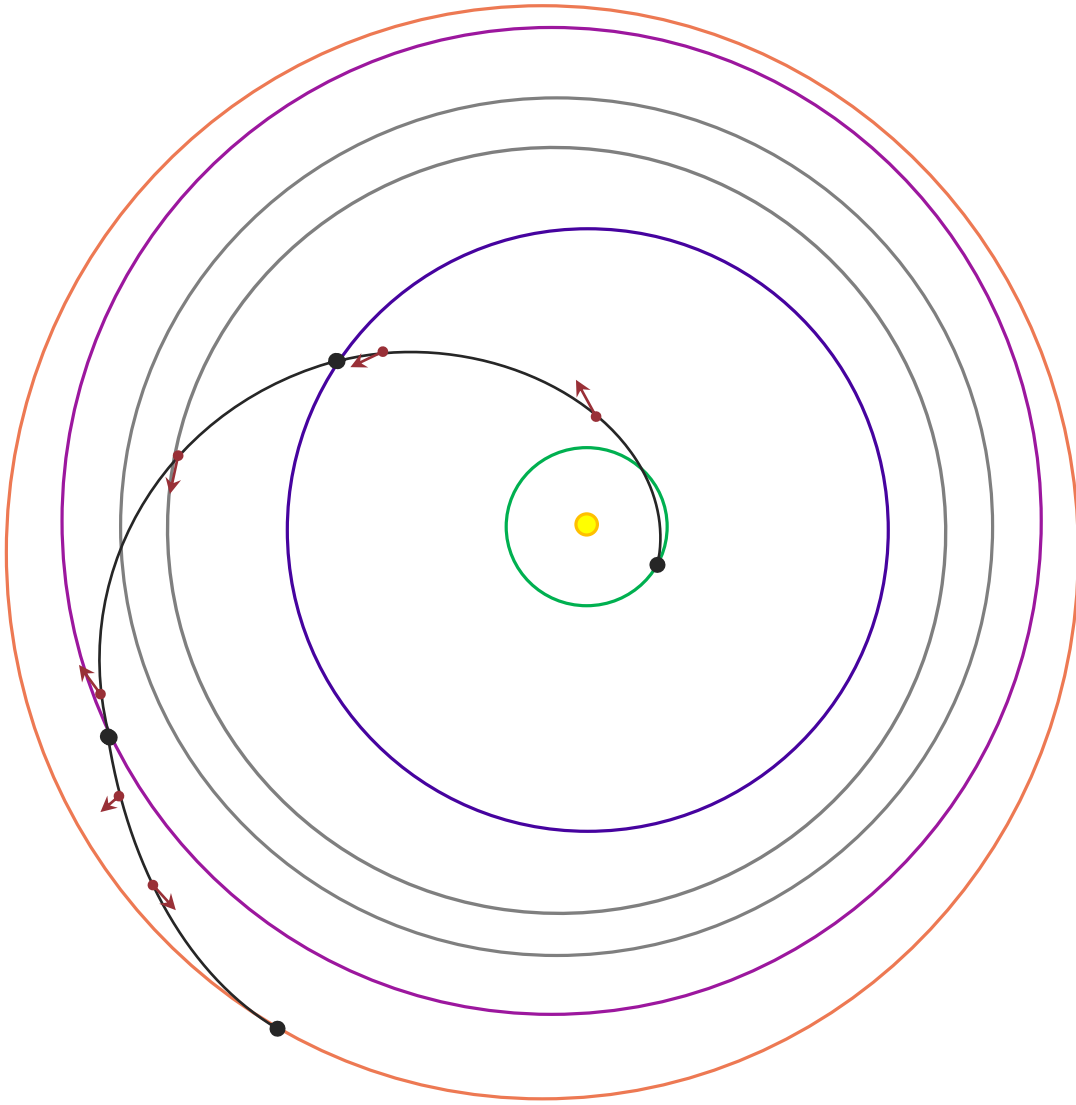
- For each asteroid in high mass delivered set
 - Find lowest ΔV Lambert solution from Earth
 - Must depart within first year
 - Converge full fidelity flyby
 - Two ΔV s per leg
- Loop
 - Estimate ΔV to all other asteroids in high mass set
 - Lambert solver
 - Select minimum n Lambert ΔV asteroids
 - Solve n full fidelity OCPs with these asteroids as next target
 - Two ΔV s per leg
 - Kepler propagator shooting scheme
 - Upper bound on tour TOF
 - Lowest ΔV solution becomes new sequence
- Repeat until TOF limit reached

Sequence Construction



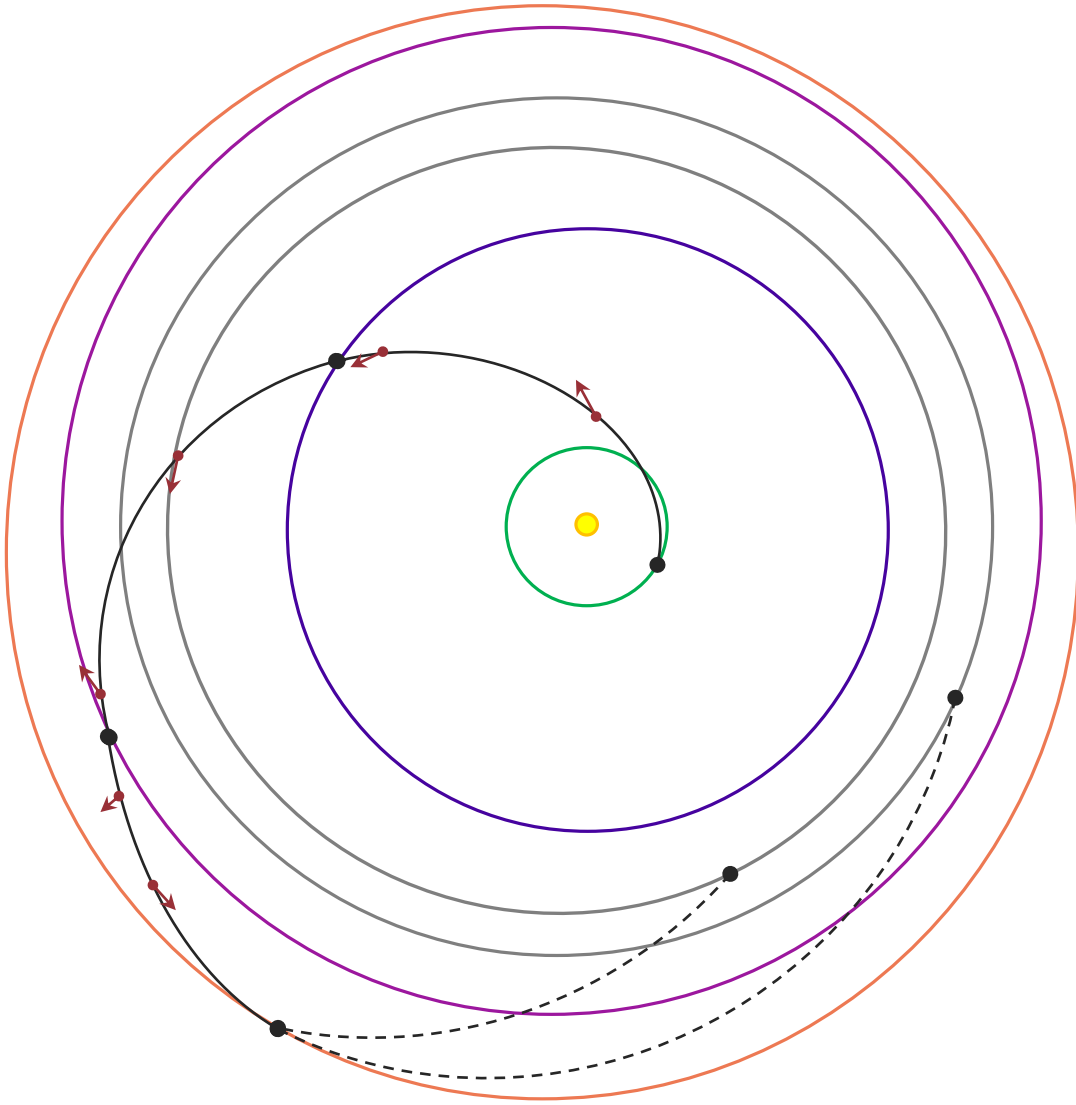
- For each asteroid in high mass delivered set
 - Find lowest ΔV Lambert solution from Earth
 - Must depart within first year
 - Converge full fidelity flyby
 - Two ΔV s per leg
- Loop
 - Estimate ΔV to all other asteroids in high mass set
 - Lambert solver
 - Select minimum n Lambert ΔV asteroids
 - Solve n full fidelity OCPs with these asteroids as next target
 - Two ΔV s per leg
 - Kepler propagator shooting scheme
 - Upper bound on tour TOF
 - Lowest ΔV solution becomes new sequence
- Repeat until TOF limit reached

Sequence Construction



- For each asteroid in high mass delivered set
 - Find lowest ΔV Lambert solution from Earth
 - Must depart within first year
 - Converge full fidelity flyby
 - Two ΔV s per leg
- Loop
 - Estimate ΔV to all other asteroids in high mass set
 - Lambert solver
 - Select minimum n Lambert ΔV asteroids
 - Solve n full fidelity OCPs with these asteroids as next target
 - Two ΔV s per leg
 - Kepler propagator shooting scheme
 - Upper bound on tour TOF
 - **Lowest ΔV solution becomes new sequence**
- Repeat until TOF limit reached

Sequence Construction



➤ For each asteroid in high mass delivered set

➤ Find lowest ΔV Lambert solution from Earth

- Must depart within first year

➤ Converge full fidelity flyby

- Two ΔV s per leg

➤ Loop

➤ Estimate ΔV to all other asteroids in high mass set

➤ Lambert solver

➤ Select minimum n Lambert ΔV asteroids

➤ Solve n full fidelity OCPs with these asteroids as next target

- Two ΔV s per leg

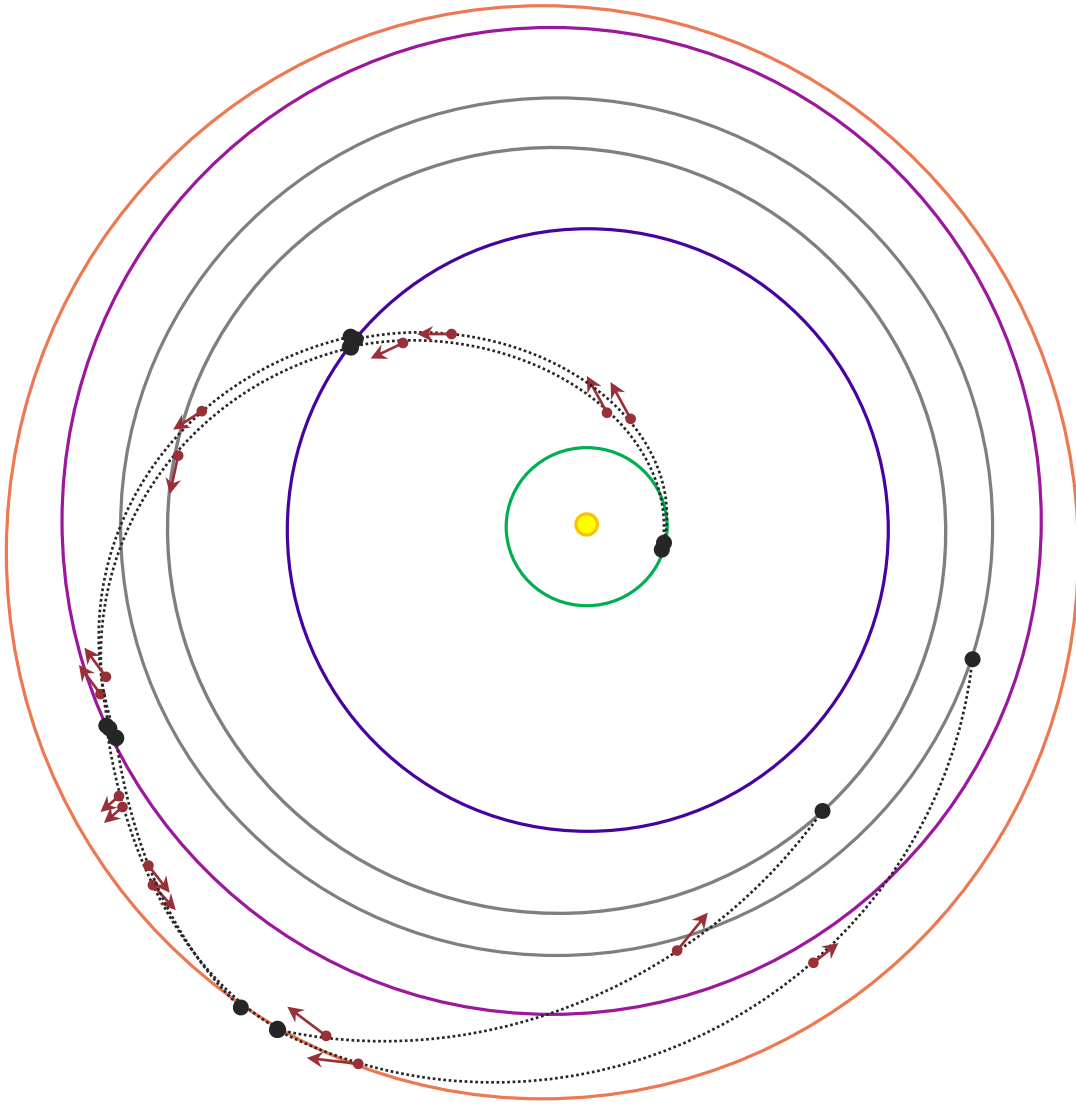
- Kepler propagator shooting scheme

- Upper bound on tour TOF

➤ Lowest ΔV solution becomes new sequence

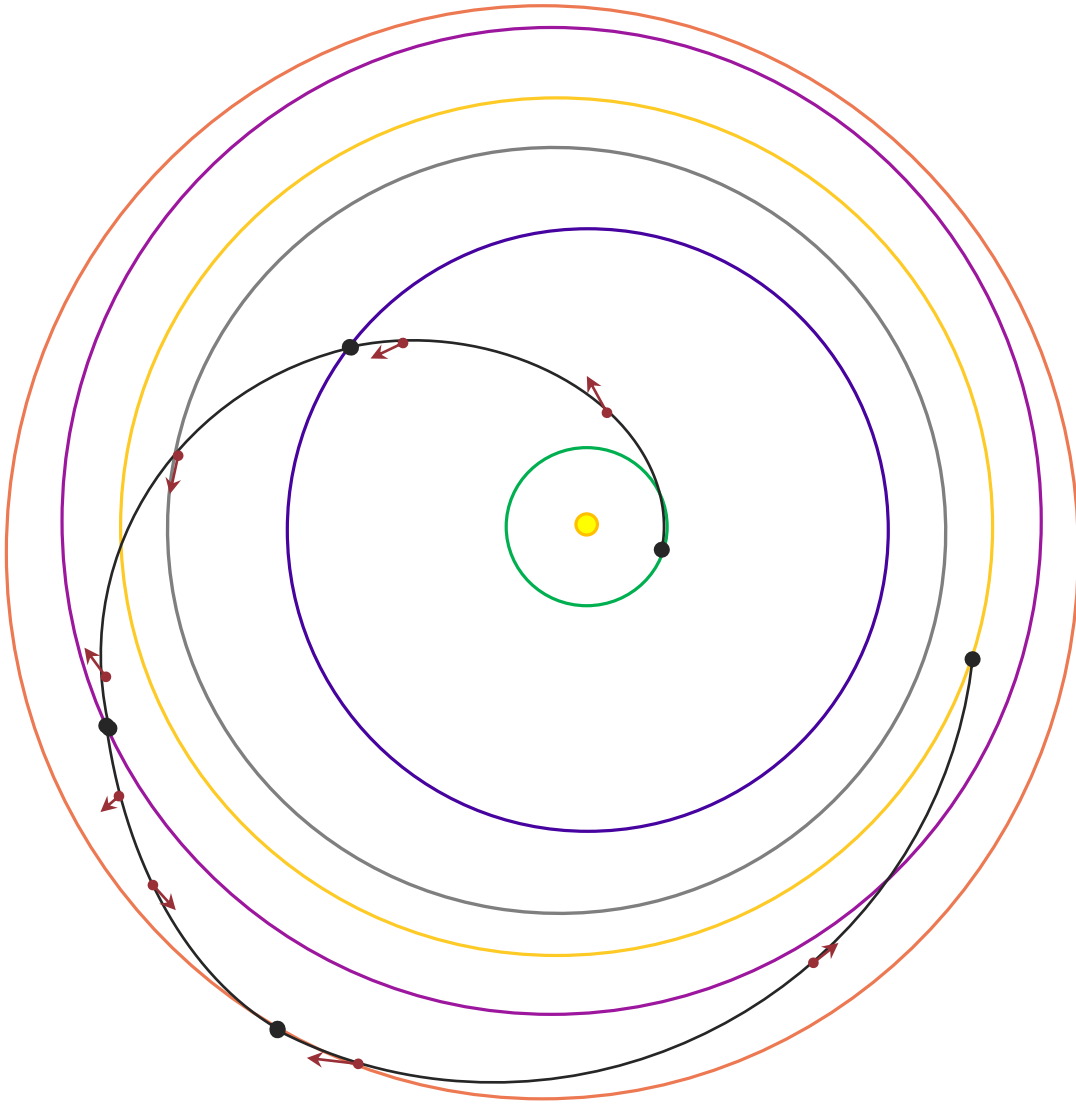
➤ Repeat until TOF limit reached

Sequence Construction



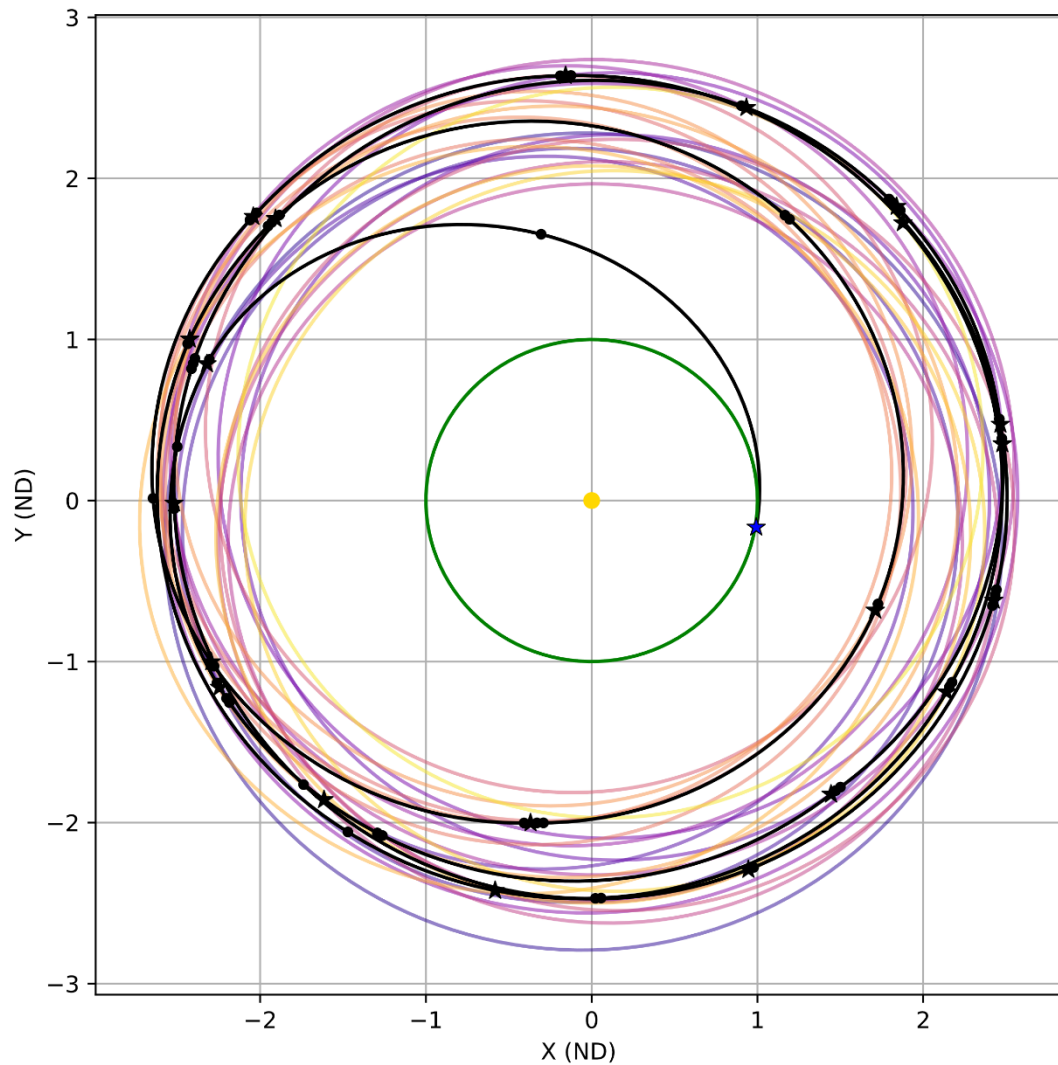
- For each asteroid in high mass delivered set
 - Find lowest ΔV Lambert solution from Earth
 - Must depart within first year
 - Converge full fidelity flyby
 - Two ΔV s per leg
 - Loop
 - Estimate ΔV to all other asteroids in high mass set
 - Lambert solver
 - Select minimum n Lambert ΔV asteroids
 - Solve n full fidelity OCPs with these asteroids as next target
 - Two ΔV s per leg
 - Kepler propagator shooting scheme
 - Upper bound on tour TOF
 - Lowest ΔV solution becomes new sequence
- Repeat until TOF limit reached

Sequence Construction



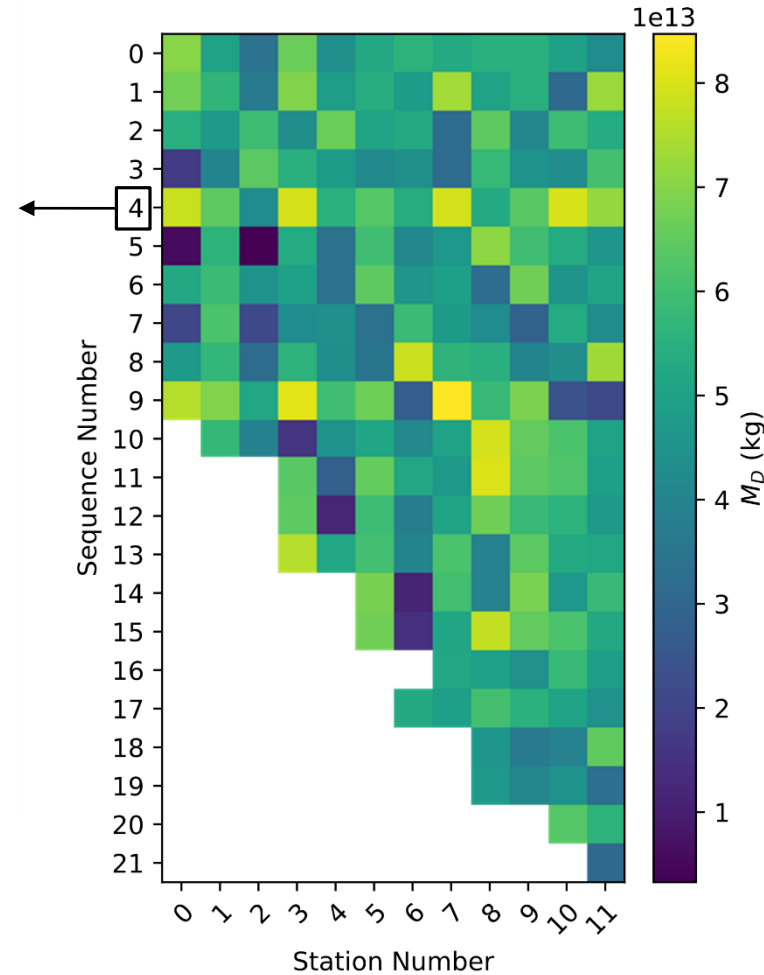
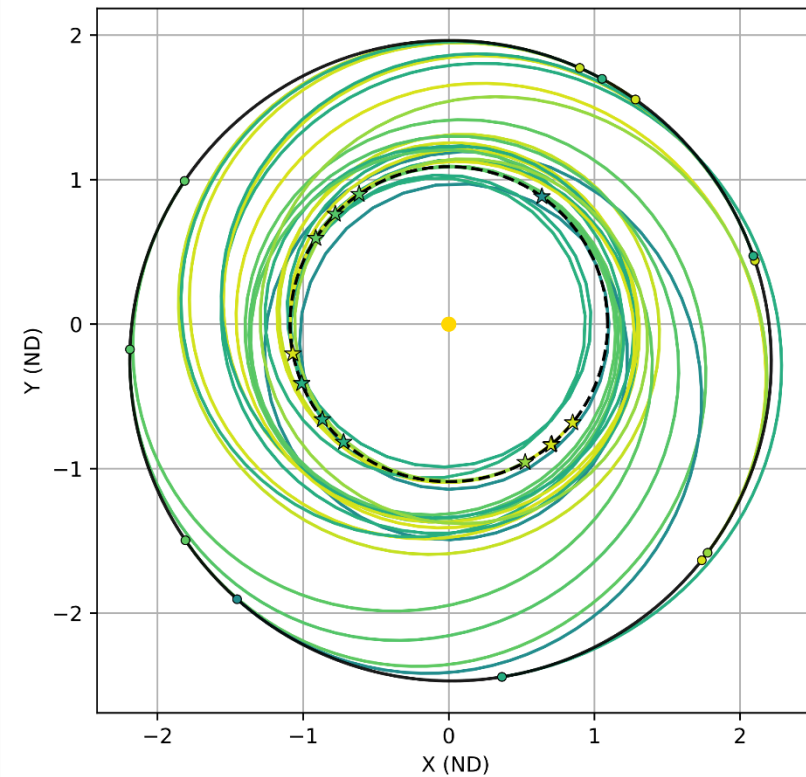
- For each asteroid in high mass delivered set
 - Find lowest ΔV Lambert solution from Earth
 - Must depart within first year
 - Converge full fidelity flyby
 - Two ΔV s per leg
 - Loop
 - Estimate ΔV to all other asteroids in high mass set
 - Lambert solver
 - Select minimum n Lambert ΔV asteroids
 - Solve n full fidelity OCPs with these asteroids as next target
 - Two ΔV s per leg
 - Kepler propagator shooting scheme
 - Upper bound on tour TOF
 - **Lowest ΔV solution becomes new sequence**
 - **Repeat until TOF limit reached**
- Deterministic wrt: target # flybys, asteroid set, TOF limit, n
 - Calculate all possible solutions

Sequence Construction Results



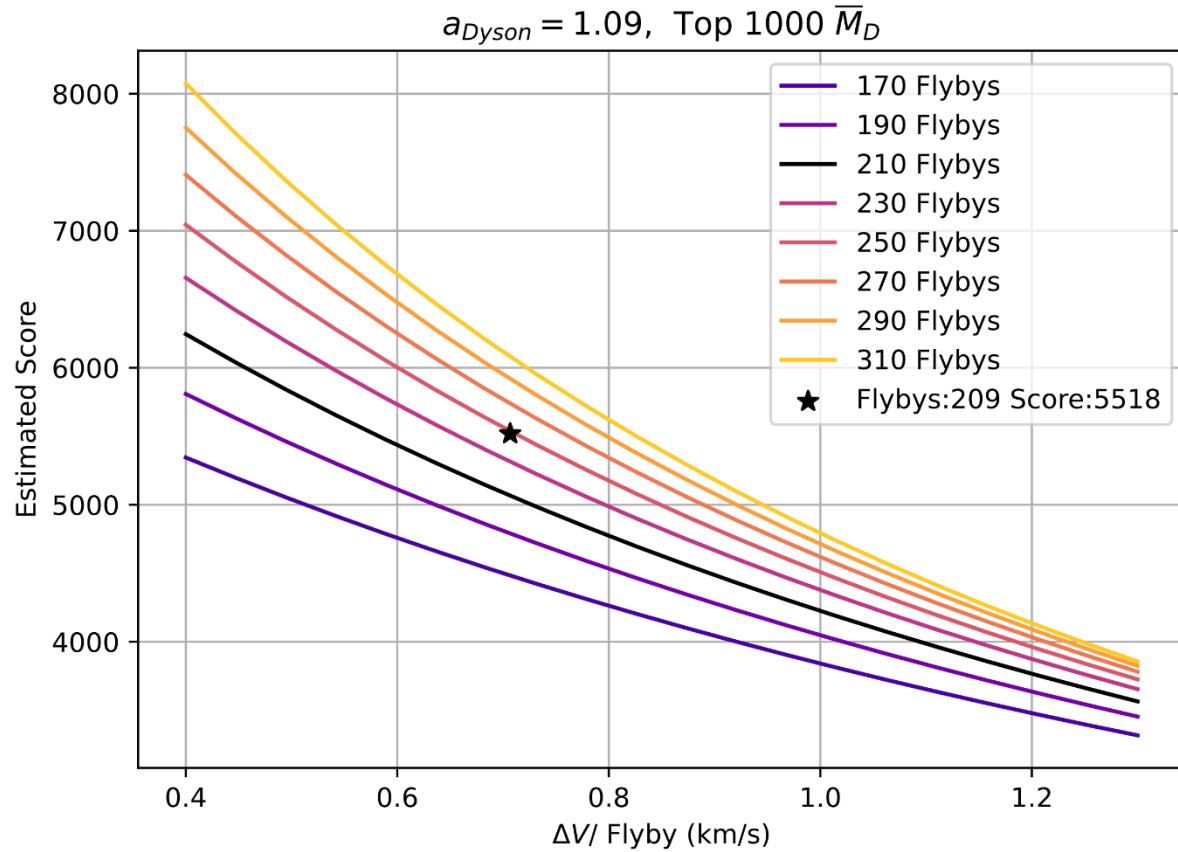
- Computed 1000s of sequences through high-mass sets
 - Targeted 20 – 24 flybys per tour
 - $n=8$ breadth at each stage in beam search
 - 6-10 search instances running on 2 desktops
 - 16-20 full solutions per minute at peak throughput
 - Approx. 3000 OCPs per minute (170-180 per full solution)
- 20% met ΔV target of .75 km/s per flyby
 - Pass on to transfer construction phase

Transfer Construction



- For each asteroid in sequence compute transfers to as many stations as possible
 - Evenly spaced build times over final 9 years
 - Maximize bin-packing mobility of asteroids in solution construction phase

Solution Construction



- Combine 10 tours into final solution
 - Delete least efficient duplicate asteroid (if any)
 - Select-best station for each asteroid
 - Bin-pack to increase minimum station mass
- Final solution scored better than estimated
 - Better than average mass delivered

Final Solution / Conclusion

Video Removed due
to size restrictions

- Final score 5525 w/ bonus
 - 209 asteroids delivered
 - 1.10×10^{15} kg minimum mass
- Possible improvements to our solution
 - Increase the length of lambert sequences in tour search
 - More asteroids per tour at same ΔV
 - Add in light asteroids that almost encounter a mothership
- Really enjoyed the competition, can't wait for GTOC 12

