2021.12

11th Global Trajectory Optimization Competition: Results Found At NUAA

ASTL-NUAA

Nanjing University of Aeronautics and Astronautics

NUAA 航天学院

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ANALYSIS

Multi-targets flyby impulse sequence optimization

Continuous-thrust time-optimal trajectory optimization

Programming algorithm based on greedy principle

Make the total ΔV lower

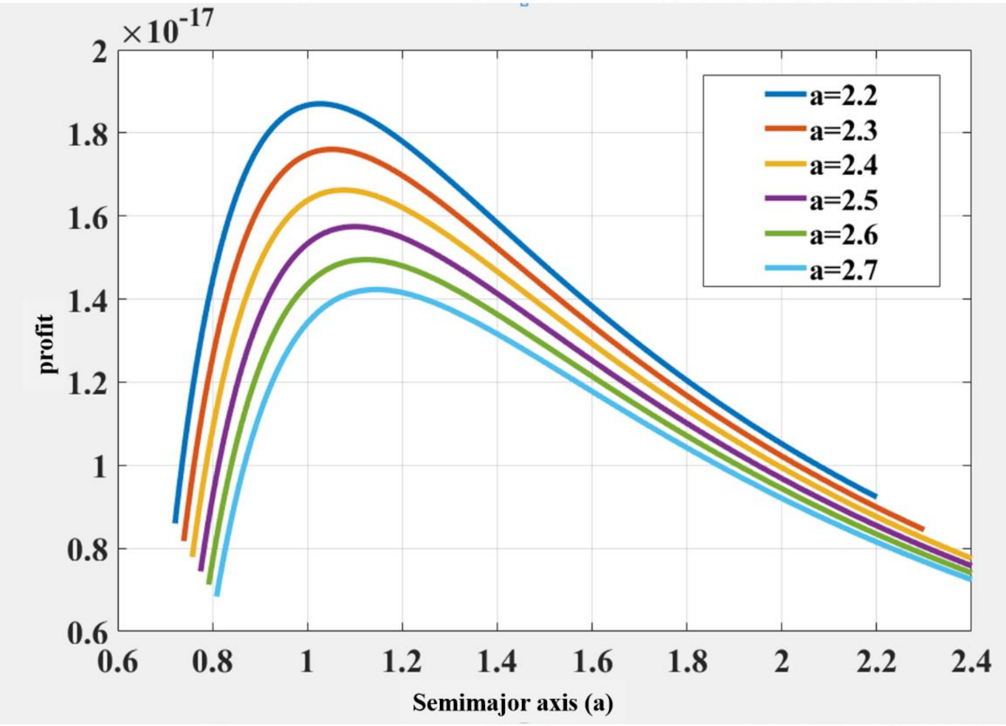
Optimize orbit elements of “Dyson Ring”

Final solution

Initial solution

Improving

ANALYSIS



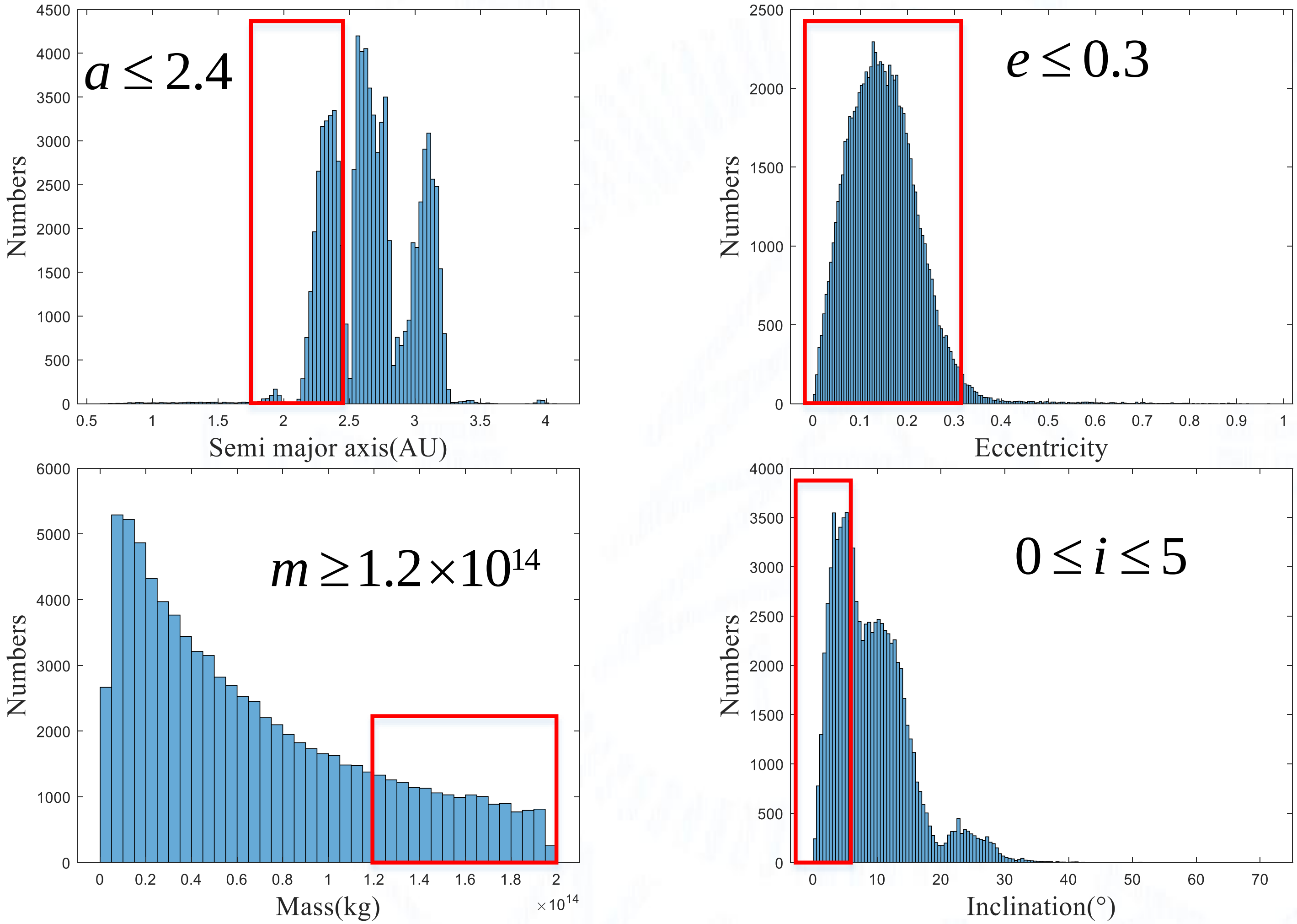
$$C = \frac{1}{2}v^2 - \frac{\mu}{r}$$

$$J = \frac{M}{a^2_{Dyson}}$$

The semi major axis of station is about 1 to 1.2 AU.

So, we assumed it is 1.1 AU.

ANALYSIS



1045
asteroids

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Dynamic model

The dynamic model considered in the problem is as follow:

$$\begin{cases} \dot{\boldsymbol{r}} = \boldsymbol{v} \\ \dot{\boldsymbol{v}} = -\dfrac{\mu}{r^3}\boldsymbol{r} \end{cases}$$

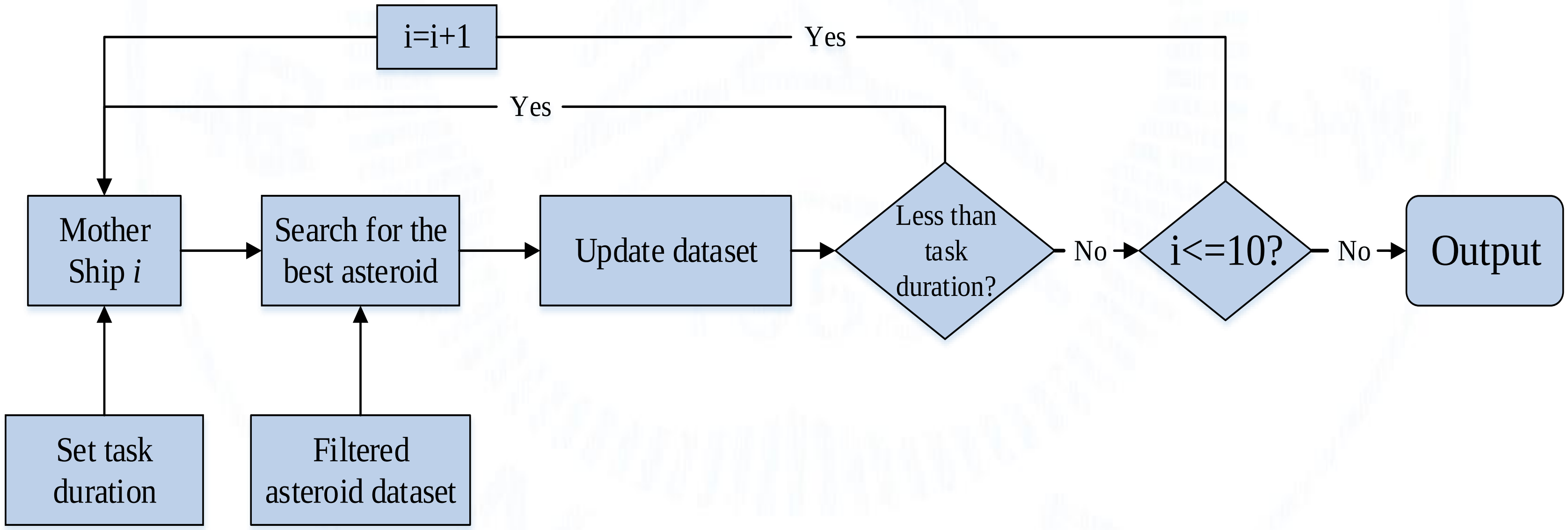
The Mother Ship should meet the following conditions , when flying over an asteroid

$$\begin{cases} \left\| \boldsymbol{r}^{mother}(t) - \boldsymbol{r}^{ast}(t) \right\| \le 1\text{km} \\ \left\| \boldsymbol{v}^{mother}(t) - \boldsymbol{v}^{ast}(t) \right\| \le 2\text{km/s} \end{cases}$$

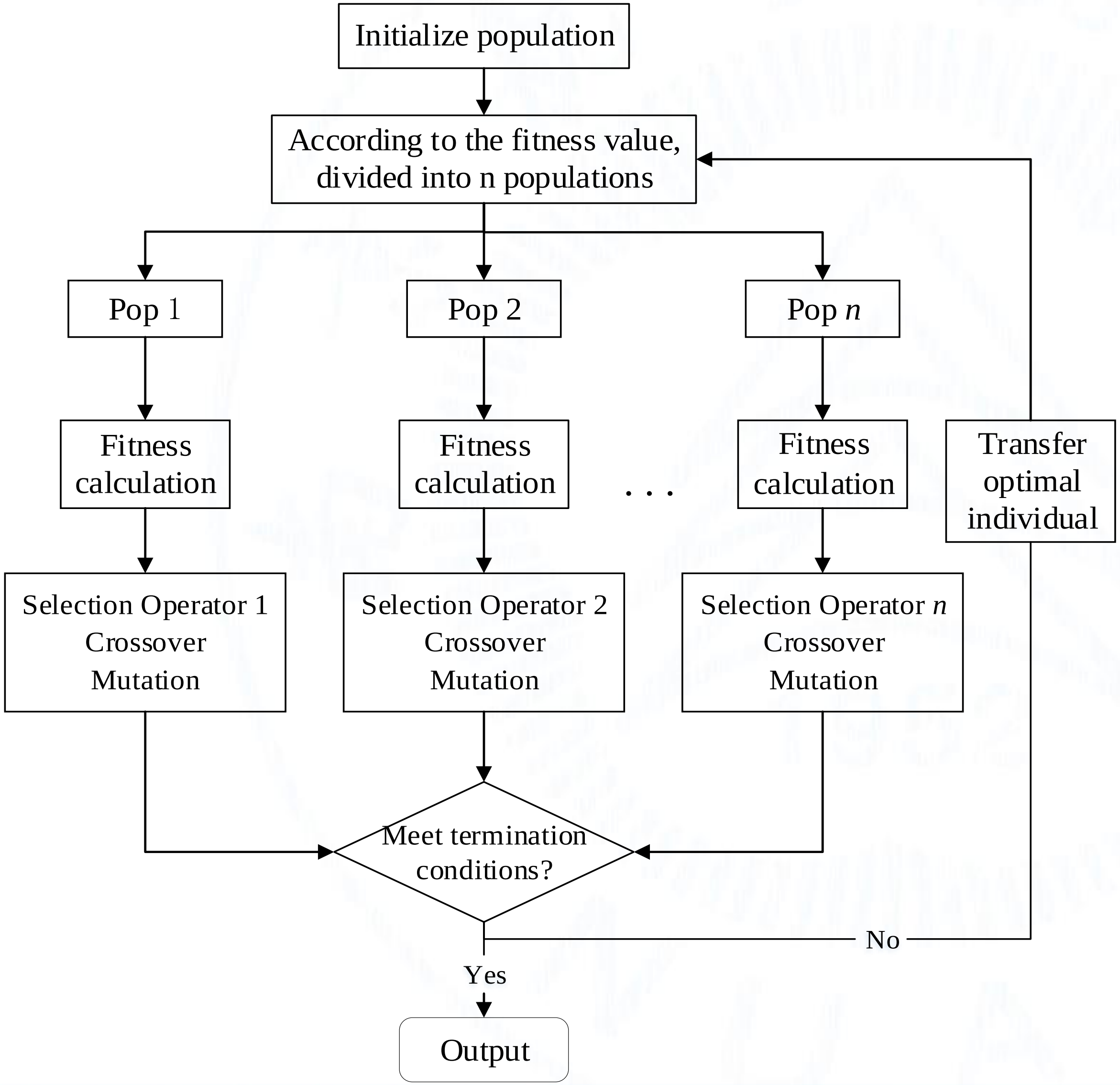
In the J2000 heliocentric ecliptic coordinate , the position error can be ignored, so, it can be treated as a two body **Lambert problem** with relative velocity difference.

Strongly nonlinear, NP-hard.

Genetic algorithm based on the **greedy principle**



MULTI-TARGETS FLYBY SEQUENCE OPTIMIZATION



Optimization variables:

Asteroid number N

Waiting time t_w

Transfer time t_f

Parameters:

Iteration number :1500

Binary encoding

Encoding bit length:100

Population number: 100

Community number: 6

Crossover probability: 0.9

Mutation probability: 0.5

Selection operators :

Roulette

Tournament

Random sampling

Design of Fitness Function

$$fit_{\min}=\omega_1\left(\frac{50+dv_i+dv_e}{50+dV_{\max}}\right)^2+\omega_2\frac{t_f+t_w}{T}+\omega_3\frac{m}{m_{\max}}$$

$$fit_{\min}=\omega_1\left(1+\frac{dv_i+dv_e}{50}\right)^2+\omega_2\frac{t_f+t_w}{T}$$

$$\Rightarrow\quad fit_{\min}=0.99\left(\frac{50+dv_i+dv_e}{50+dV_{\max}}\right)^2+0.01\frac{t_f+t_w}{T}$$

$$fit_{\min}=\omega_1\frac{\left(1+\frac{dv_i+dv_e}{50}\right)^2}{10^{-16}m}+\omega_2\frac{t_f+t_w}{T}$$

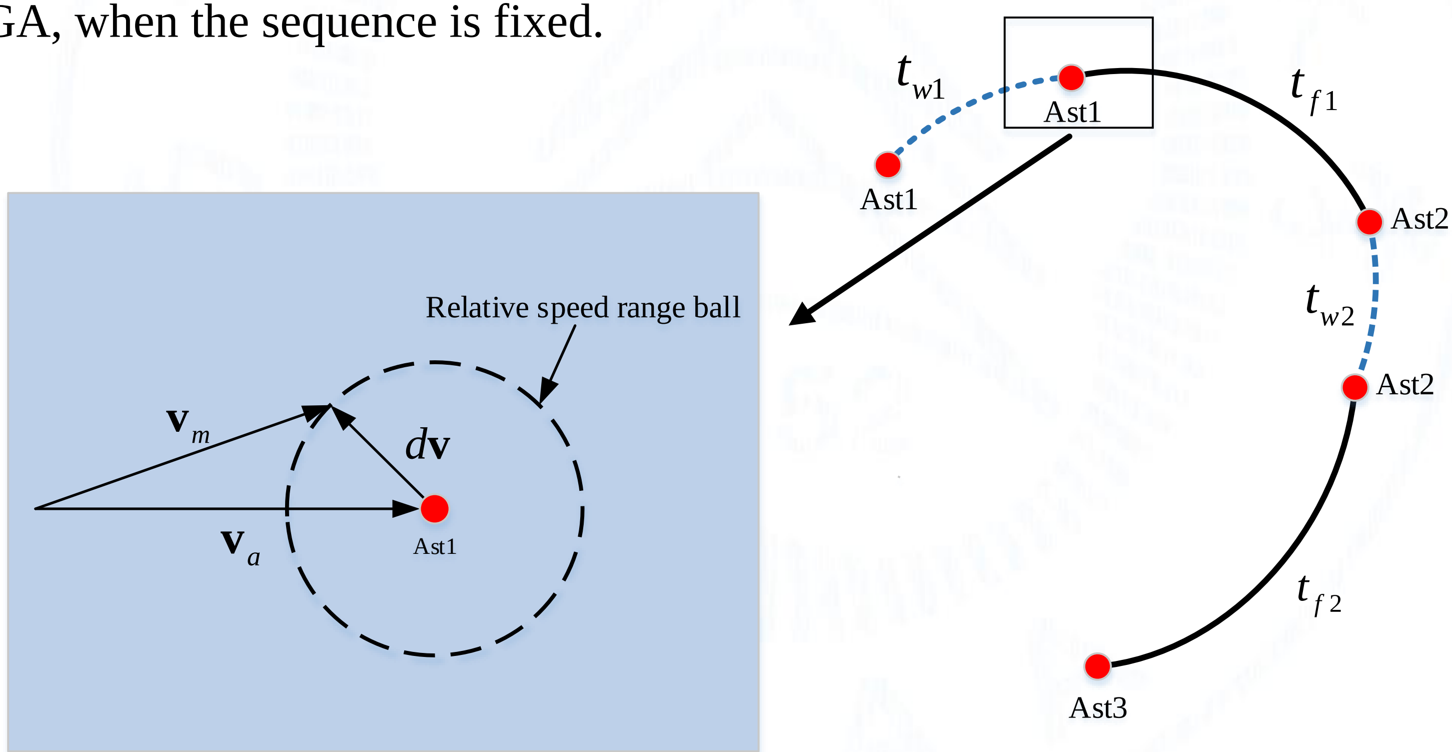
Fitness function is designed into these three types, which may have the independent variables, such as velocity increment, time and asteroid mass. After adjustment and comparison, the fitness is fixed.

Result of the first problem

Mother Ship	Number of Asteroids visited	Impulses Times	Total Impulse(km/s)
1	20	26	21.244
2	20	28	20.743
3	24	29	29.964
4	24	27	29.325
5	21	23	26.941
6	22	29	22.514
7	20	25	24.634
8	22	29	23.582
9	19	22	21.713
10	21	33	30.234

Impulse Re-optimization

In the previous results, the relative velocity is fixed to be 2 km/s for reduce the optimization variables. A new series of optimization variables, [d**v**, *tw*, *tf*], are adopted for the GA, when the sequence is fixed.



Indirect method

Equinoctial elements

$$p = a\left(1-e^2\right) \qquad h = \tan(i/2)\cos\Omega$$
$$f = e\cos(\omega+\Omega) \qquad k = \tan(i/2)\sin\Omega$$
$$g = e\sin(\omega+\Omega) \qquad L = \Omega+\omega+v$$

Dynamic model

$$\dot{\boldsymbol{x}} = \boldsymbol{M\alpha} + \boldsymbol{D}$$

Costate equation

$$\dot{\lambda} = -\frac{\partial H}{\partial \boldsymbol{x}} = -\left(\lambda^T \frac{\partial \boldsymbol{M}}{\partial \boldsymbol{x}}\boldsymbol{\alpha} + \lambda^T \frac{\partial \boldsymbol{D}}{\partial \boldsymbol{x}}\right)$$

Shooting equation

$$\begin{bmatrix} \boldsymbol{x}\left(t_f\right)-\boldsymbol{x}_f \\ \left\|\boldsymbol{\lambda}\left(t_0\right)\right\|-1 \\ H\left(t_f\right)-\lambda_x\left(t_f\right)\dot{\boldsymbol{x}}_f \end{bmatrix} = \boldsymbol{0}$$

rendezvous

Indirect method

Equinoctial elements

$$p = a\left(1-e^2\right) \qquad h = \tan(i/2)\cos\Omega$$

$$f = e\cos(\omega+\Omega) \qquad k = \tan(i/2)\sin\Omega$$

$$g = e\sin(\omega+\Omega) \qquad L = \Omega+\omega+v$$

Dynamic model

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Shooting equation

$$\begin{bmatrix} \left(\boldsymbol{x}\left(t_f\right) - \boldsymbol{x}_f\right)\circ \boldsymbol{B}_x \\ \lambda\left(t_f\right)\circ \boldsymbol{B}_\lambda \\ H\left(t_f\right) \end{bmatrix} = \boldsymbol{0}$$

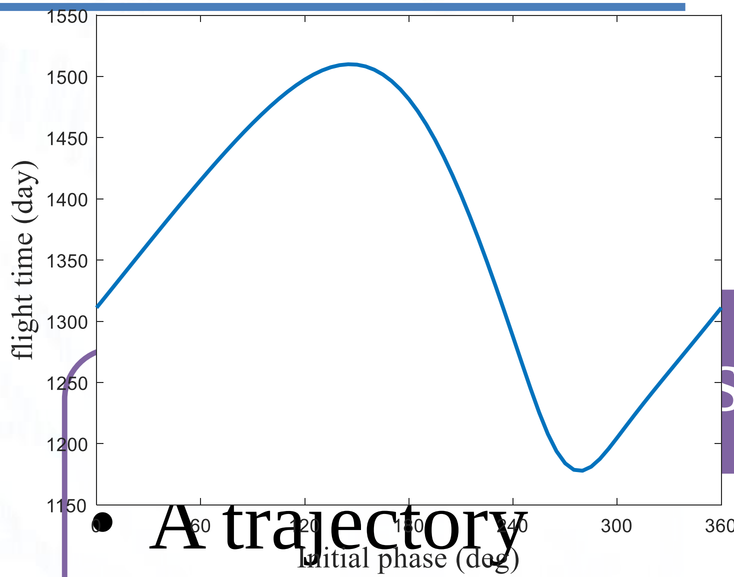
Insert orbit

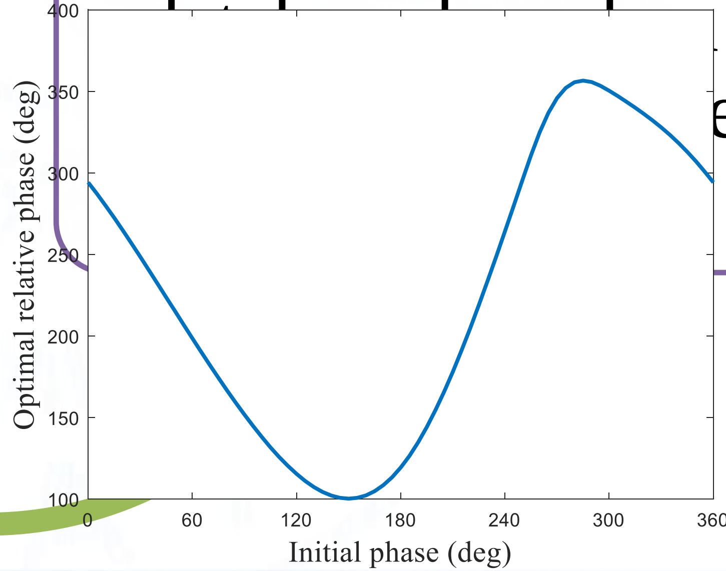
Indirect method

- discretize the asteroid orbits in phase
- the initial database.

Insert orbit

- A trajectory



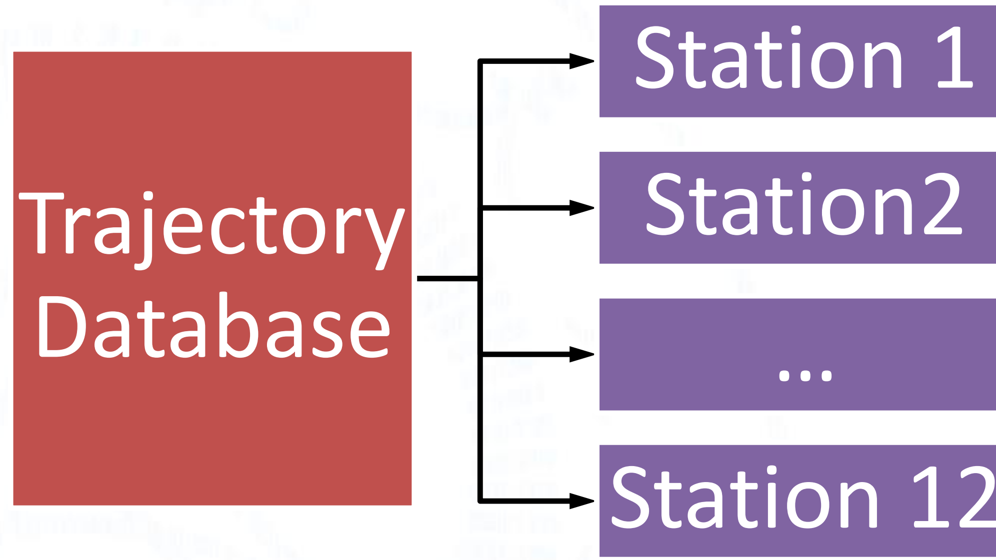


PROGRAMMING ALGORITHM BASED ON GREEDY PRINCIPLE

Determine

1.Asteroid is sent to which station

2.The start and end time of station constructions



the time constraint:

$$t_{i-end} + 90 \le t_{i+1-star} \quad (days)$$

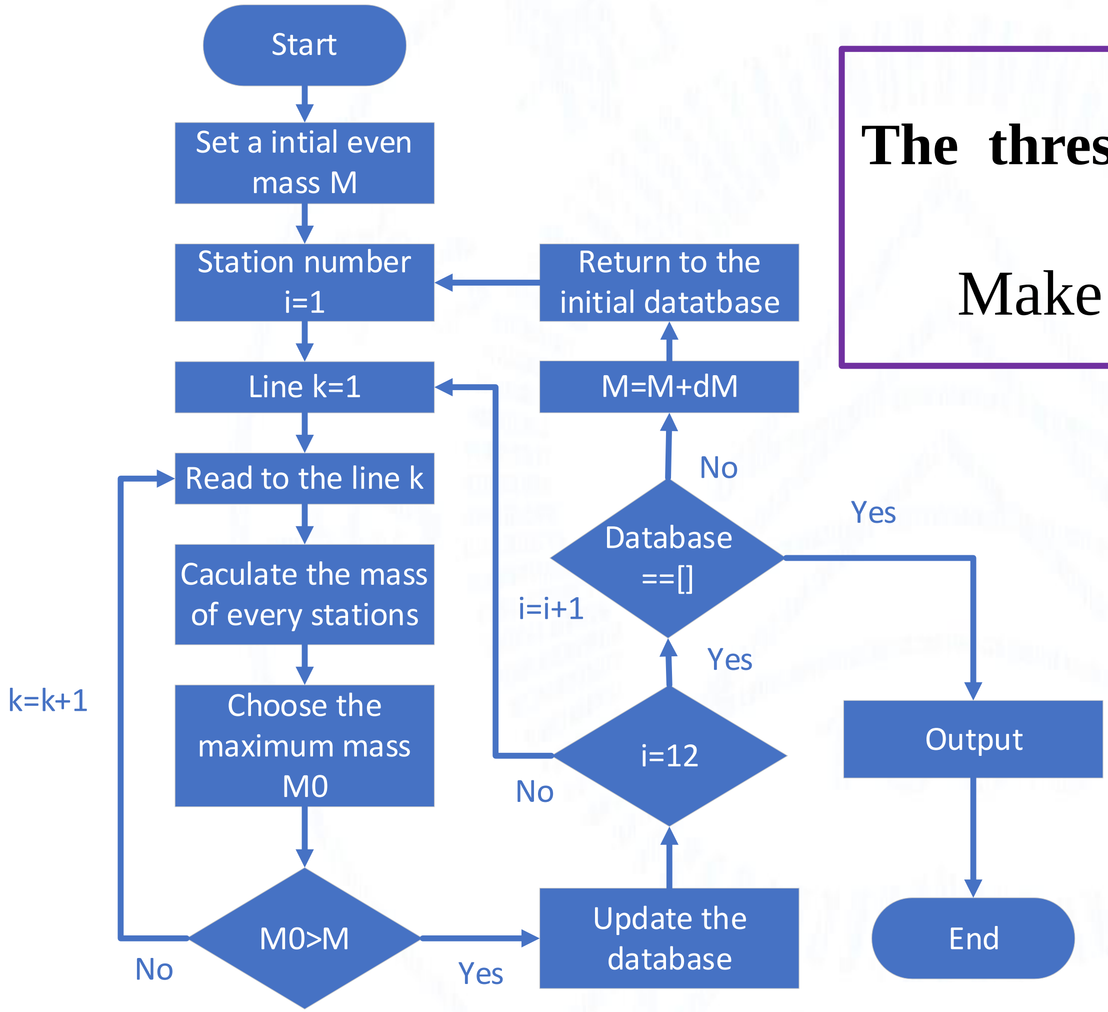
→ Sort the database with arrival time

Maximize the minimum mass of stations:

Max（M_{min}）

→ Make the station mass average

PROGRAMMING ALGORITHM BASED ON GREEDY PRINCIPLE



Algorithm flowchart

The threshold value of stations mass M :

Make the mass distribution average

Greedy principle:

Building the station whose mass reaches M fastest.

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RESULTS

ASTL-NUAA	7	Nov 07, 2021 12:08 AM UTC	Nov 07, 2021 12:08 AM UTC	1.03068e+15, 213	3735.160200

Station	start（JD）	ending（JD）	M（kg）	Building
1	101321. 02537	101669. 47069	1. 0514e+15	9
2	100712. 15409	101223. 56348	1. 0454e+15	8
3	99893. 87524	100168. 61019	1. 0384e+15	6
4	99141. 47788	99438. 90271	1. 0501e+15	4
5	100279. 44797	100605. 42114	1. 0510e+15	5
6	102703. 48538	103013. 11937	1. 0381e+15	12
7	99569. 02925	99790. 23103	1. 0590e+15	7
8	97891. 22694	98482. 11152	1. 0415e+15	2
9	97695. 61553	97788. 00911	1. 0303e+15	1
10	102179. 20612	102477. 51018	1. 0357e+15	11
11	101797. 09119	102086. 49942	1. 0502e+15	10
12	98734. 20003	98898. 33828	1. 0635e+15	3

CONTENTS

TO BE IMPROVED

- Global optimization

- The orbit is fixed with a=1.1 AU, and the inclination is an average value of all the candidate asteroids.

- Phases of stations need optimization.

- A four impulse transfer maybe save the velocity increment.

航天学院

THANKS