Methods and Results from the Team BIT&ITNS

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Report Contents

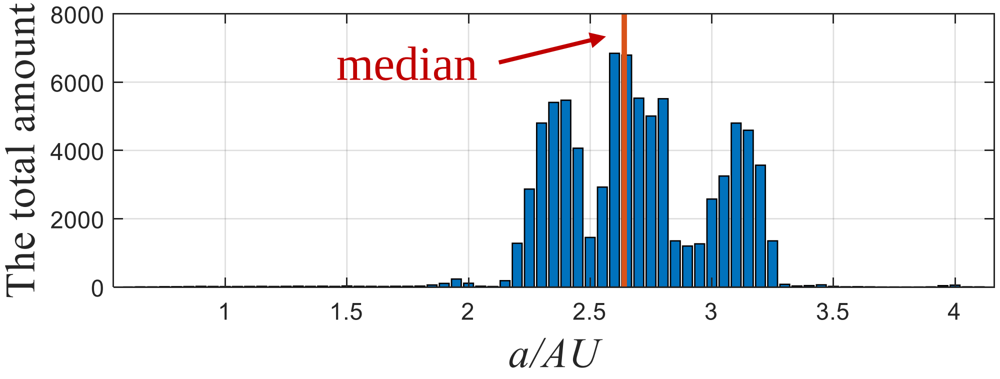
1 Preliminary analysis

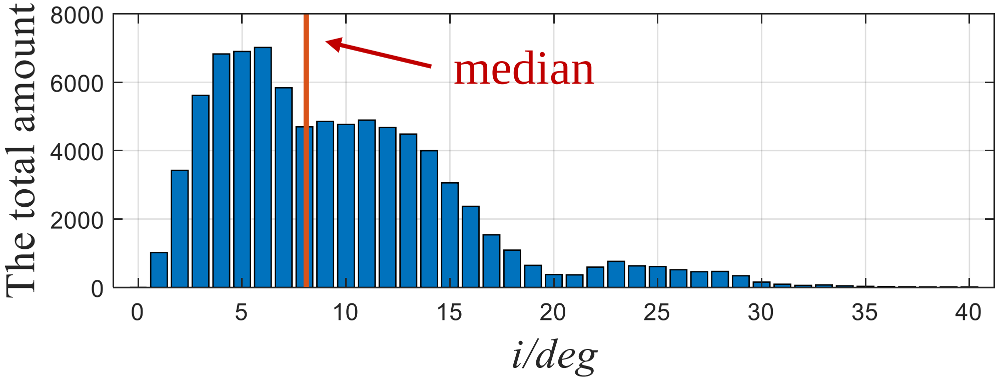
Candidate analysis

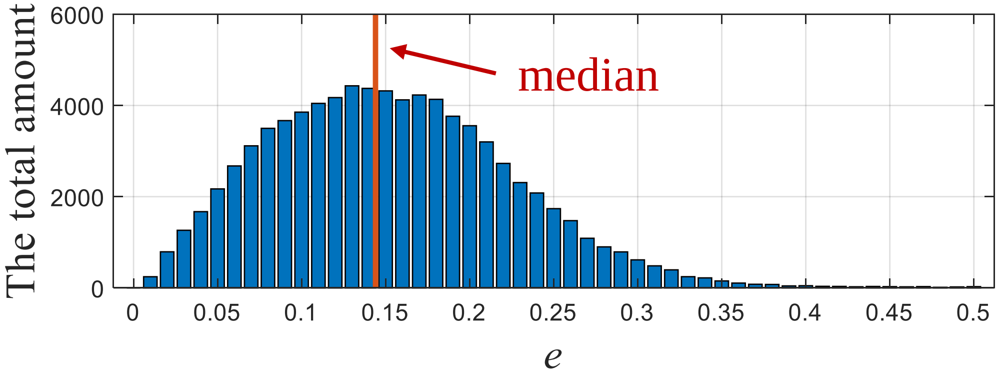
☐ The **semi-major** axis is concentrated in the range of 2.2-3.3.

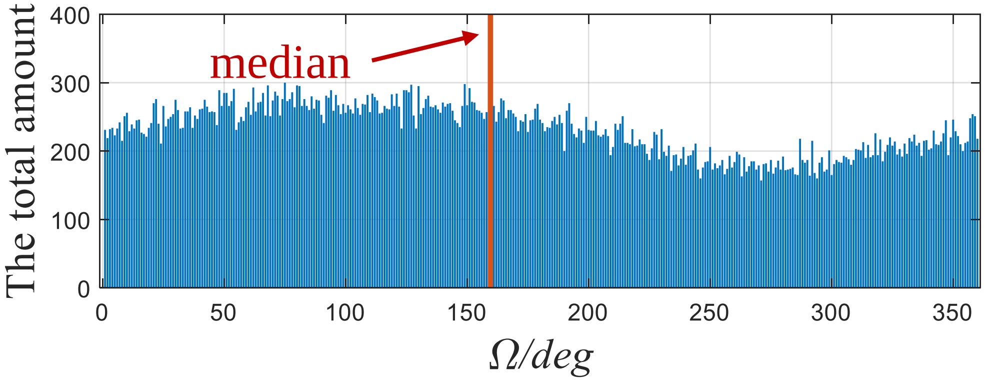
☐ The **eccentricity** is concentrated in the range of 0-0.3.

☐ The **inclination** is concentrated in the range of 0-20°.









1 Preliminary analysis

Candidate analysis

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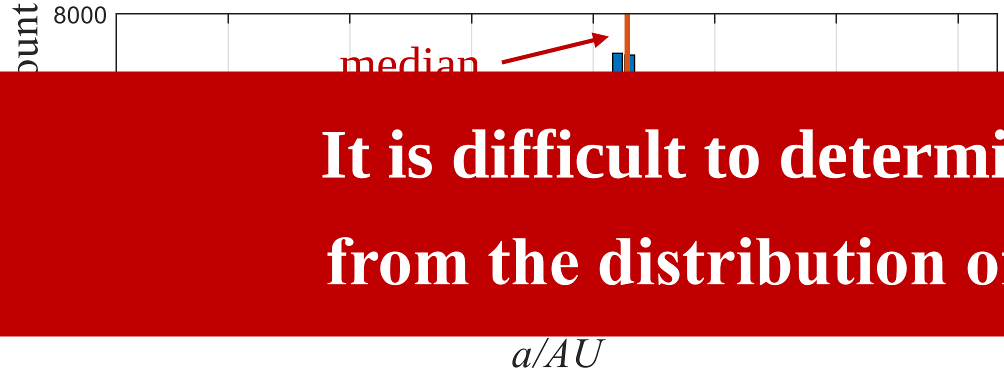
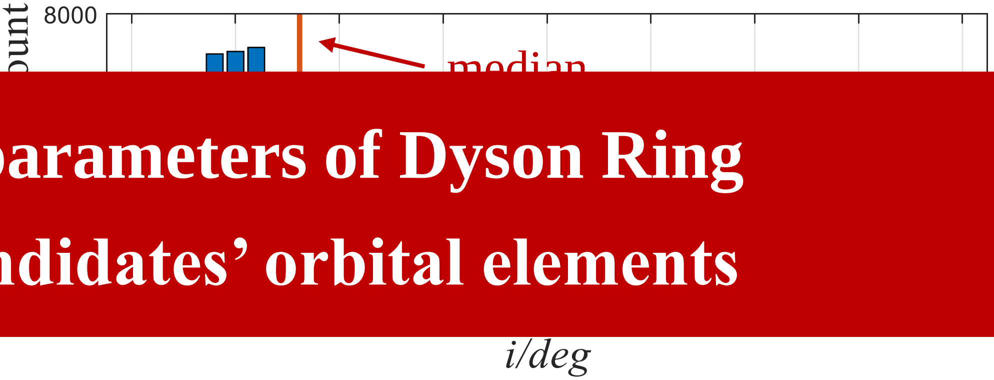
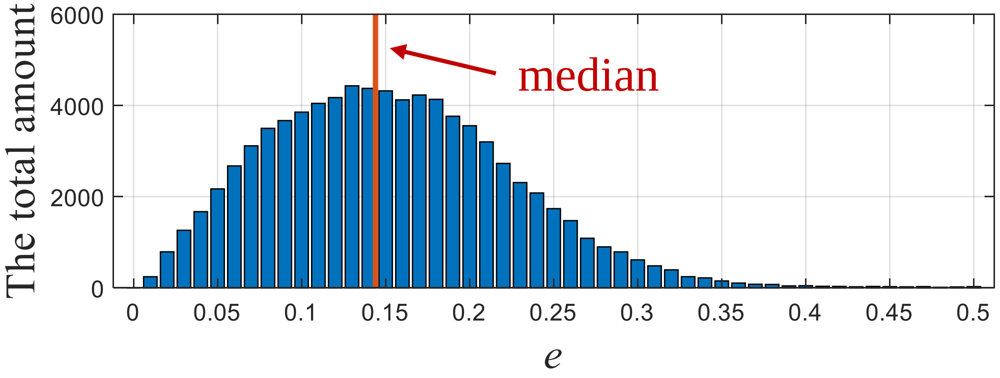
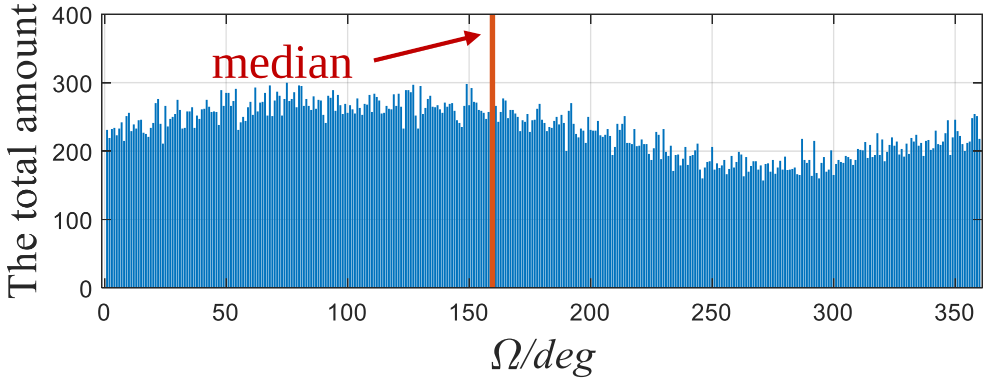




It is difficult to determine the parameters of Dyson Ring from the distribution of the candidates’ orbital elements





1 Preliminary analysis

Performance index analysis

$$J = B \cdot \frac{10^{-10} \cdot M_{\min}}{a_{Dyson}^{2} \sum_{k=1}^{10} \left(1 + \Delta V_{k}^{Total} / 50\right)^{2}}$$

☐ Design the Dyson Ring orbit elements.

Semi-major axis? inclination?

☐ Minimize the sum of ΔV.

Mother Ship flyby sequence? Local optimization method?

☐ Maximize $M_{\min}$, $M_{\min} = \min\{M_j \mid j = 1, 2, \ldots, 12\}$.

Asteroid departure epoch?   Station building time?

- **Design the Dyson Ring orbit elements**
- **Design the mothership flyby sequence**
- **Design the asteroid departure epoch**

2 Design the Dyson Ring elements

Grid search

$a = 1.0 \sim 1.4\text{AU}, \quad i = 0 \sim 14°$

Free phase time-optimal problem for each asteroid

Choose *a*, *i* according to the characteristic mass

Indirect Trajectory Optimization Using Modified Equinoctial Elements

MEE equation $$\boldsymbol{x} = \boldsymbol{M}\frac{T}{m}\boldsymbol{\alpha} + \boldsymbol{D}$$

Hamiltonian $$H = \boldsymbol{\lambda}^T\left(\boldsymbol{M}\frac{T}{m}\boldsymbol{\alpha} + \boldsymbol{D}\right)$$

Costate equation $$\boldsymbol{\lambda} = -\frac{\partial H}{\partial \boldsymbol{x}} = -\left(\boldsymbol{\lambda}^T\frac{\partial \boldsymbol{M}}{\partial \boldsymbol{x}}\frac{T}{m}\boldsymbol{\alpha} + \boldsymbol{\lambda}^T\frac{\partial \boldsymbol{D}}{\partial \boldsymbol{x}}\right)$$

Optimal thrust direction $$\boldsymbol{\alpha}^* = -\frac{\left(\boldsymbol{\lambda}^T\boldsymbol{M}\right)^T}{\left\|\boldsymbol{\lambda}^T\boldsymbol{M}\right\|}$$

***L*(*t*$_f$) free**

Solved by *cminpack* solver

$$\begin{cases} p(t_f) - p_f = 0 \\ f(t_f) - f_f = 0 \\ g(t_f) - g_f = 0 \\ h(t_f) - h_f = 0 \\ k(t_f) - k_f = 0 \\ \lambda_L(t_f) = 0 \\ H(t_f) + 1 = 0 \end{cases}$$

Shooting function

2 Design the Dyson Ring elements

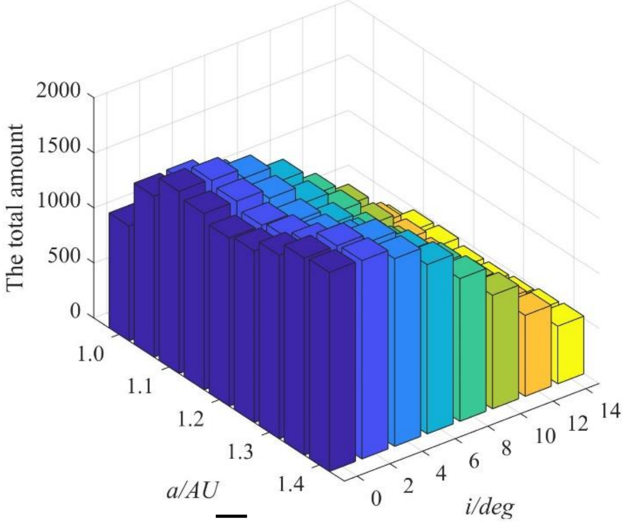
Grid search

a = 1.0~1.4AU, i = 0~14°

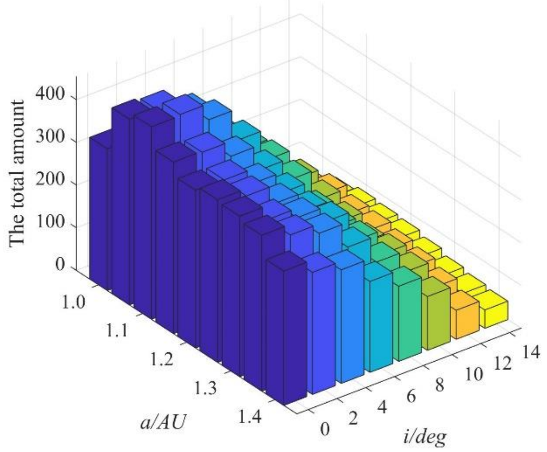
Free phase time-optimal problem for each asteroid

Choose a, i according to the characteristic mass

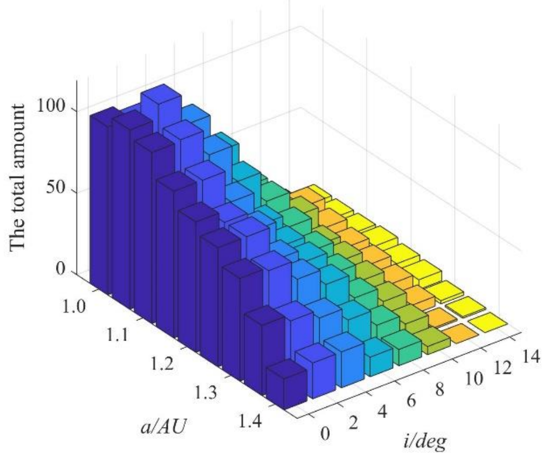
- **Characteristic mass:** $$\bar{M}_f = \frac{10^{-10}\cdot M_f}{a_{Dyson}^2} = \frac{10^{-10}}{a_{Dyson}^2}\left(1-\alpha\Delta t_{\min}\right) m_0^{ast}$$



$$\bar{M}_f \geq 4000$$



$$\bar{M}_f \geq 5000$$



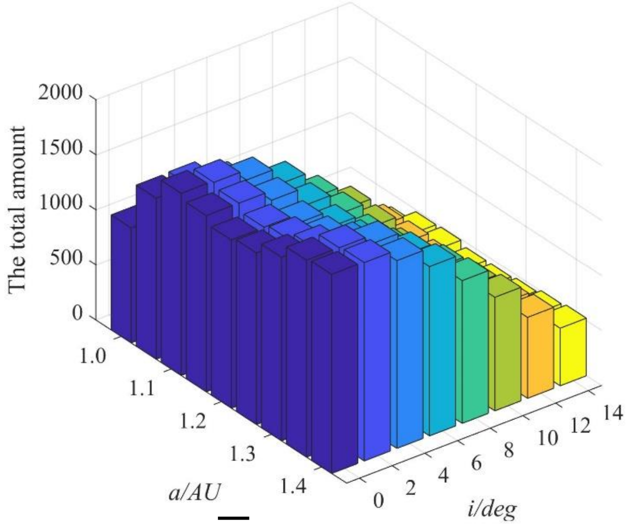
$$\bar{M}_f \geq 6000$$

Assumption: At a suitable departure epoch, the transfer time of the asteroid to each station is near the same.

2 Design the Dyson Ring elements

Choose *a*, *i* according to the characteristic mass

- ✓ The number of asteroids decreases as the orbital inclination increases.
- ✓ The semi-major axis of 1.1 contains more big quality asteroids.



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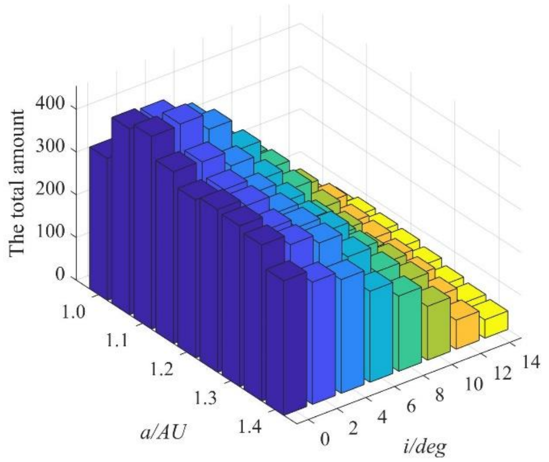

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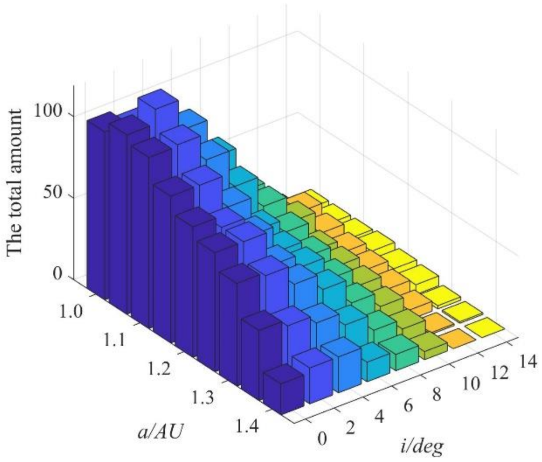

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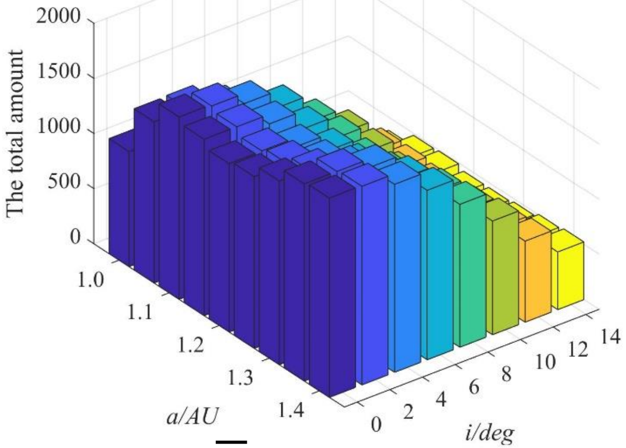
Assumption: At a suitable departure epoch, the transfer time of the asteroid to each station is near the same.

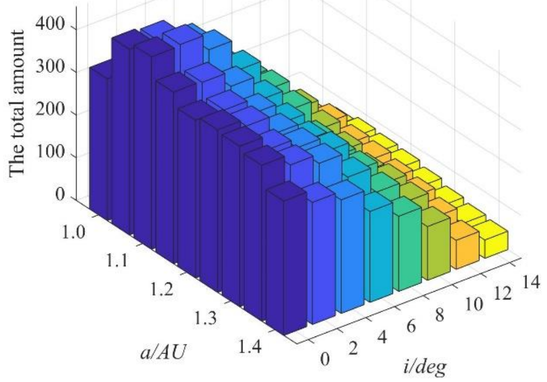
2 Design the Dyson Ring elements

Choose *a*, *i* according to the characteristic mass

Dyson Ring elements: a = 1.1AU, i = 0°



$$\bar{M}_f \geq 4000$$



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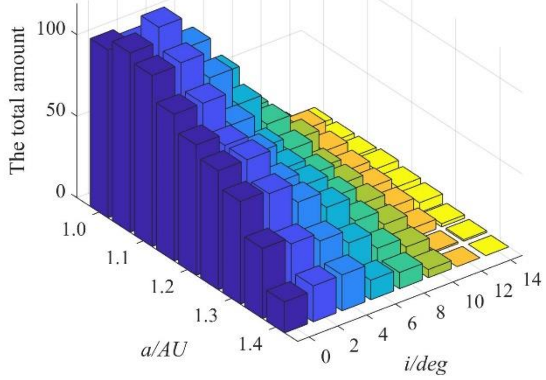


$$\bar{M}_f \geq 6000$$

Assumption: At a suitable departure epoch, the transfer time of the asteroid to each station is near the same.

3 Design the flyby sequence

Design the flyby sequence

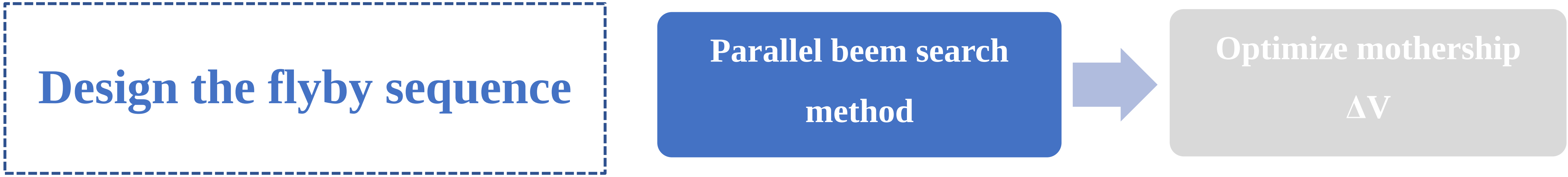
Parallel beem search method

→

Optimize mothership ΔV

- ✓ **The number of $\bar{M}_f \geq 4000$ is: 1685 ($a = 1.1$AU, $i = 0°$)**

3 Design the flyby sequence



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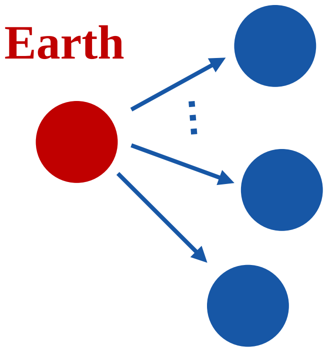

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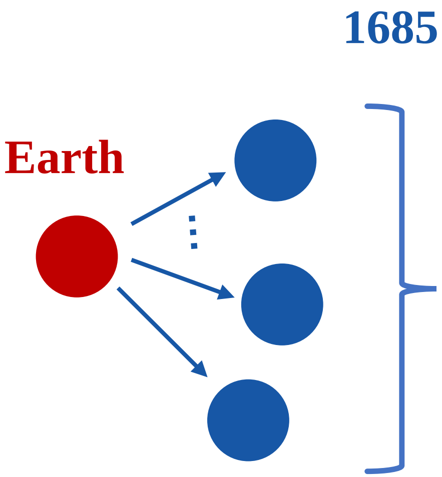

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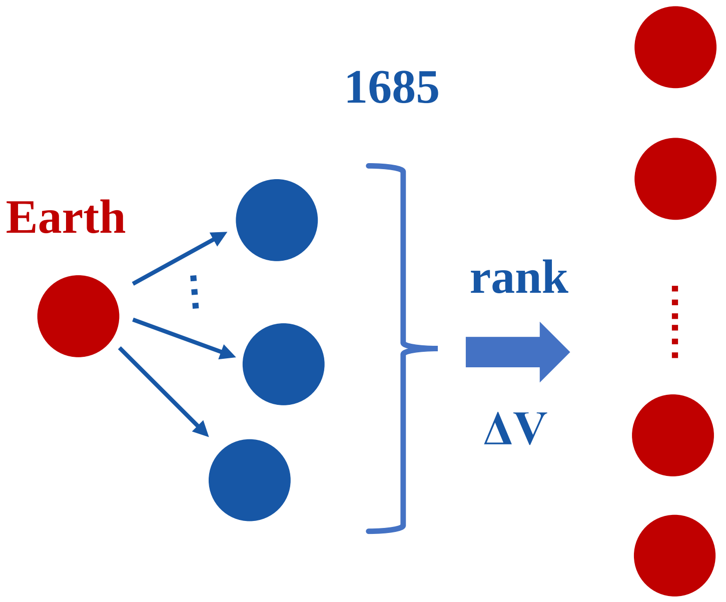

Beam width = 50

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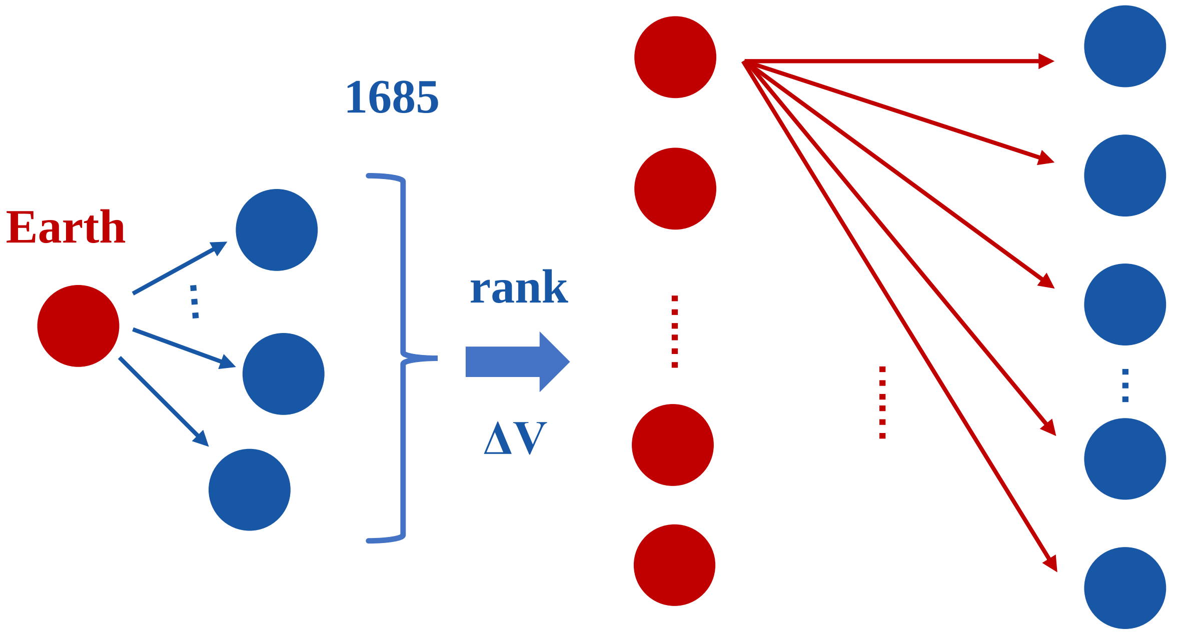

Beam width = 50

1685

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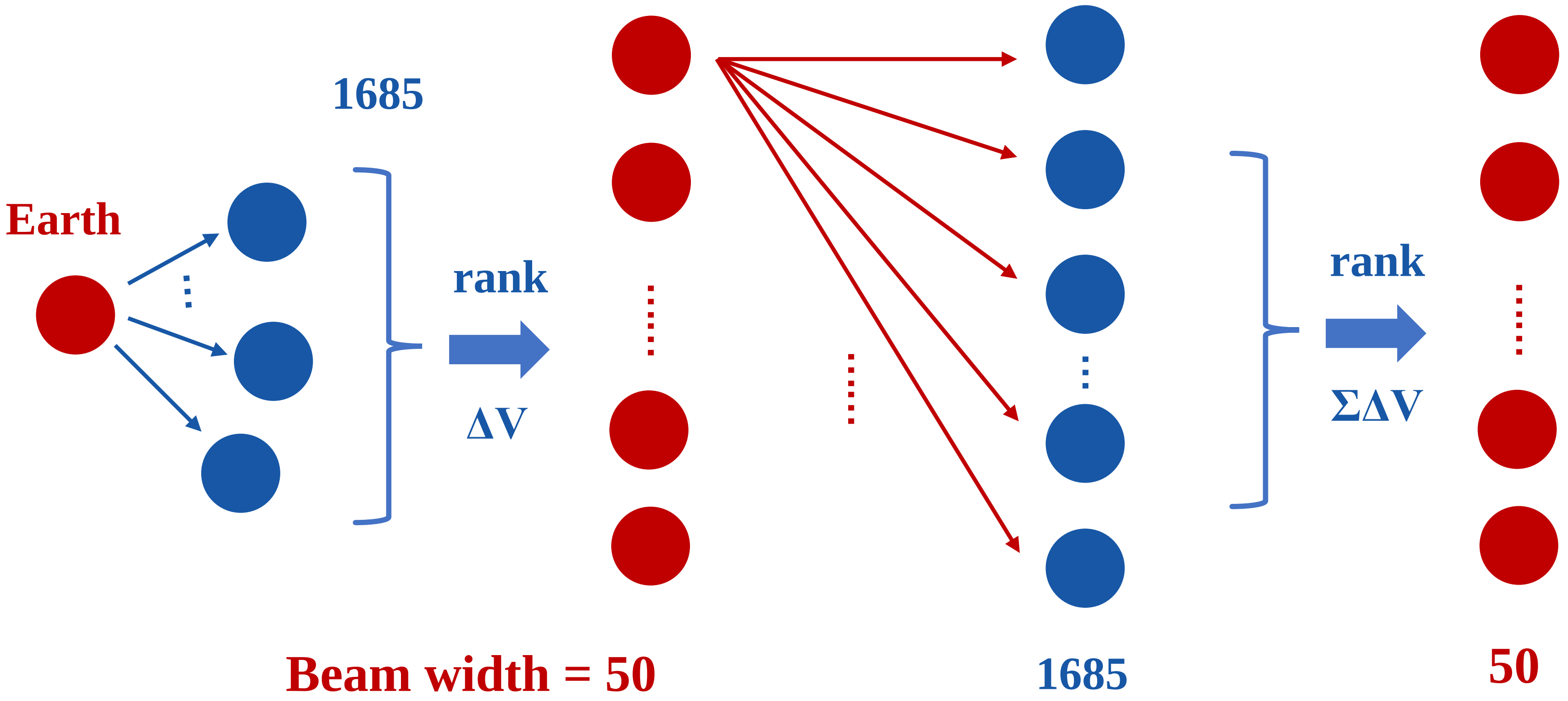

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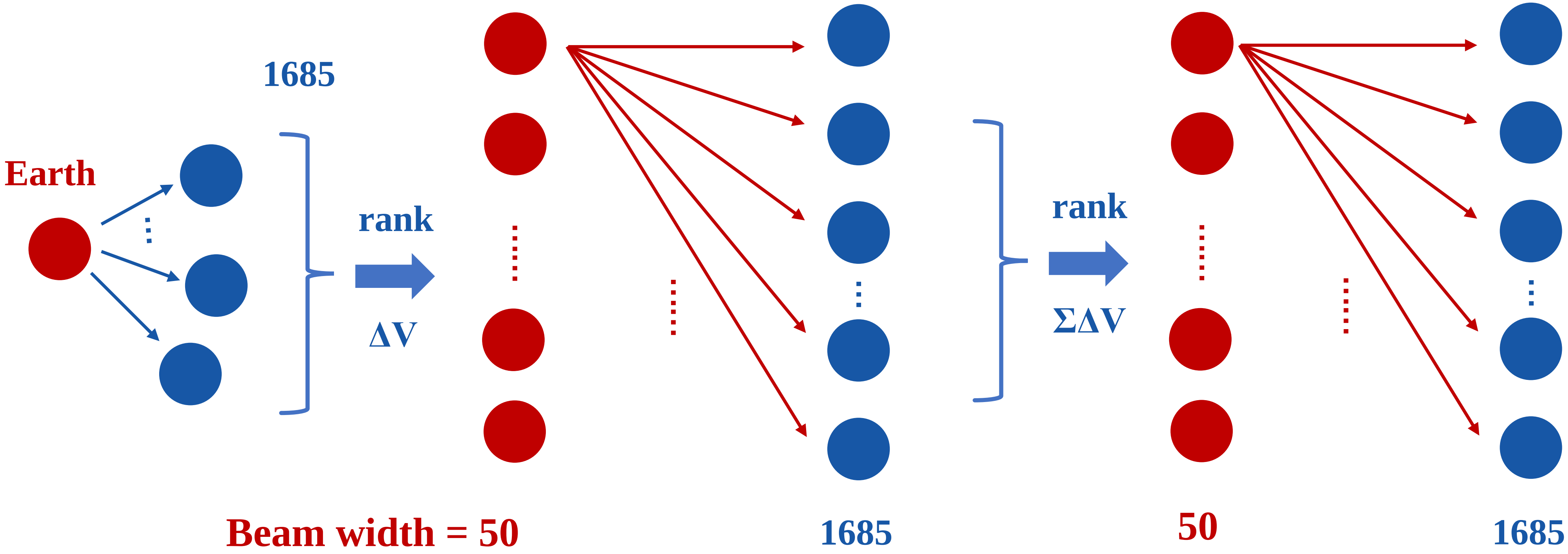

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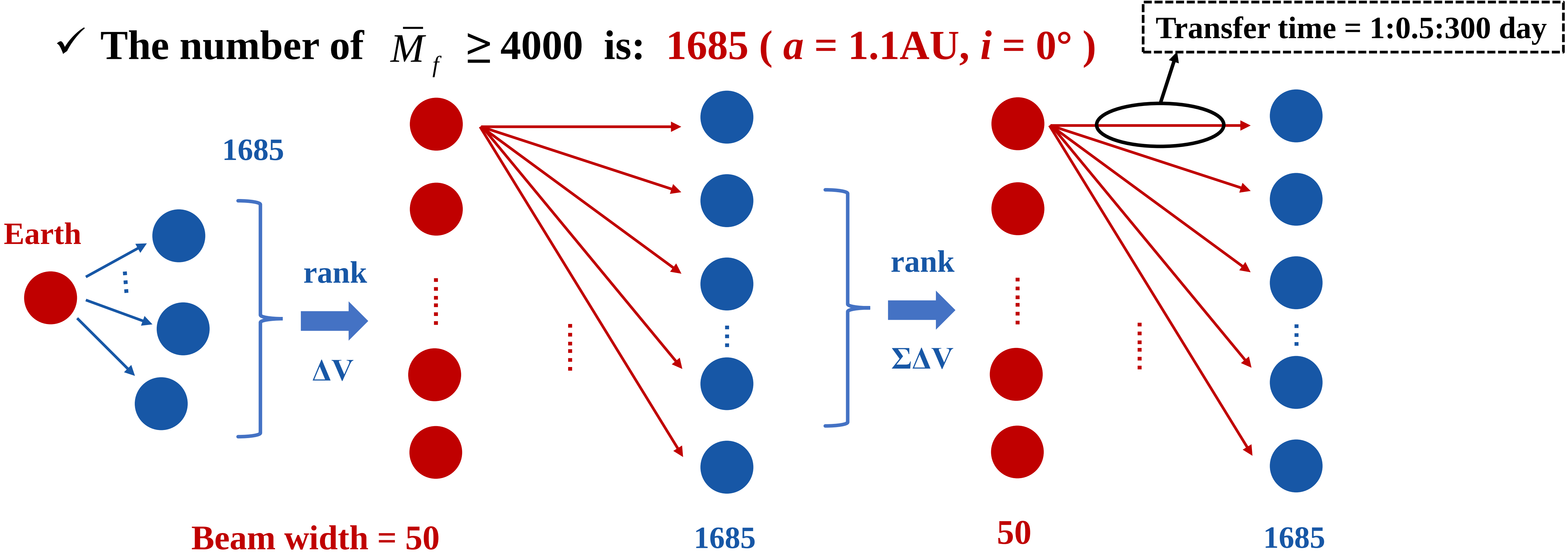

3 Design the flyby sequence

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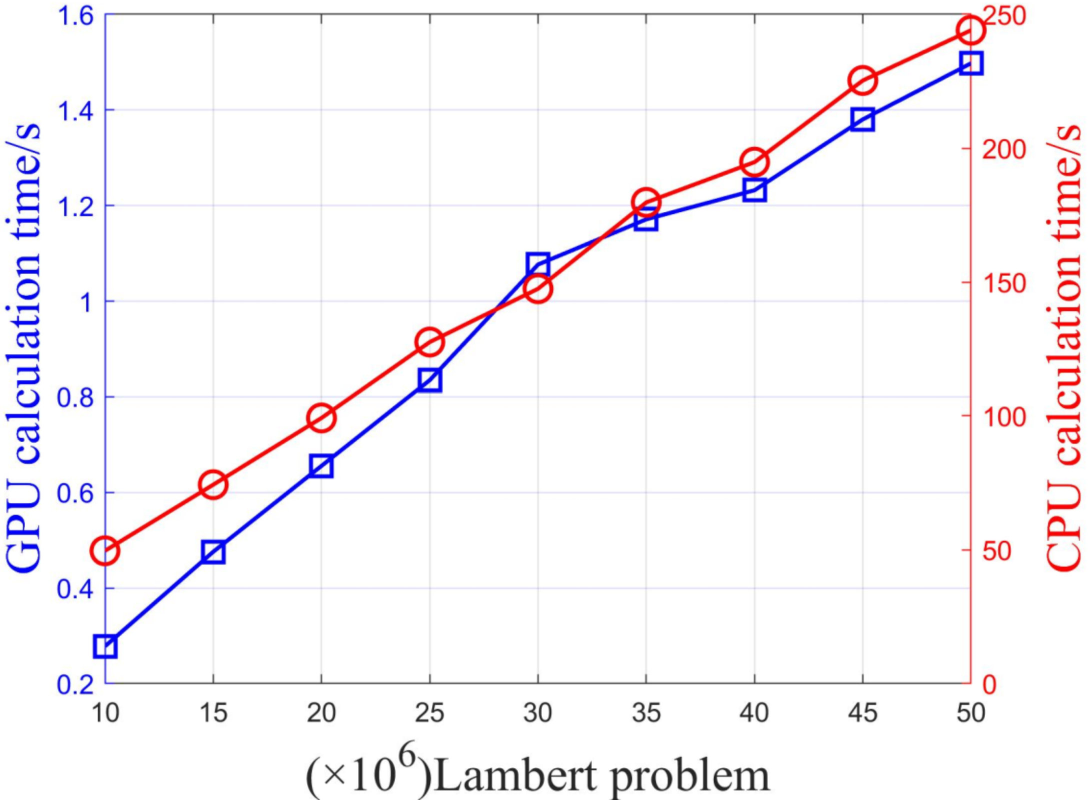
Parallel beem search method

Optimize mothership ΔV

- ☐ For each step, the amount of lambert problems = 50×600×1685 = 50,550,000.
- ☐ Using GPU parallel computing technique.
- ☐ For each step, the GPU calculation time is less than 1.6s, while the CPU calculation time would up to 250s.



3 Design the flyby sequence

Design the flyby sequence

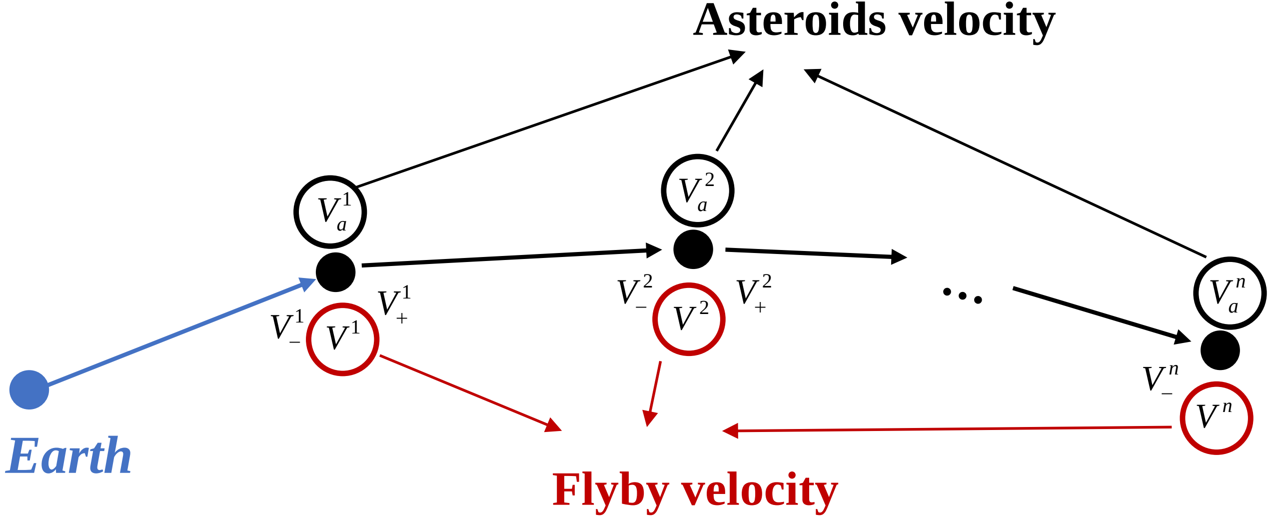
Parallel beem search method

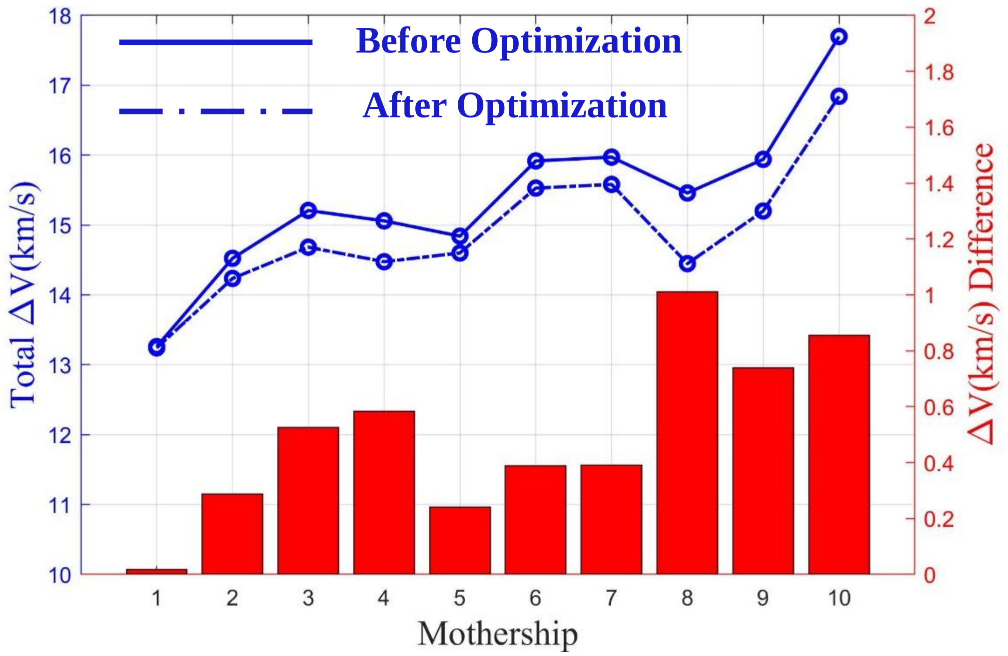
Optimize mothership ΔV

- ✓ **Optimize the flyby sequence total ΔV by sequential quadratic programming.**

$$\min \quad \sum_{i=1}^{n-1}\left(\left\|V^{i}-V_{-}^{i}\right\|+\left\|V_{+}^{i}-V^{i}\right\|\right)+\left\|V^{n}-V_{-}^{n}\right\|+\left\|V_{+}^{E}-V^{E}\right\|$$

$$s.t. \quad \left\|V^{i}-V_{a}^{i}\right\| \leq 2 \qquad i=1,2,\ldots,n$$



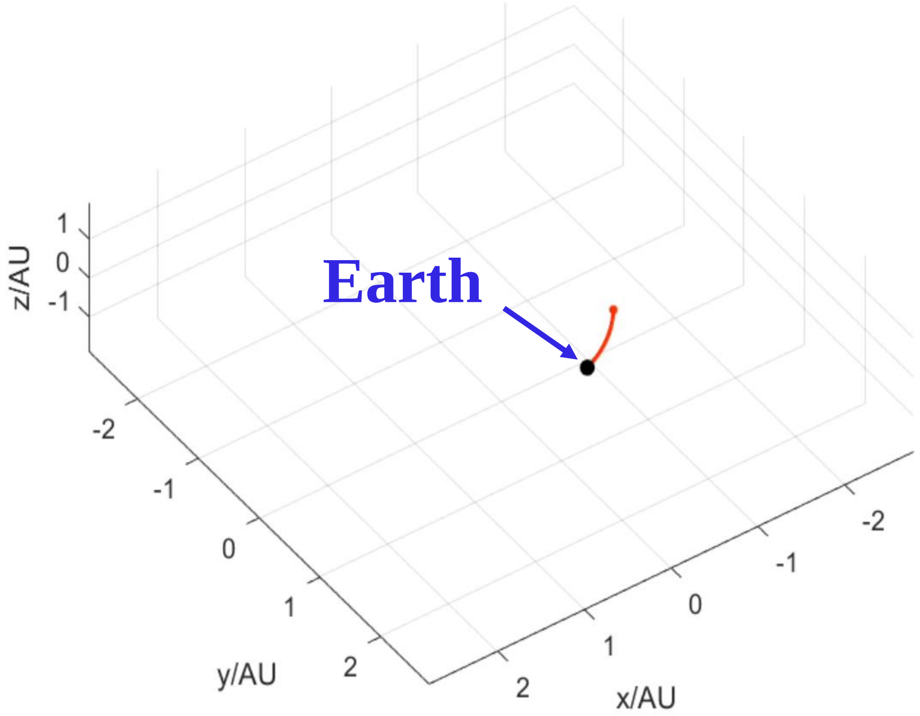


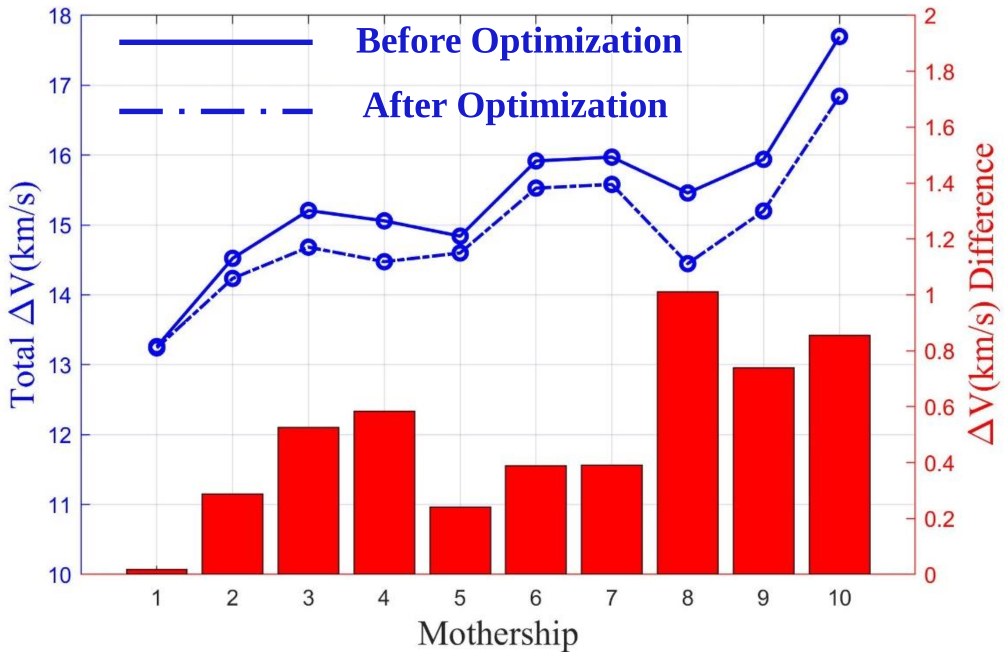
3 Design the flyby sequence

Design the flyby sequence

Parallel beem search method → **Optimize mothership ΔV**

- ✓ **Mothership 1 flyby sequence： 22 Asteroids, ΔV = 13.28km/s**





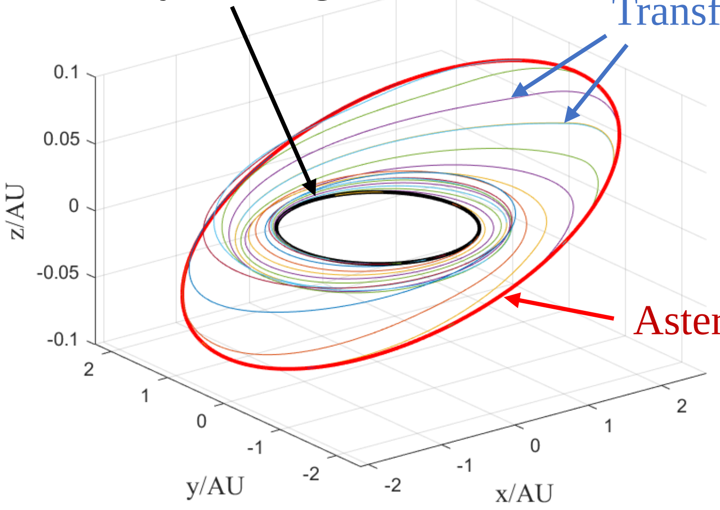
4 Design the departure time

Attainable station calculation

Coordinate descent method

Particle swarm optimization

- ✓ For each flyby asteroid, generate a database of arrival phases.
- ✓ Given the building time of station *i*, determine whether the asteroid can be reached by the intersection point.



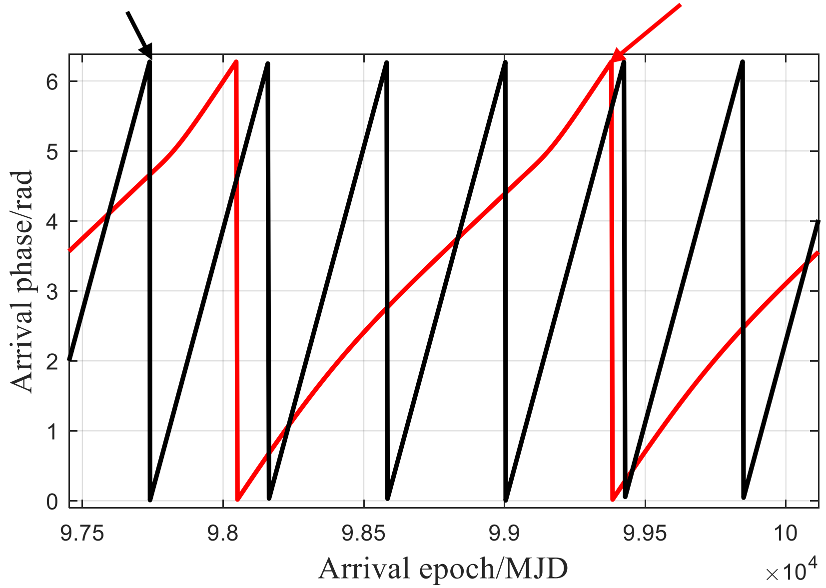
Dyson Ring orbit

Transfer orbit

Asteroid orbit

Station *i* phases

Asteroid arrival phases



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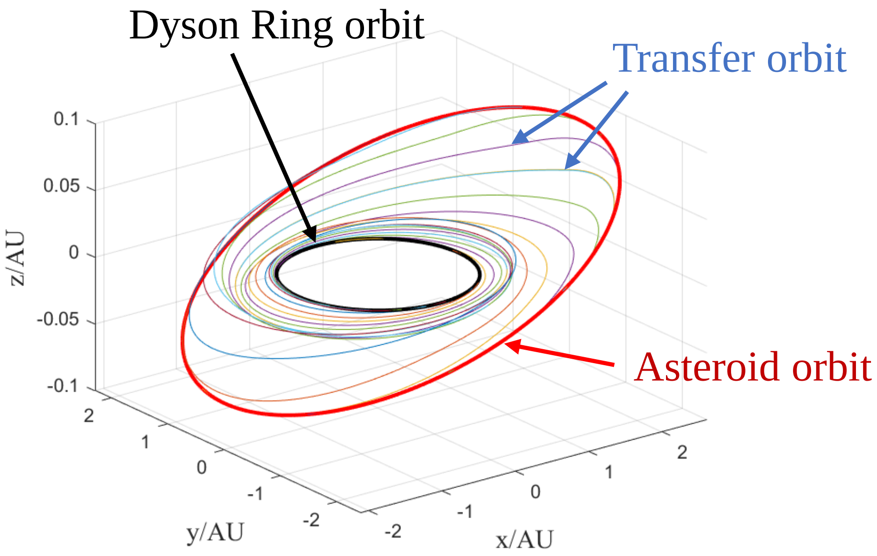

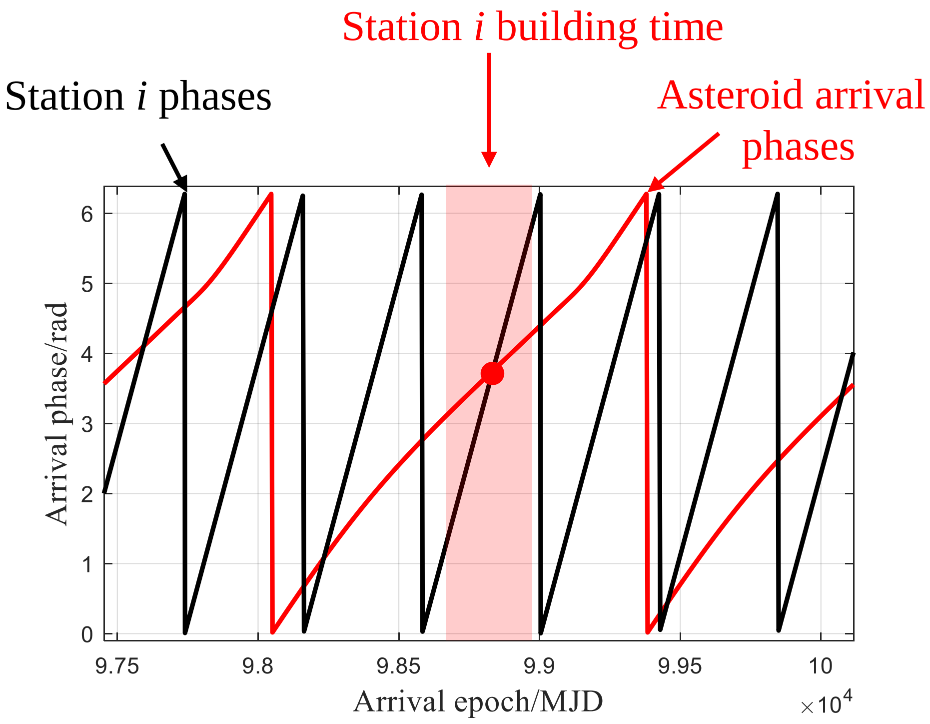

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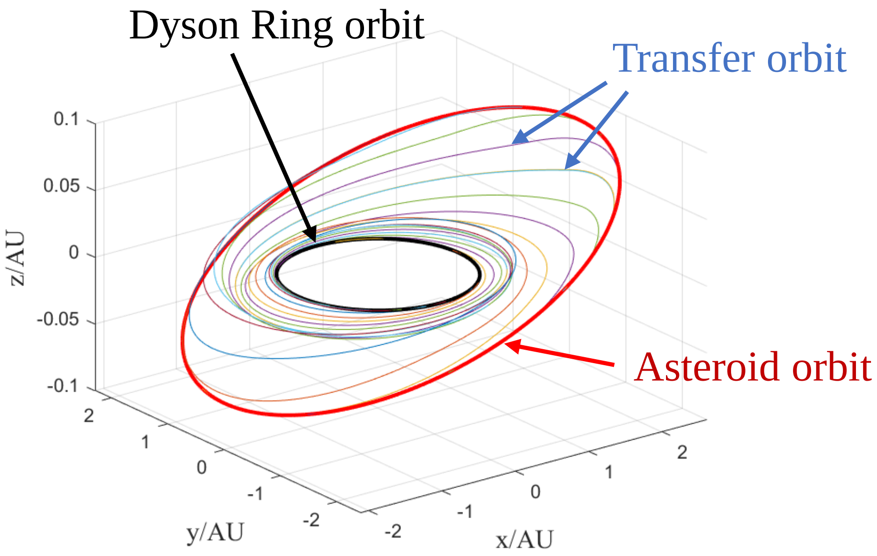

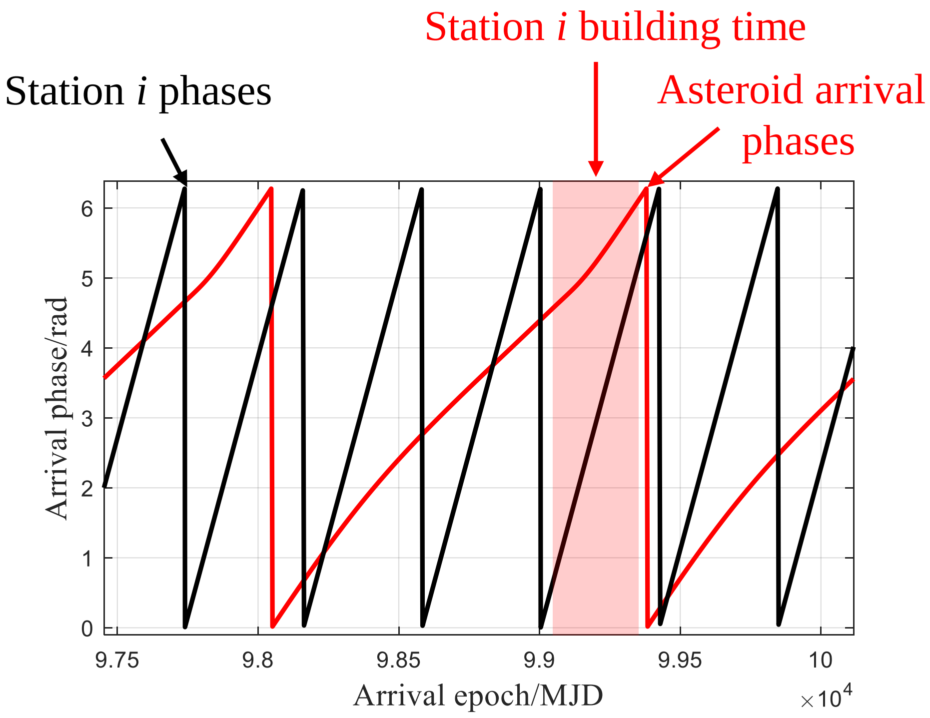

4 Design the departure time

Attainable station calculation

Coordinate descent method

Particle swarm optimization

- ✓ Given the building time of station 1-12, generate station reachability table.
- ✓ Maximize the minimum column.

```
While $\sum M_{\min}^{p} > \sum M_{\min}^{p-1}$
    for $i$ = 1:$n$
        for $j$ = 1:12
            Find $j^{※}$ maximizes table;
        end
        Assignment[$i$] = $j^{※}$;
    end
end
```

Station

Flyby Asteroids

	1	2	…	11	12
1	m_1^1	0	⋯	m_1^{11}	0
2	0	m_2^2	⋯	0	m_2^{12}
3	0	0	⋯	m_3^{11}	m_3^{12}
4	m_4^1	m_4^2	⋯	m_4^{11}	0
…	⋯	⋯	⋯	⋯	⋯
…	⋯	⋯	⋯	⋯	⋯
…	⋯	⋯	⋯	⋯	⋯
…	⋯	⋯	⋯	⋯	⋯
n	0	m_n^2	⋯	m_n^{11}	0

4 Design the departure time

Attainable station calculation

Coordinate descent method

Particle swarm optimization

- ✓ Given the building time of station 1-12, generate station reachability table.
- ✓ Maximize the minimum column.
- ✓ Initial phase and building time as optimization variables.

$$\varphi_1$$

$$MJD_1^0, MJD_2^f, \ldots, MJD_{12}^0, MJD_{12}^f$$

		Station				
		1	**2**	**…**	**11**	**12**
Flyby Asteroids	**1**	m_1^1	0	$\cdots$	m_1^{11}	0
	2	0	m_2^2	$\cdots$	0	m_2^{12}
	3	0	0	$\cdots$	m_3^{11}	m_3^{12}
	4	m_4^1	m_4^2	$\cdots$	m_4^{11}	0
	…	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	…	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	…	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	…	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	n	0	m_n^2	$\cdots$	m_n^{11}	0

4 Design the departure time

- ✓ The particle swarm optimization is used to determine the station building time, initial phase, and the departure epoch of each flyby asteroid.
- ✓ Establish the following indirect method to determine the initial costate variables and transfer time for each asteroid.

Indirect Trajectory Optimization Using Modified Equinoctial Elements

MEE equation $\boldsymbol{x} = \boldsymbol{M}\frac{T}{m}\boldsymbol{\alpha} + \boldsymbol{D}$

Hamiltonian $H = \boldsymbol{\lambda}^T\left(\boldsymbol{M}\frac{T}{m}\boldsymbol{\alpha} + \boldsymbol{D}\right)$

Costate equation $\boldsymbol{\lambda} = -\frac{\partial H}{\partial \boldsymbol{x}} = -\left(\boldsymbol{\lambda}^T\frac{\partial \boldsymbol{M}}{\partial \boldsymbol{x}}\frac{T}{m}\boldsymbol{\alpha} + \boldsymbol{\lambda}^T\frac{\partial \boldsymbol{D}}{\partial \boldsymbol{x}}\right)$

Optimal thrust direction $\boldsymbol{\alpha}^* = -\frac{\left(\boldsymbol{\lambda}^T\boldsymbol{M}\right)^T}{\left\|\boldsymbol{\lambda}^T\boldsymbol{M}\right\|}$

***L*(t_f) = *L*$_D$(t_f)**

⇨

Solved by *cminpack* solver

$$\begin{cases} p\left(t_f\right) - p_f = 0 \\ f\left(t_f\right) - f_f = 0 \\ g\left(t_f\right) - g_f = 0 \\ h\left(t_f\right) - h_f = 0 \\ k\left(t_f\right) - k_f = 0 \\ L\left(t_f\right) - L_D\left(t_f\right) = 0 \\ H\left(t_f\right) + 1 = 0 \end{cases}$$

Shooting function

4 Design the departure time

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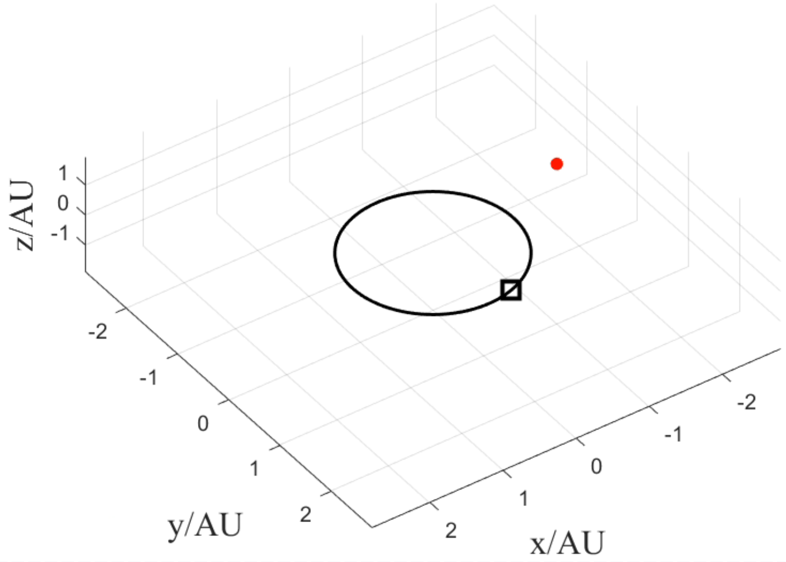
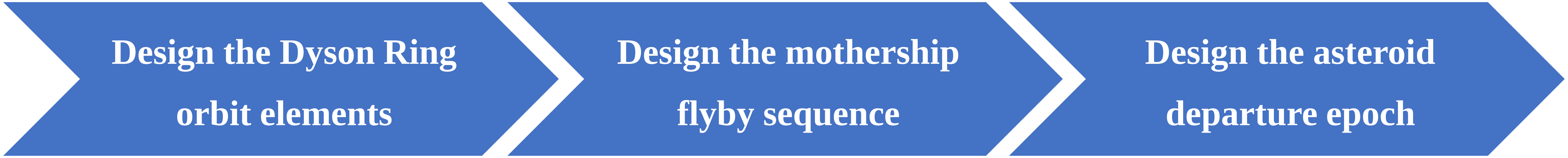


5 Conclusions

- ✓ Submissions Score: 3532.7 Mmin:8.003e+14 Dyson ring *a* = 1.0AU *i* = 8°
- ✓ Latest Score: 5063.8 Mmin:1.032e+15 Dyson ring *a* = 1.1AU *i* = 0°



Design the Dyson Ring orbit elements

Design the mothership flyby sequence

Design the asteroid departure epoch

- ■ The Dyson Ring orbital elements were not optimized in the submitted version due to the large computational load.
- ■ The use of the optimized Dyson Ring orbital elements yields a much smaller ΔV of each mothership, i.e., the average ΔV is around 26km/s in the submitted version, while it can be reduced to 16km/s after applying the optimized orbital elements.

5 Conclusions

Further improvement and optimization:

- The departure epoch of each mothership is the same in current design, and the departure epoch of motherships should be optimized.
- The stations are built from 1 to 12 by increasing the phase angle of 30 degree, the order of station building needs to be optimized.
- The multi-impulse transfers local optimization problem will be solved by deep neural networks estimation in the future study.

Thanks