GTOC 11: Results And Methods From The HIT Team

Harbin Institute of Technology

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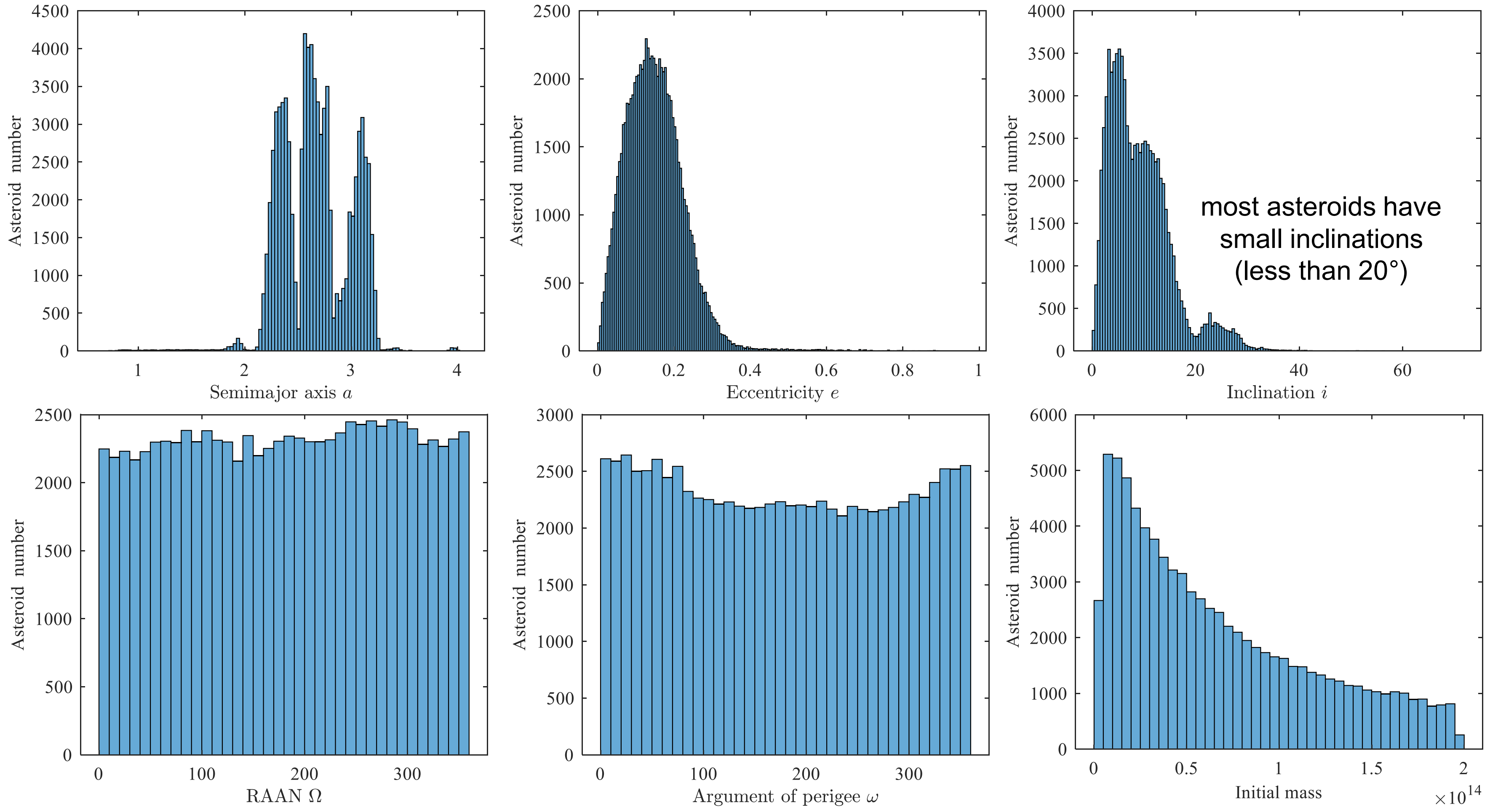
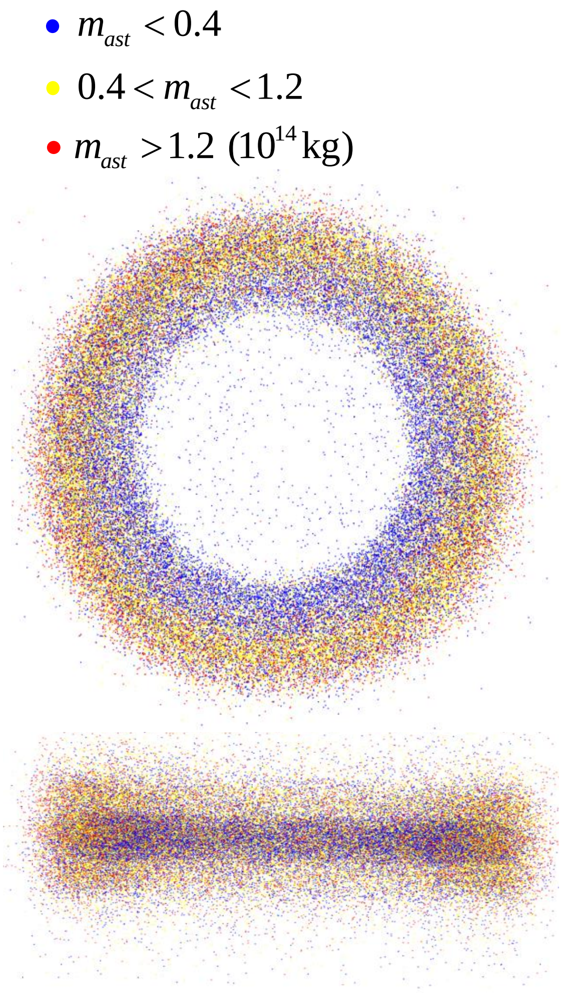
1. Problem analysis

Analysis of asteroid parameters





The RAAN and the argument of perigee of the asteroids are approximately uniformly distributed in space, and to eliminate their influence, the inclination of Dyson ring is chosen to be 0 in our strategy.

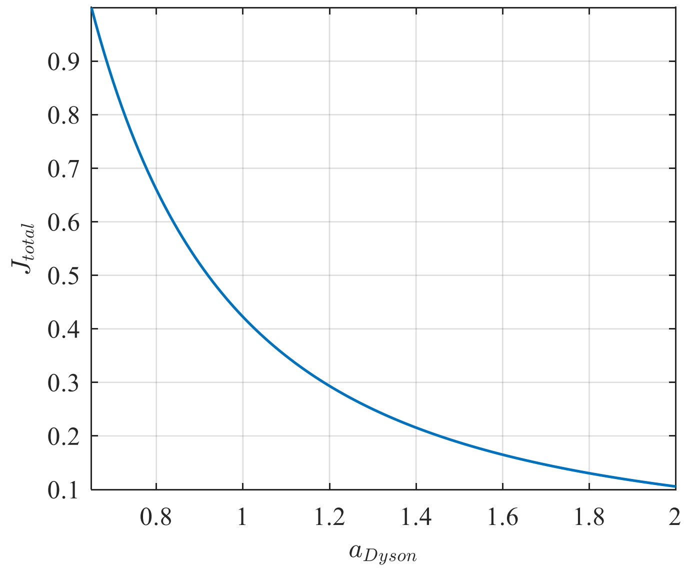
1. Problem analysis

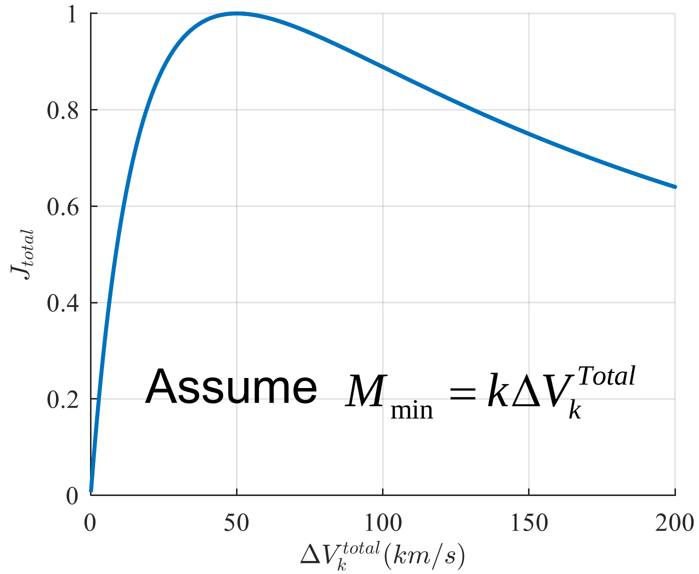
Analysis of performance index

$$J_{total} = \frac{10^{-10} \cdot M_{\min}}{a_{Dyson}^2 \sum_{k=1}^{10} \left(1 + \Delta V_k^{Total} / 50\right)^2}$$

$$m^{ast}(\Delta t) = m_0^{ast} - 6 \times 10^{-9} \cdot m_0^{ast} \cdot \Delta t$$

Each ATD can work for up to 5.28 years





We first solve the problem optimally at a given Dyson ring radius, and subsequently try to traverse the Dyson ring radius to find the optimal solution.

Although there is no limit on the impulse magnitude in the competition, the performance index starts to decrease when it exceeds 50 km/s.

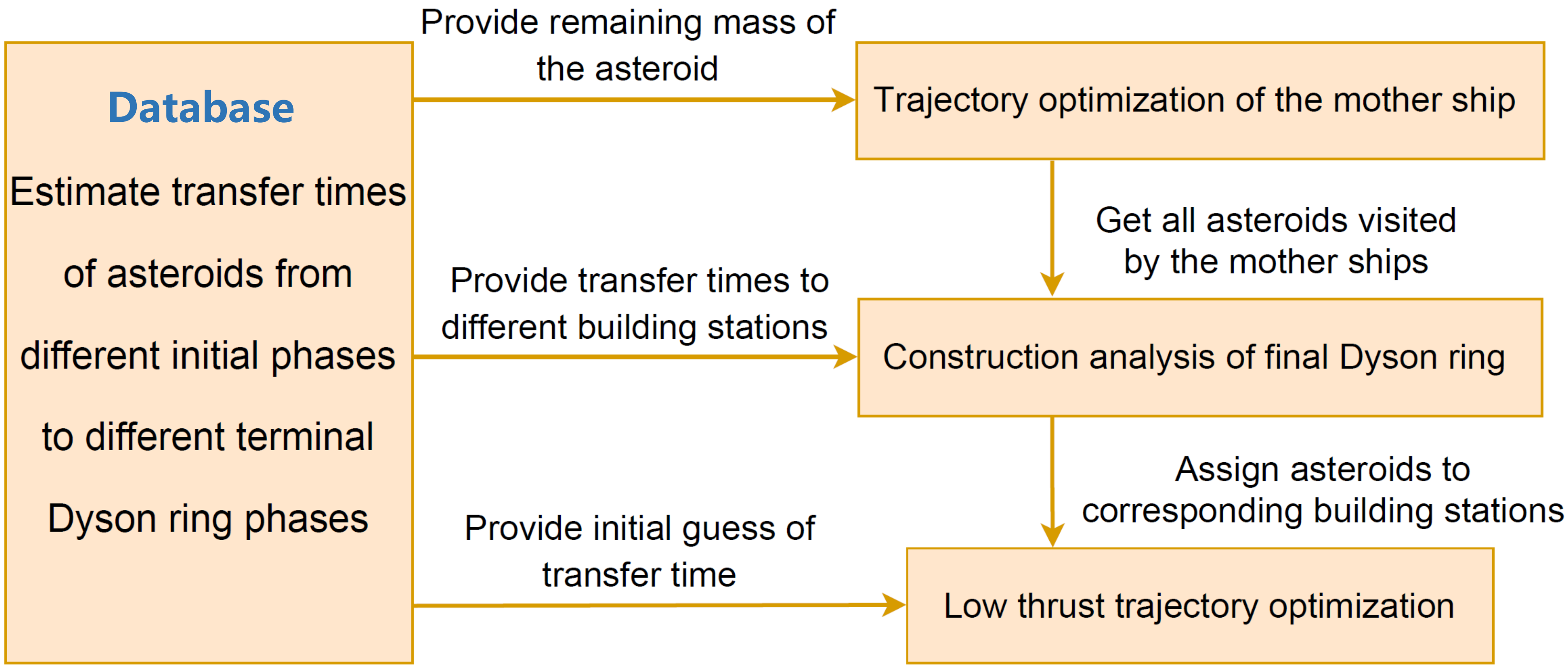
Part 02

Solving process

2. Solving process

Methods Framework

For a given radius of the Dyson ring, the whole solving process is divided into the following four parts, in which the estimation of the asteroid transfer time is the basis for the other three parts.



2. Solving process

1 Estimation of the asteroid transfer time: build the database

Due to the huge number of asteroids provided by the competition, and the uncertainty of both the departure phase (ATD activation phase) and the Dyson ring phase of the terminal arrival, it is not practical to solve the transfer times of all asteroids by traversal. Therefore, we use an estimation method with the following steps:

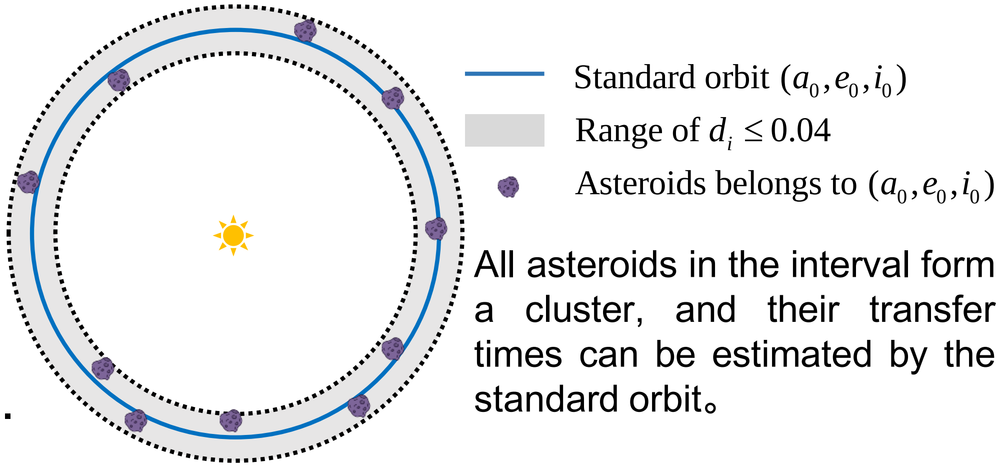
Define the standard orbit

We select the semimajor axis a_0, eccentricity e_0, and inclination i_0 at equal step size to form a series of so-called standard orbit (a_0, e_0, i_0). Define the distance between an asteroid orbit (a_i, e_i, i_i) and a certain standard orbit (a_0, e_0, i_0) as

$$d_i = \sqrt{\left(\frac{a_i - a_0}{a_0}\right)^2 + (e_i - e_0)^2 + (i_i - i_0)^2} = \sqrt{\left(\frac{\Delta a}{a_0}\right)^2 + \Delta e^2 + \Delta i^2}$$

If $d_i \le 0.04$, we have $\text{Asteroid } (a_i, e_i, i_i) \in \text{Standard orbit } (a_0, e_0, i_0)$

Accordingly, all asteroids are divided into a series of clusters.



All asteroids in the interval form a cluster, and their transfer times can be estimated by the standard orbit。

2. Solving process

1 Estimation of the asteroid transfer time: build the database

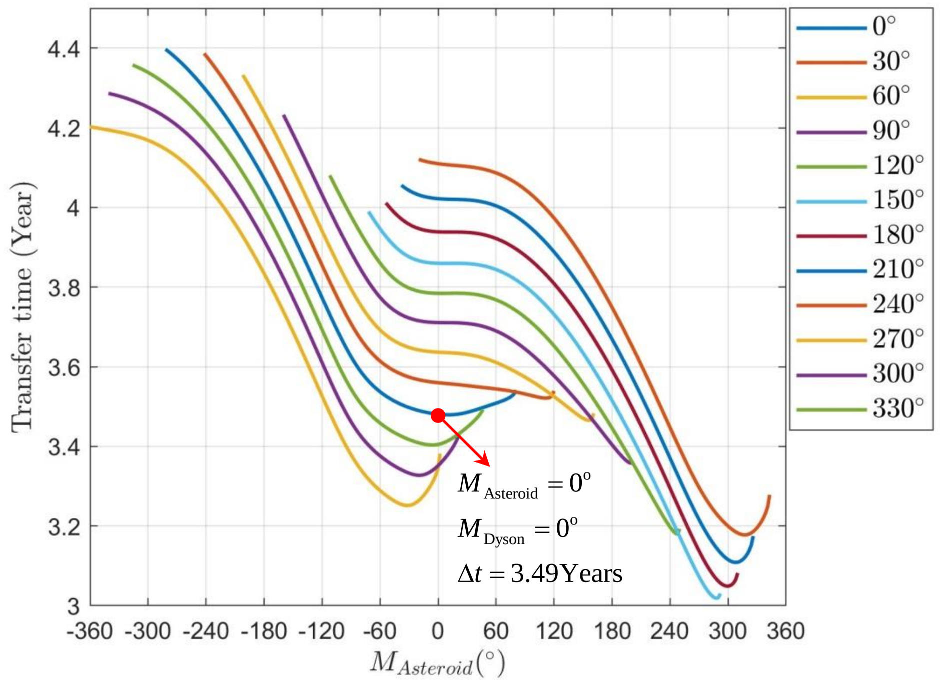
Example $R_{Dyson} = 1.1\ \text{AU}$

	a_0(AU)	e_0	$i_0(^{\circ})$
Maximum	2.0	0	0
Minimum	3.0	0.25	10
Step size	0.1	0.05	2
Number	11	6	6

There are a total of 11×6×6=396 standard orbits and the corresponding number of orbital clusters. 220 orbital clusters are left after removing those containing less than 100 asteroids.

For each standard orbit, the optimal transfer time from different initial phases to different Dyson ring phases (discretization) are obtained by the optimization algorithm.

$M_{\text{Asteroid}} = 0:2^{\circ}:360^{\circ}, M_{\text{Dyson}} = 0:30^{\circ}:360^{\circ}$



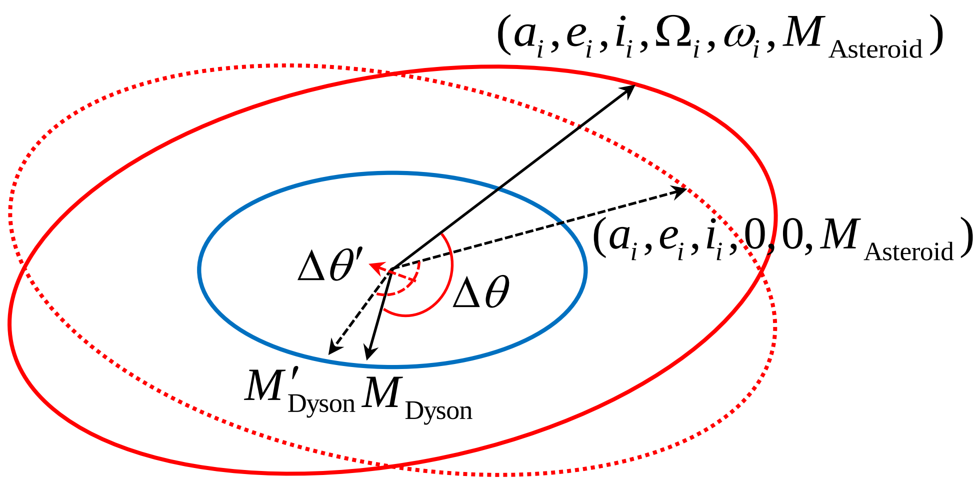
For any initial asteroid phase M_{Asteroid} and terminal Dyson ring phase M_{Dyson}, the transfer time Δt_0 can be obtained by two-dimensional interpolation.

2. Solving process

1 Estimation of the asteroid transfer time: build the database

Estimation of the transfer time

For an asteroid i, the orbital elements is $(a_i, e_i, i_i, \Omega_i, \omega_i, M_i)$. The initial departure phase and the Dyson ring phase of the terminal arrival are M_{Asteroid} and M_{Dyson}. The transfer time can be estimated by the following steps:



$$\Delta\theta' = \Delta\theta \Rightarrow M'_{\text{Dyson}} = F(M_{\text{Dyson}}, M_{\text{Asteroid}}, i_i, \Omega_i, \omega_i)$$

$$\Delta t_{true} : (a_i, e_i, i_i, \Omega_i, \omega_i, M_{\text{Asteroid}}) \rightarrow M_{\text{Dyson}}$$

$$\Delta t'_{true} : (a_i, e_i, i_i, 0, 0, M_{\text{Asteroid}}) \rightarrow M'_{\text{Dyson}}$$

$$\Delta t_{true} \approx \Delta t'_{true}$$

Find the standard orbit that the asteroid belongs to

$$\text{Asteroid } (a_i, e_i, i_i) \in \text{Standard orbit } (a_0, e_0, i_0)$$

The true transfer time of the asteroid can be estimated by linear interpolation of the transfer time of the standard orbit

$$\Delta t_{true} \approx \Delta t'_{true} = \Delta t_0 + k_a \Delta a + k_e \Delta e + k_i \Delta i$$

The results of numerical simulation show that the error of the above method is less than 5%.

2. Solving process

2 Trajectory optimization of the mother ship

Whole task: $$J_{total} = \frac{10^{-10} \cdot M_{\min}}{a_{Dyson}^2 \sum_{k=1}^{10} \left(1 + \Delta V_k^{Total} / 50\right)^2}$$

$$\Downarrow$$

k-th mother ship: $$J_k = \frac{10^{-10} \cdot M_k}{\left(1 + \Delta V_k^{total} / 50\right)^2}$$

The sum of the remaining masses of all asteroids visited by the k-th mother ship.

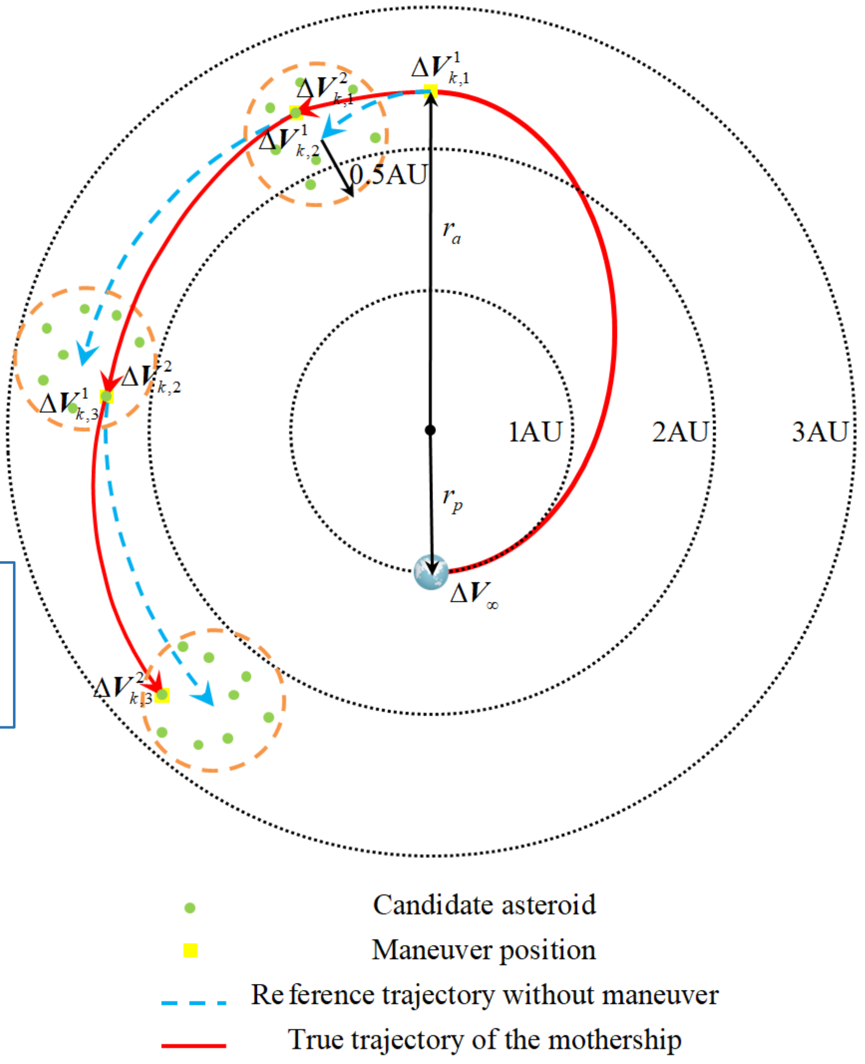
$$\Downarrow$$

k-th mother ship, i-th asteroid: $$J_{k,i} = \frac{10^{-10} \cdot m_i^{ast}}{(1 + \Delta V_{k,i} / \xi)^2}$$

The remaining mass of the i-th asteroid visited by the k-th mother ship (database).

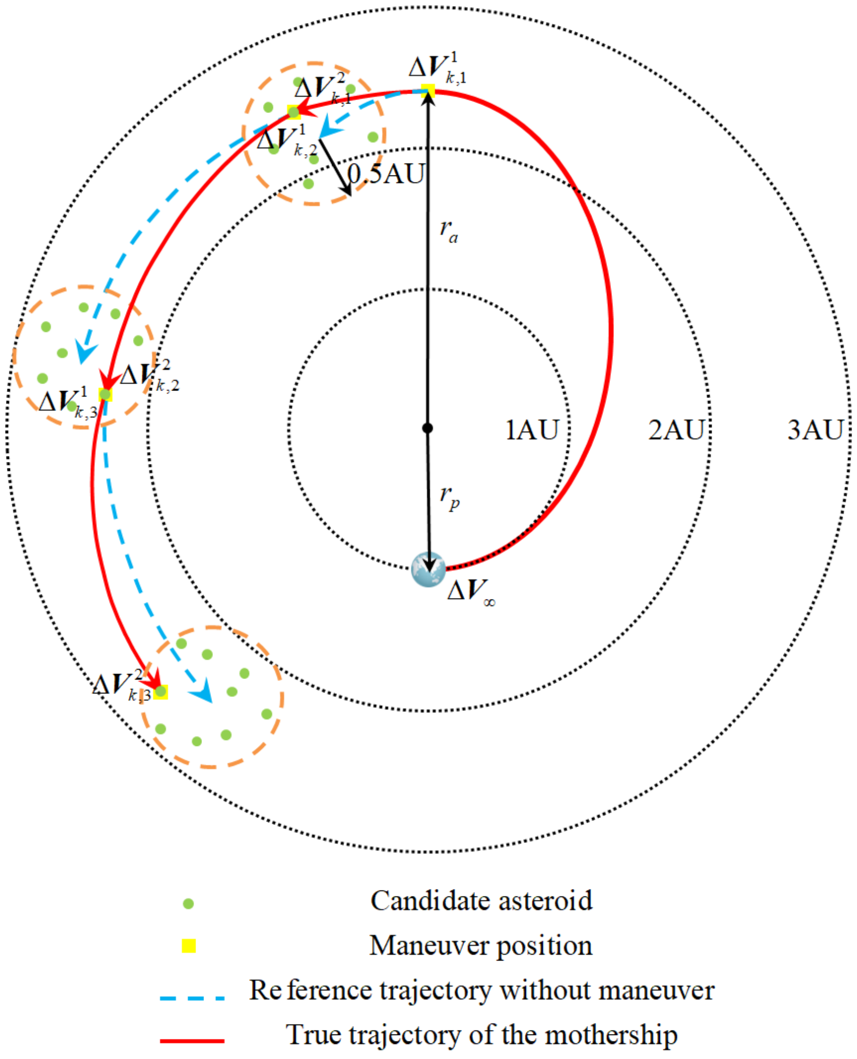
The impulse magnitude consumed for visiting the i-th asteroid

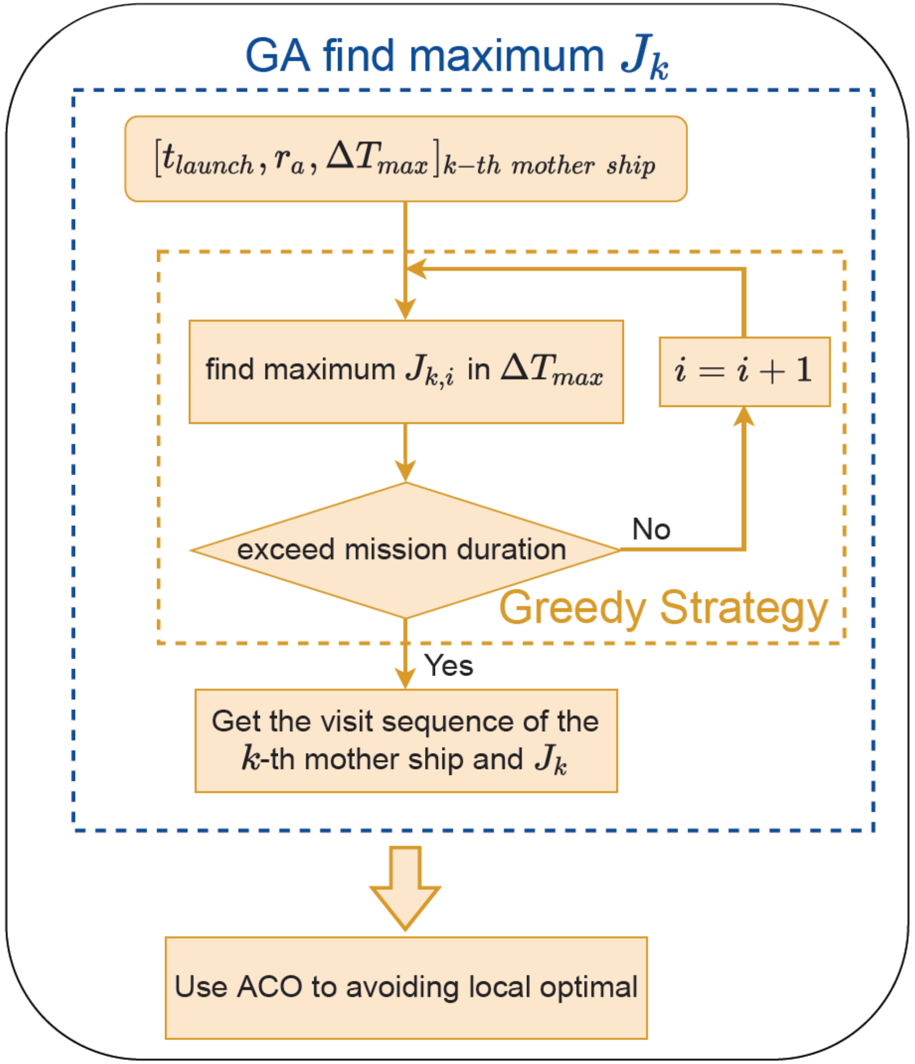
Expected impulse magnitude $\xi \in [0.5 \sim 1]$



2. Solving process

2 Trajectory optimization of the mother ship





Based on two-impulse lambert rendezvous

$$J_k = \frac{10^{-10} \cdot M_k}{\left(1 + \Delta V_k^{total} / 50\right)^2}$$

$$J_{k,i} = \frac{10^{-10} \cdot m_i^{ast}}{(1 + \Delta V_{k,i} / \xi)^2}$$

GA find minimum ΔV_k^{total}

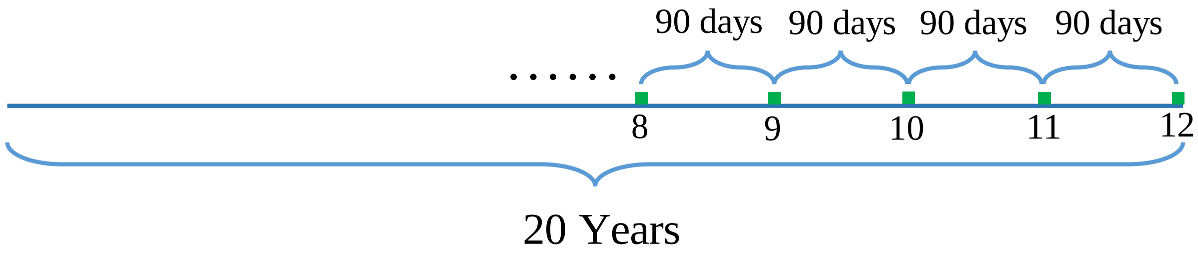
Optimize the visit time and the number of impulses between two consecutive flyby asteroids

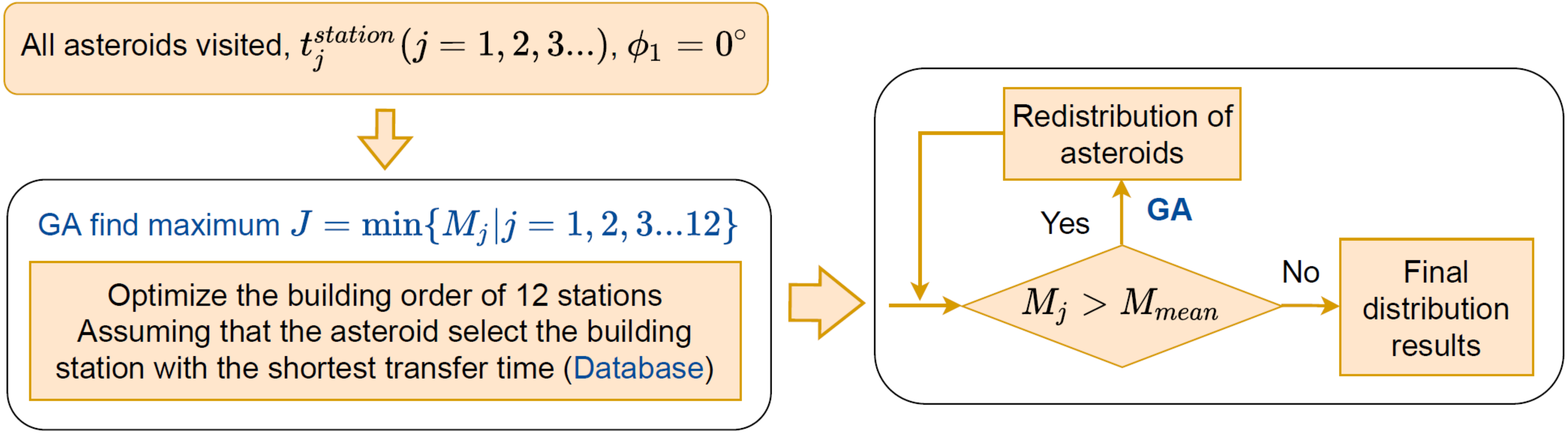
Based on the solved visit sequence

2. Solving process

3 Construction analysis of final Dyson ring

We assume that each building station is completed instantaneously and the corresponding asteroids arrive at the same time. The last building station is completed at the last moment of the mission.

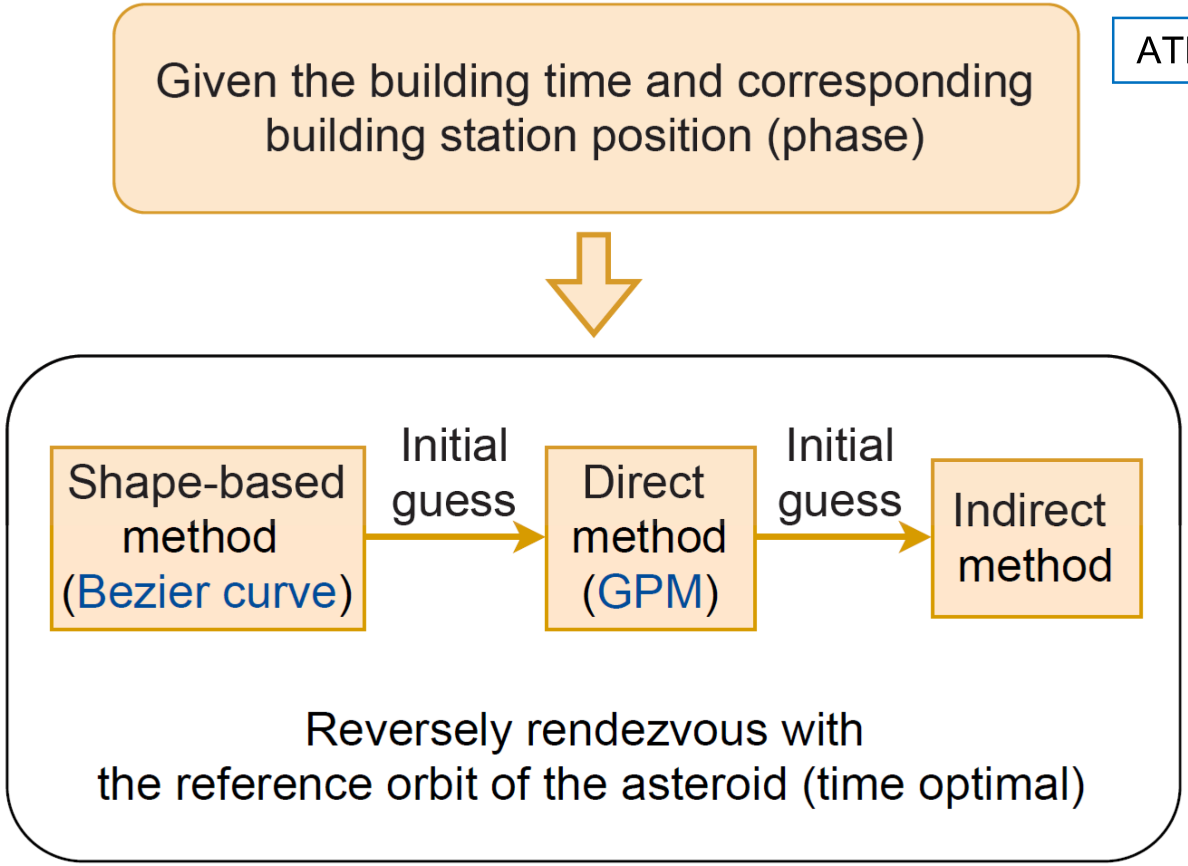


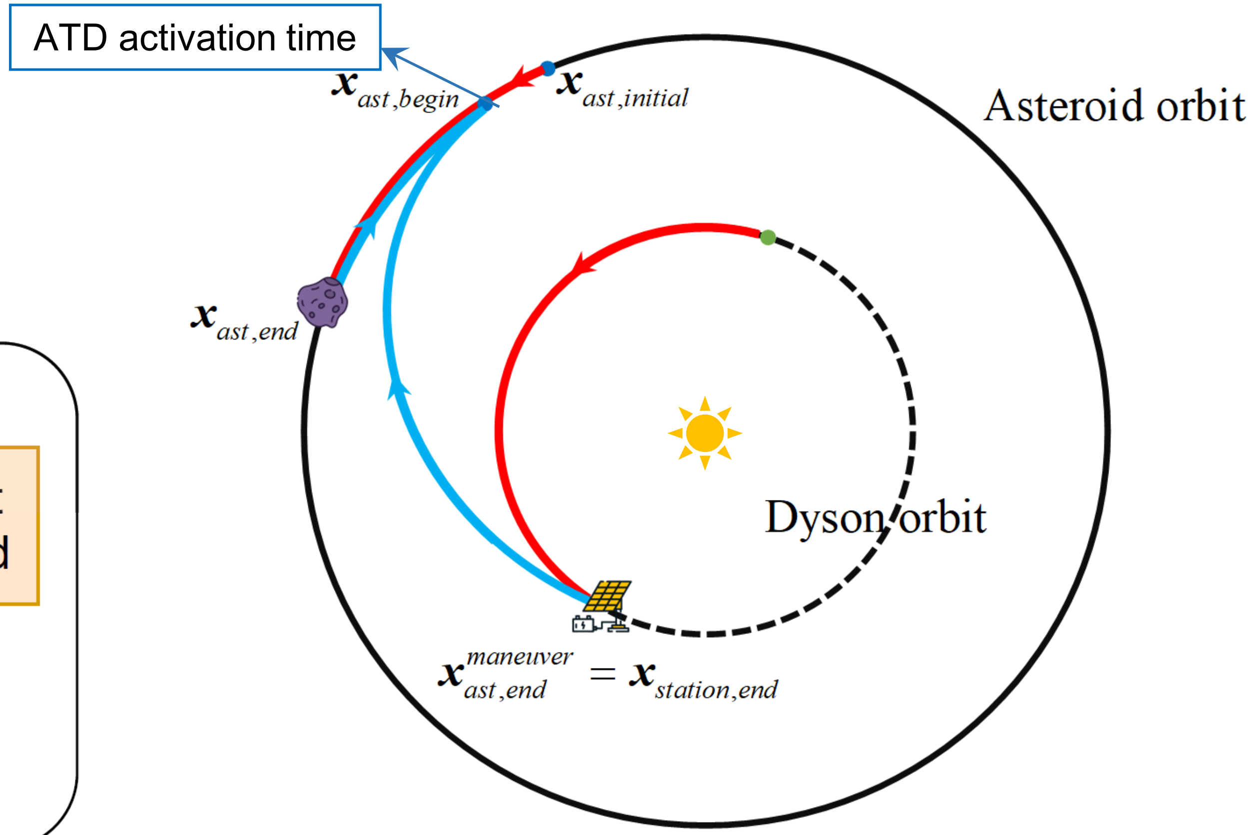


2. Solving process

4 Low thrust trajectory optimization

In order to ensure that each building station is completed instantaneously and the corresponding asteroids arrive at the same time, we need to solve for the time when each asteroid starts ATD.





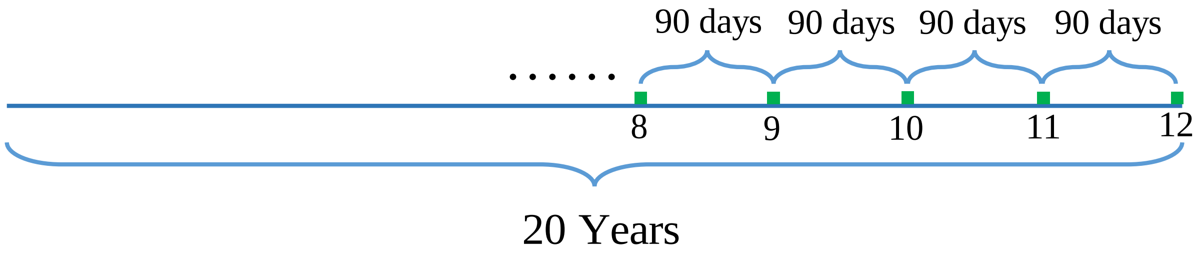
Part 03

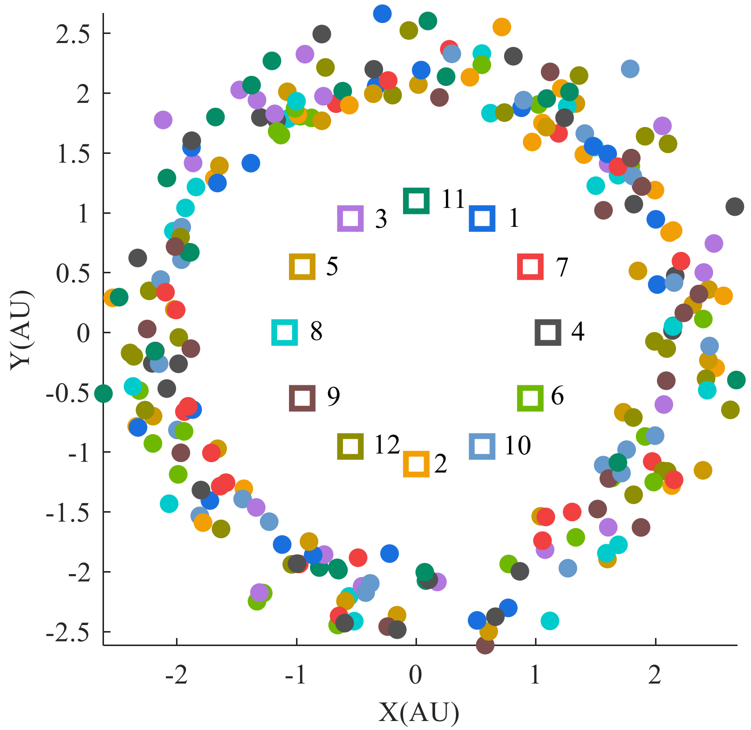
Final Result

3. Final Result

Details

We traversed all the Dyson ring radii between $R_{Dyson} = 0.9:0.05:1.4$ AU and finally chose $R_{Dyson} = 1.1\text{AU}$. Other orbital elements of the Dyson ring are all 0. Each building station is built in an instant, and the building sequence is shown on the right figure.





The number of asteroids for each building station and the corresponding remaining masses are

Building order	1	2	3	4	5	6	7	8	9	10	11	12
Asteroid number	20	19	19	22	22	21	21	20	19	22	22	23
Remaining mass (e15kg)	1.0966	1.0871	1.0864	1.0857	1.1060	1.0855	1.0906	1.1119	1.0881	1.1317	1.1013	1.0897

$R_{Dyson} = 1.1$ AU

$Score = 5208.3463$

$M_{min} = 1.08546 \times 10^{15}$ kg

$N_{Asteroid} = 250$

Thanks to NUDT&XSCC and all colleagues for a amazing competition!

T H A N K Y O U F O R W A T C H I N G