The GTOC Series: Analytic Scaffolding that Simplifies Global Trajectory Optimisation

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Eleventh Global Trajectory Optimisation Competition

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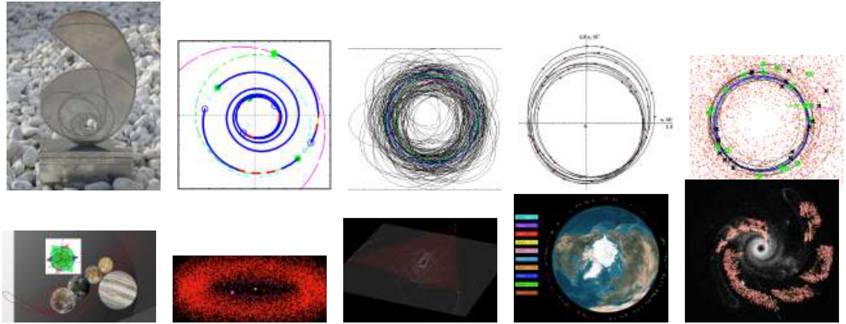
The Global Trajectory Optimisation Competition

- Conceived by Dario Izzo of the European Space Agency's Advanced Concepts Team, in 2005, as a recurring event
- Vehicle for investigating both innovative mission design methods and innovative mission designs



Images of winning solutions GTOC 1 through X (credit: winners, organizers, ACT)

JPL Team Members over the years GTOC1–GTOCX

Anastassios Petropoulos, Theresa Kowalkowski, Daniel Parcher, Paul Finlayson, Ed Rinderle, Matthew Vavrina, Jon Sims, Ryan Russell, Try Lam, Powtawche Williams, Gregory Whiffen, Nathan Strange, Jennie Johannesen, Chen-Wan Yen, Carl Sauer, Seungwon Lee, Steven Williams, Damon Landau, Brent Buffington, Eugene Bonfiglio, Daniel Grebow, Jeffrey Parker, Juan Arrieta, Rodney Anderson, Eric Gustafson, Gregory Lantoine, Nitin Arora, Drew Jones, Juan Senent, Mark Jesick, Aline Zimmer, Julie Bellerose, Thomas Pavlak, Mar Vaquero, Jeffrey Stuart, Austin Nicholas, Javier Roa, Timothy McElrath, Ralph Roncoli, David Garza, Nicholas Bradley, Zahi Tarzi, Frank Laipert, Mark Wallace

44 people in all

The Solution and the Problem

$$\begin{bmatrix} \dot{\vec{r}} \\ \dot{\vec{v}} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} \vec{v} \\ \vec{g}(r,t) + \vec{a}(m,\vec{r},t,p) \\ \dot{m}(p) \end{bmatrix}$$

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$$\min J = \int_{t_0}^{t_f} f(\vec{r}, \vec{v}, \vec{a}) dt$$

The Oldest Special-Case Solution

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$$
\begin{aligned}
r &= \frac{a(1-e^2)}{1+e\cos\theta} \\
\mu(t_f - t_0) &= F(a,\ r_1 + r_2,\ c;\ n)
\end{aligned}
$$

Shape-Solutions for Non-Zero, Derived a?

- Cassini Oval

$$r_1 r_2 = k$$

Shape-Solutions for Non-Zero, Derived *a*?

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$$r = k_0 \mathrm{e}^{q\theta + k_1 \sin(k_2\theta + \phi)}$$

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$$r = k_0 \mathrm{e}^{q\theta + k_1 \sin(k_2\theta + \phi)}$$

- Pinkham's spiral

$$r = \frac{p_s \mathrm{e}^{q\theta}(1 + q^2)}{1 + \mathrm{e}^{q\theta}(1 + q^2) k \cos(\theta - \omega)}$$

What about "Cost" Estimates?

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  ΔV expressed in terms of a series in initial and target semimajor axes and of elliptic integrals of initial eccentricity. Accurate for large changes in semimajor axis. Correspondingly for eccentricity.

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- Targeted Stark solutions (Landau)

  Semi-analytically target a final position and velocity using a thrust arc with one switch in inertial thrust direction; free variables are the two directions, the acceleration magnitude, and the switch time.

What about "Cost" Estimates? (cont'd)

- Edelbaum (small changes in $a, \vec{e}, i$, as simplified by Casalino *et al.*)

$$\Delta V \approx \sqrt{(k_a \Delta a)^2 + (k_e |\Delta \vec{e}\,|)^2 + (k_i \Delta i)^2}$$

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- $\Delta œ$ from Gauss's variational equations

$$
\begin{aligned}
\frac{\mathrm{d}\Omega}{\mathrm{d}t} &= \frac{r \sin(\theta + \omega)}{h \sin i} f_h \\
\frac{\mathrm{d}i}{\mathrm{d}t} &= \frac{r \cos(\theta + \omega)}{h} f_h \\
\frac{\mathrm{d}\omega}{\mathrm{d}t} &= \frac{1}{eh} \left[-p \cos\theta f_r + (p + r) \sin\theta f_\theta\right] - \frac{r \sin(\theta + \omega) \cos i}{h \sin i} f_h \\
\frac{\mathrm{d}a}{\mathrm{d}t} &= \frac{2a^2}{h} \left(e \sin\theta f_r + \frac{p}{r} f_\theta\right) \\
\frac{\mathrm{d}e}{\mathrm{d}t} &= \frac{1}{h} \left\{p \sin\theta f_r + \left[(p + r) \cos\theta + re\right] f_\theta\right\} \\
\frac{\mathrm{d}\theta}{\mathrm{d}t} &= \frac{h}{r^2} + \frac{1}{eh} \left[p \cos\theta f_r - (p + r) \sin\theta f_\theta\right]
\end{aligned}
$$

Another Type of "Cost" Estimate - Lyapunov functions

- Proximity Quotient, Q, for any pair of orbits (Petropoulos)

$$
\begin{aligned}
Q &= (1 + W_P P) \sum_{œ} W_{œ} S_{œ} \left[\frac{d(œ, œ_T)}{\dot{œ}_{\mathrm{xx}}} \right]^2, \qquad \text{for } œ = a, e, i, \omega, \Omega \\
\frac{\mathrm{d}Q}{\mathrm{d}t} &= \sum_{œ} \frac{\partial Q}{\partial œ} \dot{œ}
\end{aligned}
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- Other Lyapunov functions
  - Scalar-weighted sum of squares of subtractive differences of classical orbit elements (Ilgen, Chang *et al.*)
  - Scalar-weighted sum of [squares of] magnitudes of differences in angular momentum and eccentricity vectors (Chang *et al.*, Gustafson *et al.*)

Averaging of Natural Dynamics: J_2 in GTOC9

- Well-known mean drift rates:

$$\begin{aligned}\dot{\Omega} &= -\frac{3}{2} J_2 \left(\frac{r_{eq}}{p}\right)^2 n \cos i \\ \dot{\omega} &= \frac{3}{4} J_2 \left(\frac{r_{eq}}{p}\right)^2 n \left(5 \cos^2 i - 1\right)\end{aligned}$$

- Mean-anomaly rate (Gurfil):

$$\dot{M} = n \left[1 + \frac{3}{2} J_2 \left(\frac{r_{eq}}{a}\right)^2 \frac{1 - \frac{3}{2}\sin^2 i}{(1 - e^2)^{\frac{3}{2}}}\right]$$

- Coupling of Gauss's equations with drift rates to estimate optimal small impulses for changes in orbital elements

Synthesis

- Judicious use of analytical repertoire greatly focuses the broad search procedures
- Care needed not to remove too many degrees of freedom just for analytical convenience
- Use reasonable padding when filtering solutions based on analytical estimates