The Antipodes

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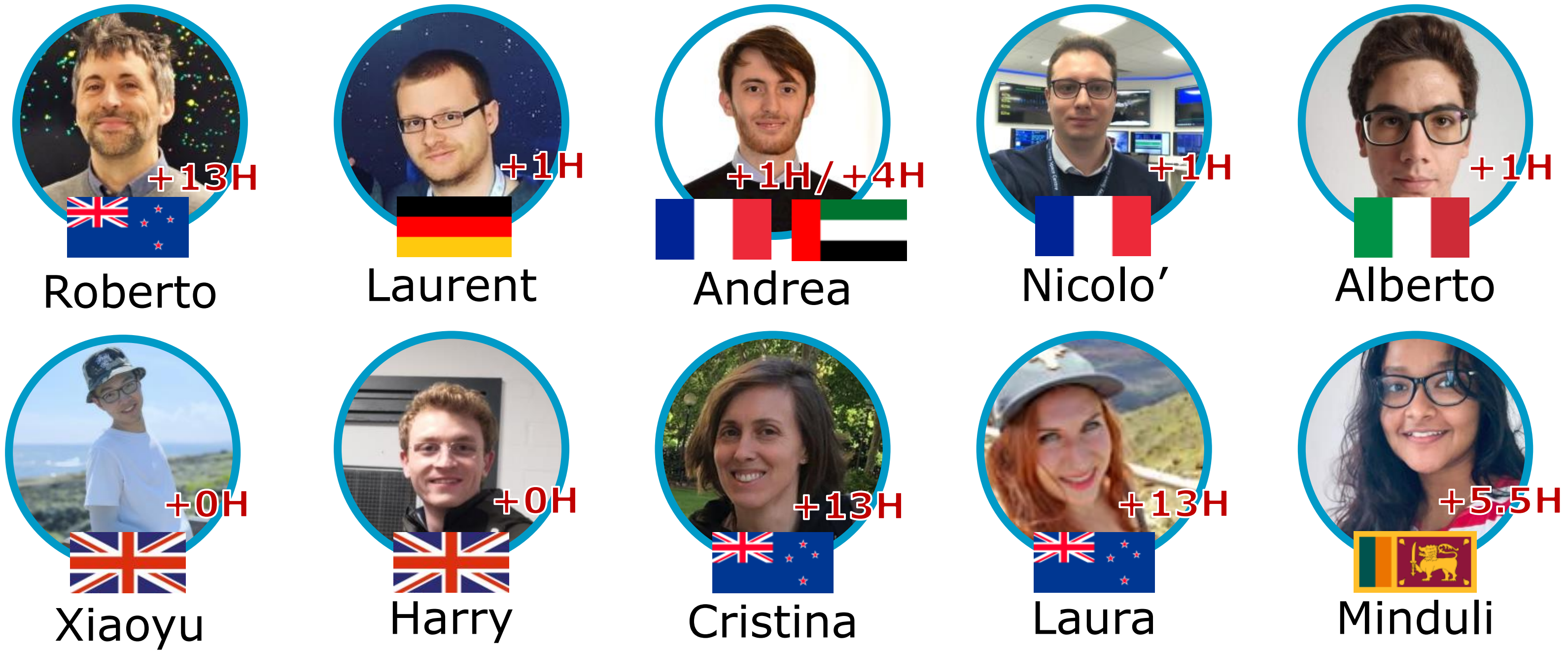
18 (and 19!) December 2021, GTOC 11 Workshop

The Team & Roles



Curiosity:

- no more than two people were co-located
- some of us have never met in person (yet)

The Approach

The Chains

The Tables

The Dispatcher

The Refinement

High-performance computing

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The Approach

Overview

Preliminary study

- Estimate of chain length
- Estimate ring location

Chains generation

- Select the asteroids
- Optimise bi-impulsive transfers

Tables generation

- Determine all the opportunities for asteroids to transfer to the stations

Dispatch

- Allocate asteroids to station
- Trim the chain and refine the Δv

Refinement & DSM

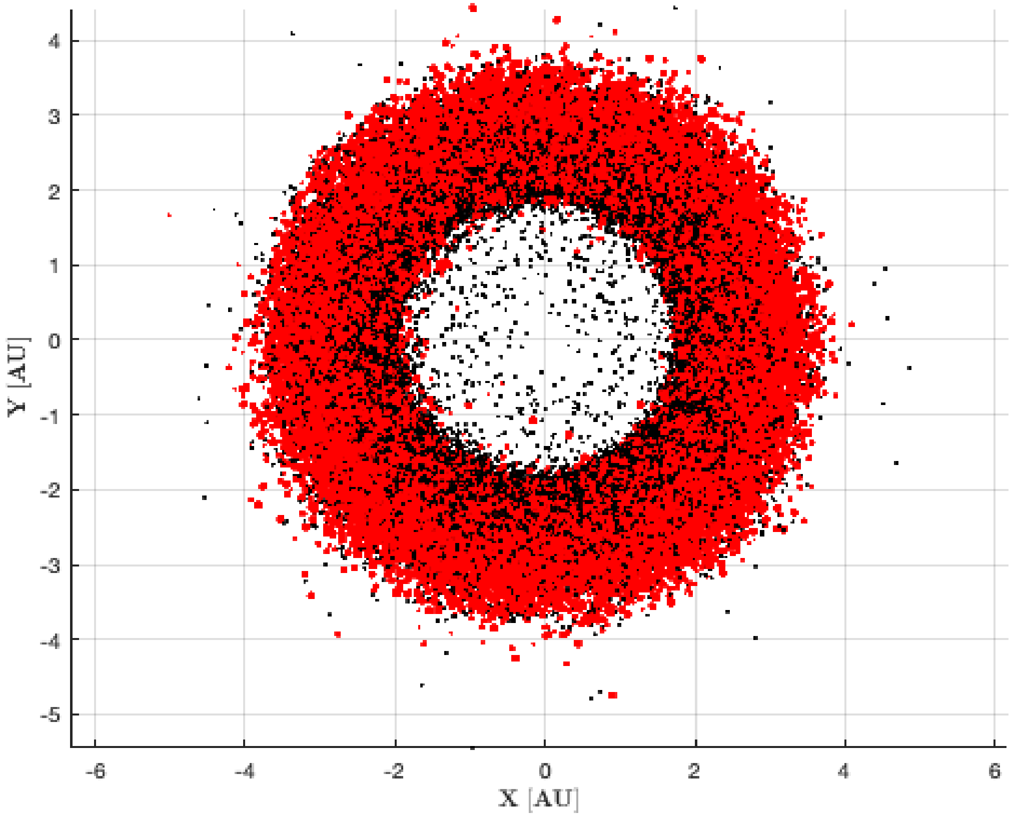
- Compute rendezvous transfers
- Add deep space maneuvers

Loop over different values of a ring

The Chains

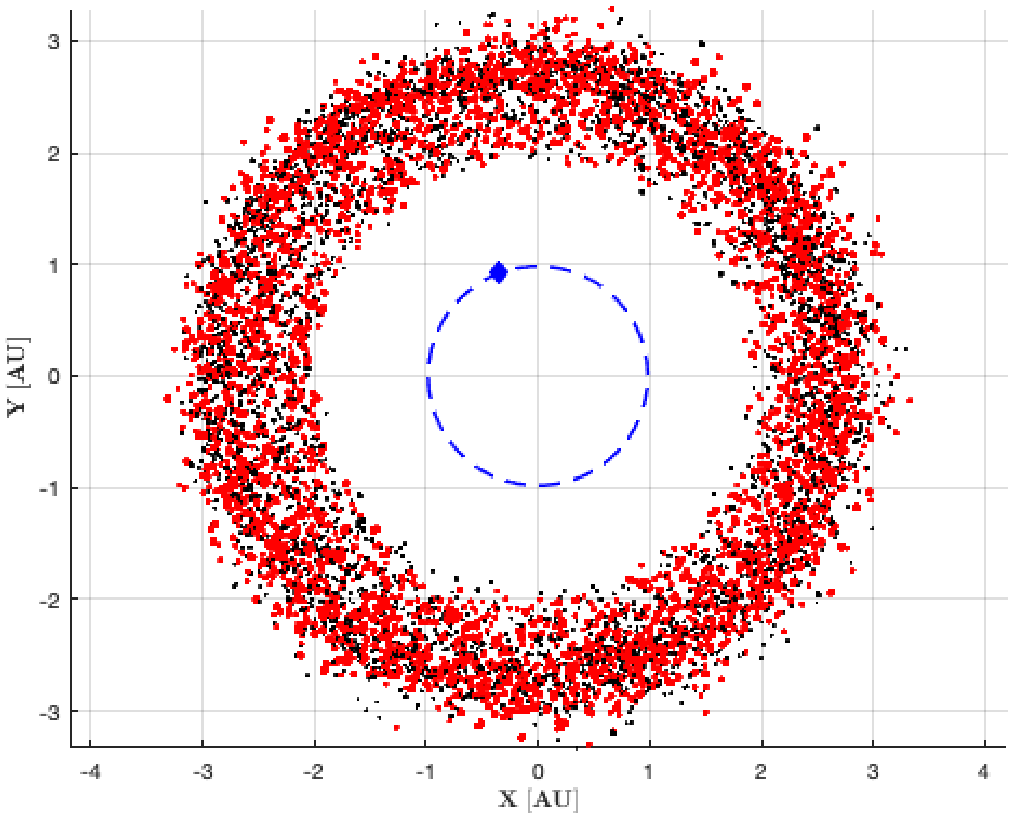
The Chains - Pruning

Raw Asteroid Catalogue



$$m_{Ast} > m_{mid}$$
$$i_{Ast} < i_{mid}$$
$$e_{Ast} < e_{mid}$$
$$a_{Ast} < 3.0 \text{ AU}$$

Pruned Asteroid Catalogue



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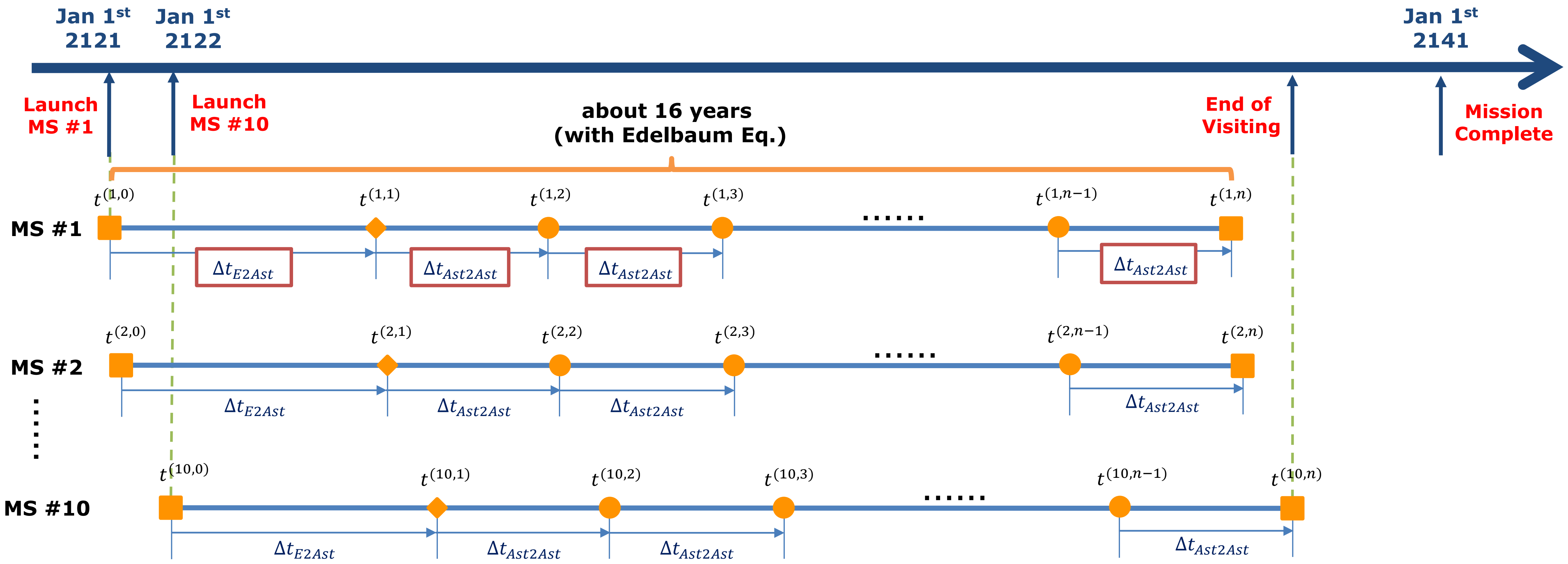
No. of Ast: 83453 → No. of Ast: 7625

The Chains – Modelling & Prel. Opt



Three design variables & Tree-Search based asteroids selection

$(\Delta t_{E2Ast}, \Delta t_{Ast2Ast}, a_{Dyson})$

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Tree Search - Transcription

The previous analysis allows to transcribe the problem from a mixed-integer formulation into a pure **combinatorial** one.

$J = f(S, T, a_{ring}, i_{ring}, \Omega_{ring})$
S: asteroids sequence
T: visiting epochs
$[a_{ring}, i_{ring}, \Omega_{ring}]$: ring parameters

Transcription

$J = f(S, \hat{T}, \hat{a}_{ring}, \hat{\imath}_{ring}, \hat{\Omega}_{ring})$
S: asteroids sequence
$\hat{T}$: fixed visiting epochs
$[\hat{a}_{ring}, \hat{\imath}_{ring}, \hat{\Omega}_{ring}]$: fixed ring parameters

S, T and ring parameters vary

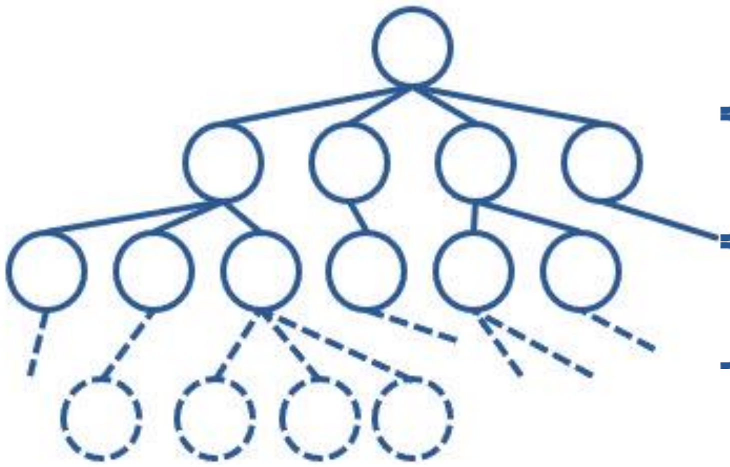
Only S varies

In this way, one can apply any of the common approaches used for combinatorial optimization. Due to the nature of GTOC problems, usually it is possible to model the search space as some form of **tree graph**.

Tree Search - Branching

On the tree graph, each **node** encodes a **trajectory** with increasing number of asteroids per tree level.

We perform the branching breadth-wise, i.e., one depth level at a time. Branching successive nodes implies solution of a **Lambert Problem (LP)**:



$$\Delta v_{leg}(i,j) = |\vec{v}_{j,departure} - \vec{v}'_{j,arrival}|$$

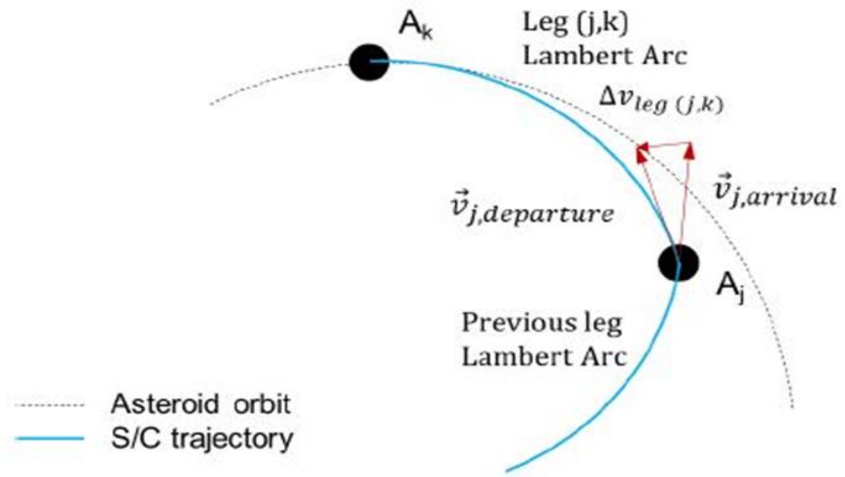
Where:

$\vec{v}'_{j,arrival} \leftarrow \vec{v}_{j,arrival} + x(\vec{v}_{Aj} - \vec{v}_{j,arrival})$

$x: |\vec{v}_{Aj} - \vec{v}'_{j,arrival}| \leq 2\ km/s$

$\vec{v}_{j,arrival}$: LP solution for previous leg

$\vec{v}_{j,departure}$: LP solution for leg (j,k)



Tree Search - Pruning

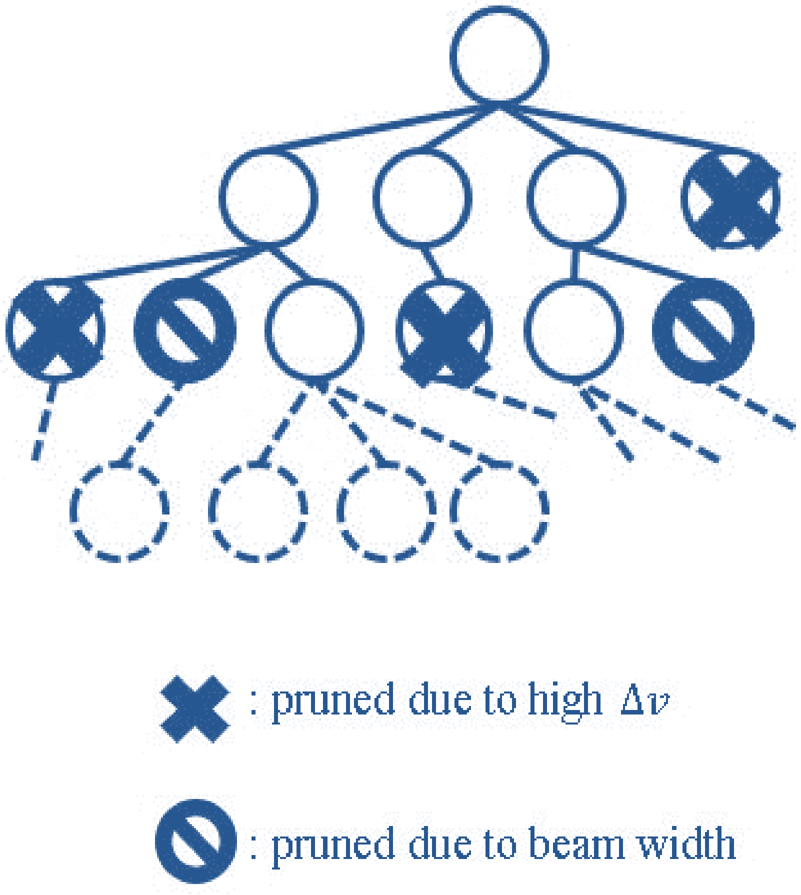
To prevent the tree expansion to become **intractable**, we applied some **pruning criteria**:

- Nodes with $\Delta v_{tot} > 30\ \mathrm{km/s}$ are no further expanded
- Nodes with $\Delta v_{leg} > 1.5\ \mathrm{km/s}$ are no further expanded
- **Beam Search** strategy (only best Beam Width options are kept for further expansion)

The function to be minimized is:

$$J = 10^{-10} \frac{m_{f_{tot}}}{a_{ring} \left(1 + \frac{\Delta v_{tot}}{50}\right)^2}$$

$m_{f_{tot}}$: total final mass at the ring per node
Δv_{tot}: accumulated Δv per node



Epochs and Ring Refinement

1. Refinement of the **encounter dates** to further lower the cost of each mothership

$J = f(\hat{S}, T, \hat{a}_{ring}, \hat{\imath}_{ring}, \hat{\Omega}_{ring})$
$\hat{S}$: fixed asteroids sequence
T: visiting epochs
$[\hat{a}_{ring}, \hat{\imath}_{ring}, \hat{\Omega}_{ring}]$: fixed ring parameters

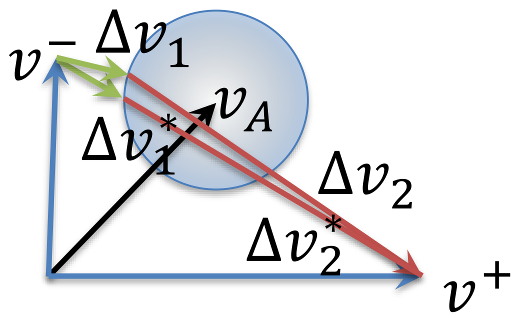
Only T varies

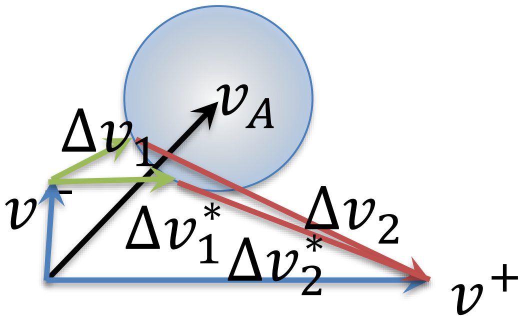
2. Refinement of the **ring parameters** to provide a better understanding of its size and orientation

$J = f(\hat{S}, \hat{T}, a_{ring}, i_{ring}, \Omega_{ring})$
$\hat{S}$: fixed asteroids sequence
$\hat{T}$: fixed visiting epochs
$[a_{ring}, i_{ring}, \Omega_{ring}]$: ring parameters

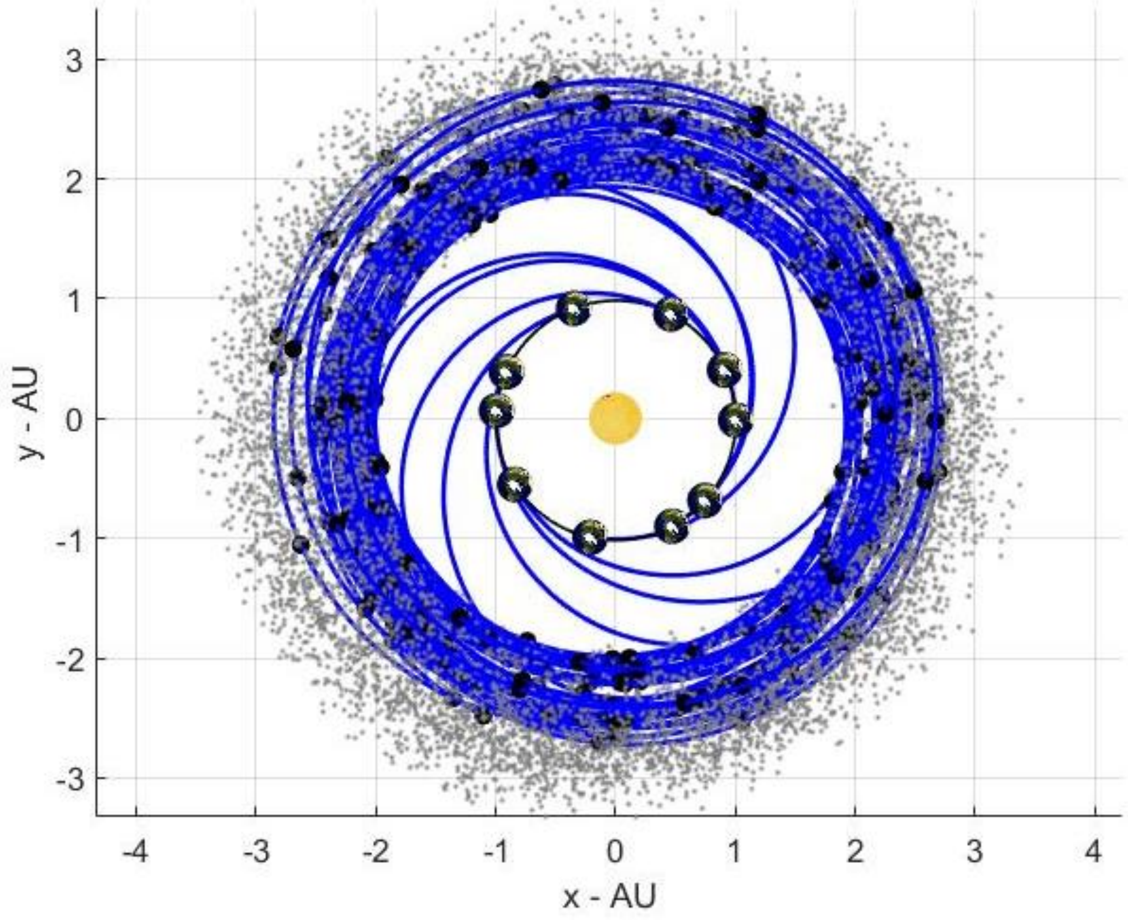
Only ring parameters vary

3. Refinement of **motherships Δv** w.r.t. asteroids approaching velocity constraint





Asteroids Chains



Computational time: ≈ 3 h, MATLAB

Chains are now built and asteroids ready to be transferred to the optimal ring

The Tables

Getting to the Dyson Orbit

1. Solve linearised energy-optimal problem with variable thrust analytically

Costates at t_0

2. Solve the nonlinear energy optimal problem with the true longitude as a free variable (L free) using the shooting method, with costates from 1. as guess.

Costates at t_0

3. Solve the time optimal problem with the true longitude as a free variable (L free) using the shooting method with costates from 2. as initial guess.

- Method used to solve the optimal control problems: indirect method

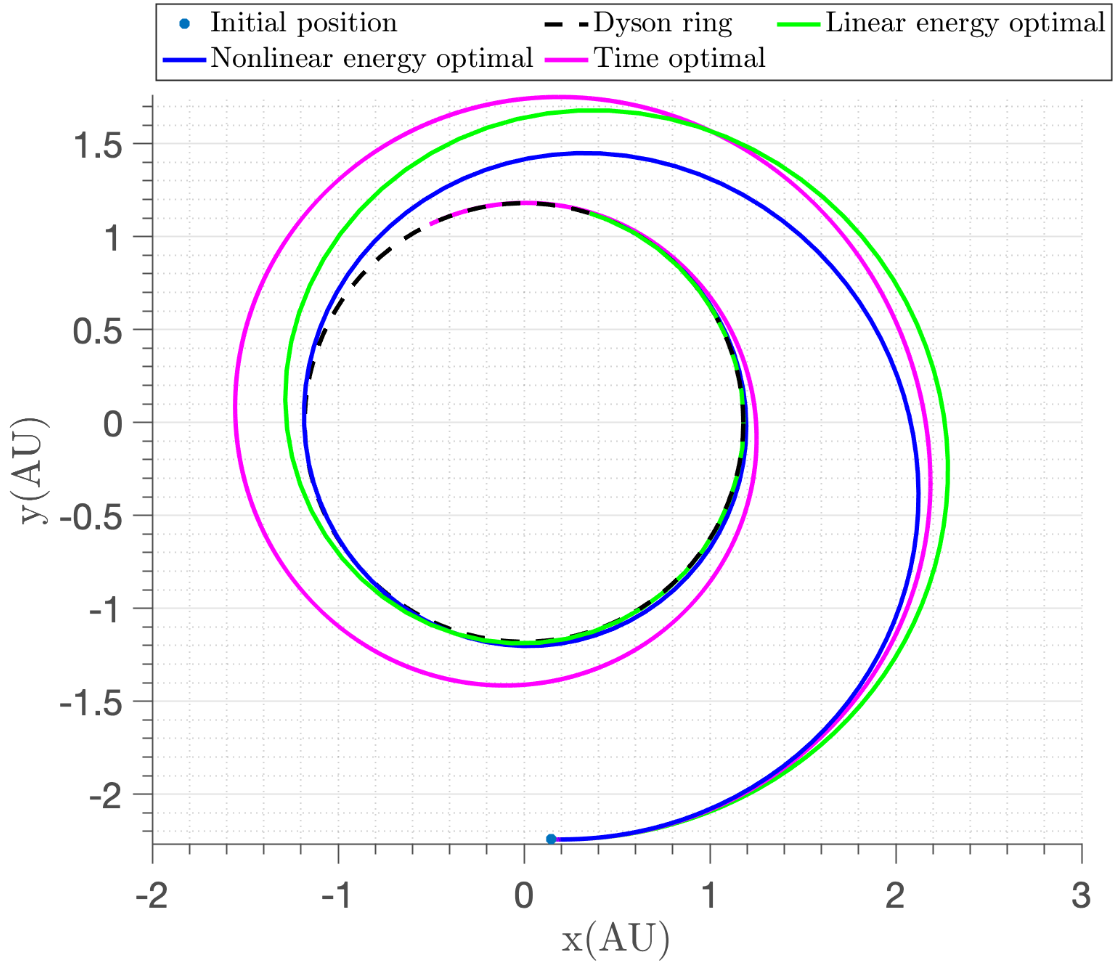


Table - Data Collation

1. $t_{0,primary} = t_{flyby} + 30\ d + k\frac{\text{Asteroid orbital period}}{16}$, k = 1:16*.
2. For each start time in $t_{0,primary}$:
   a) Solve the L free time optimal problem.
   b) Calculate the phase difference between each station and the point of arrival at the time of arrival.

Departure time	Phase difference upon arrival at station				Arrival time	Initial costates
	1	2	…	12		
$t_{0,primary}(1)$	ΔL(1,1)	ΔL(1,2)	…	ΔL(1,12)	$t_{f,primary}(1)$	$\lambda_{t_0}(1,6)$
$t_{0,primary}(2)$	ΔL(2,1)	ΔL(2,2)	…	ΔL(2,12)	$t_{f,primary}(2)$	$\lambda_{t_0}(2,6)$
…	..	…	…	…	…	…
$t_{0,primary}(16)$	ΔL(16,1)	ΔL(16,2)	…	ΔL(16,12)	$t_{f,primary}(16)$	$\lambda_{t_0}(16,6)$

*16 points were used as it provided a good tradeoff between computational time and accuracy.

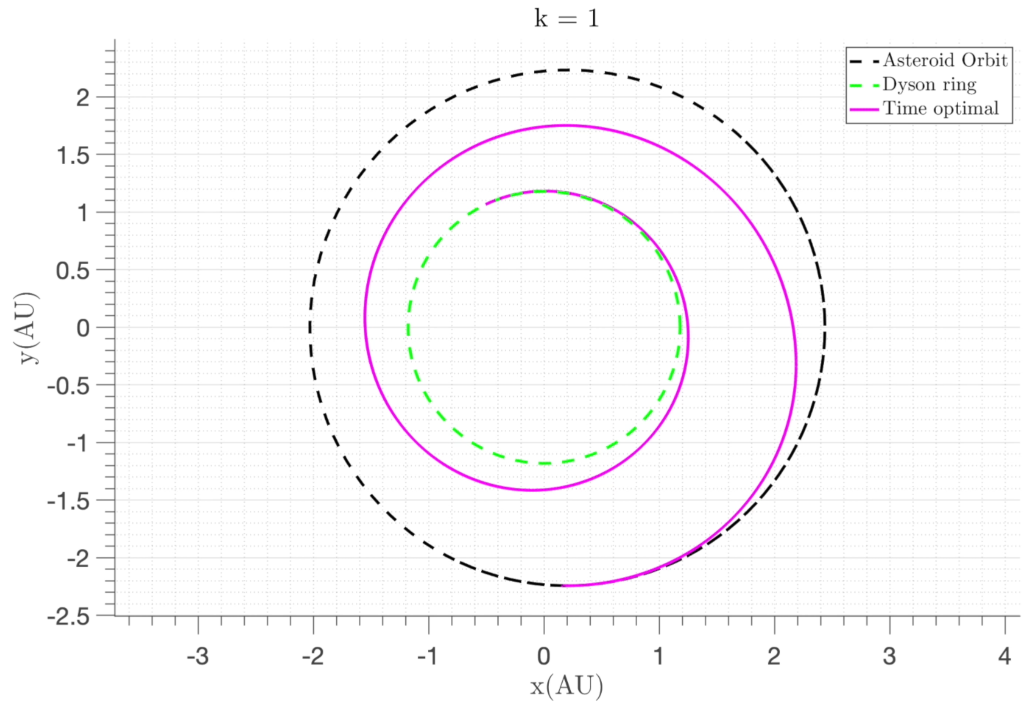


Table - Continuation

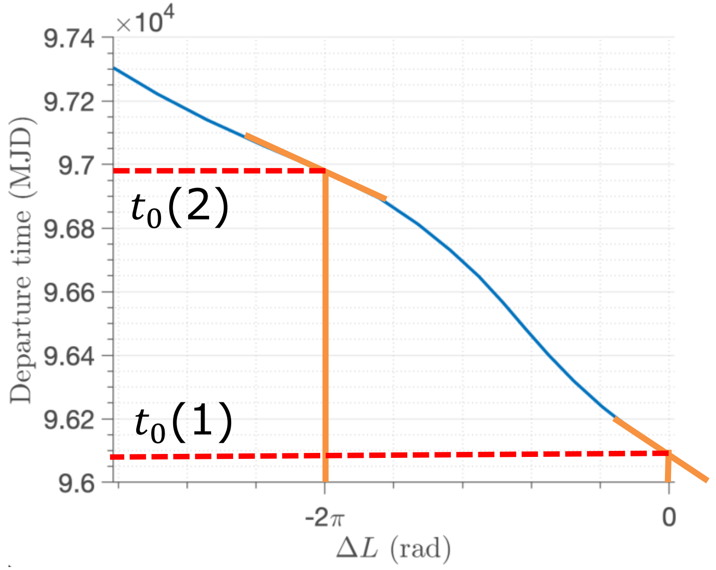
3. While $t_{f,max}$ < Mission end date (January 1st , 2141):
   a) Departure time : $t_{0,secondary} = t_{0,primary} + n_p$(Asteroid period) ($n_p$: Orbital period index)
   b) Time of flight : $tof_{secondary} = tof_{primary}$ ($L_{f,primary} = L_{f,secondary}$, $\lambda_{secondary} = \lambda_{primary}$)
   c) Arrival time : $t_{f,secondary} = tof_{secondary} + t_{0,secondary}$, calculate ΔL

	Departure time	Phase difference upon arrival at station				Arrival time	Initial costates
		1	2	…	12		
1st Asteroid period after flyby	$t_{0,primary}(1)$	ΔL(1,1)	ΔL(1,2)	…	ΔL(1,12)	$t_{f,primary}(1)$	$\lambda_{t_0}(1,6)$
	…	..	…	…	…	…	…
	$t_{0,primary}(16)$	ΔL(16,1)	ΔL(16,2)	…	ΔL(16,12)	$t_{f,primary}(16)$	$\lambda_{t_0}(16,6)$
2nd Asteroid period after flyby	$t_{0,secondary}(1)$	ΔL(17,1)	ΔL(17,2)	…	ΔL(17,12)	$t_{f,secondary}(1)$	$\lambda_{t_0}(1,6)$
	…	..	…	…	…	…	…
	$t_{0,primary}(16)$	ΔL(32,1)	ΔL(32,2)	…	ΔL(32,12)	$t_{f,secondary}(16)$	$\lambda_{t_0}(16,6)$
	…	..	…	…	…	…	…

Table - Phase Matching

4. Repeat for each asteroid period, for each station :
   a) Interpolate to get t_0, t_f and $\boldsymbol{\lambda}_{t_0}$ at which $\Delta L = 2k\pi, k \in \mathbb{Z}$

Departure time	Station 1 : Phase difference upon arrival	Arrival time	Initial costates
$t_{0,primary}(1)$	ΔL(1,1)	$t_{f,primary}(1)$	$\boldsymbol{\lambda}_{t_0}(1,6)$
…	..	…	…
$t_{0,primary}(16)$	ΔL(16,1)	$t_{f,primary}(16)$	$\boldsymbol{\lambda}_{t_0}(16,6)$



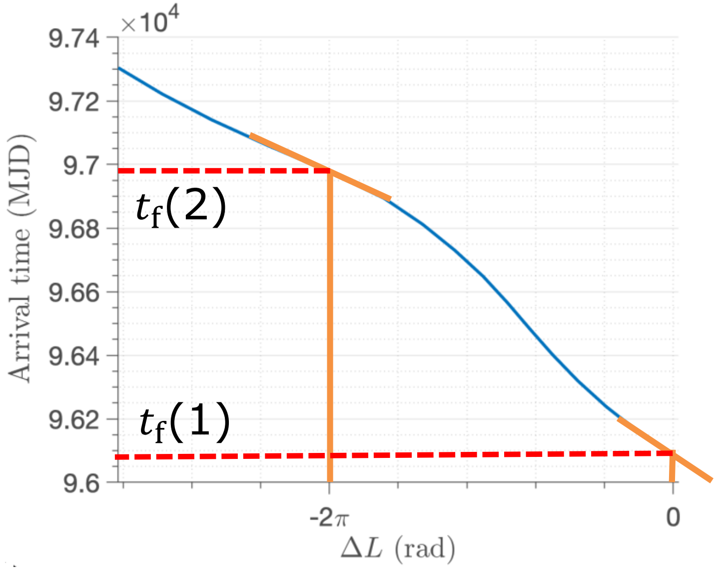
Interpolated result

Asteroid ID	xx
t_0	$t_0(1)$, $t_0(2)$
t_f	
λ_{t_0}	
m_f	

Table - Phase Matching

4. Repeat for each asteroid period, for each station :
   a) Interpolate to get t_0, t_f and $\boldsymbol{\lambda}_{t_0}$ at which $\Delta L = 2k\pi, k \in \mathbb{Z}$

Departure time	Station 1 : Phase difference upon arrival	Arrival time	Initial costates
$t_{0,primary}(1)$	ΔL(1,1)	$t_{f,primary}(1)$	$\boldsymbol{\lambda}_{t_0}(1,6)$
…	..	…	…
$t_{0,primary}(16)$	ΔL(16,1)	$t_{f,primary}(16)$	$\boldsymbol{\lambda}_{t_0}(16,6)$



Interpolated result

Asteroid ID	xx
t_0	$t_0(1)$, $t_0(2)$
t_f	$t_f(1)$, $t_f(2)$
λ_{t_0}	$\lambda_{t_0}(1,6)$, $\lambda_{t_0}(2,6)$
m_f	$m_f(1)$, $m_f(2)$

b) Calculate m_f.

$$m_f = m_0^{ast} - \alpha m_0^{ast}(t_f - t_{start})$$

Table Assembly

Asteroids	Station						
	1		2		…	12	
1	Asteroid ID	xx					
	t_0	$t_{01}^{xx}(1),\ t_{01}^{xx}(2),\ \dots$					
	t_f	$t_{f1}^{xx}(1),\ t_{f1}^{xx}(2),\dots$					
	λ_{t_0}	$\lambda_{t_01}^{xx}(1),\ \lambda_{t_01}^{xx}(2),\dots$					
	m_f	$m_{f1}^{xx}(1),\ m_{f1}^{xx}(2),\dots$					

Table Assembly

Asteroids	Station						
	1		2		…	12	
1	Asteroid ID	xx	Asteroid ID	xx	…	Asteroid ID	xx
	t_0	$t_{01}^{xx}(1),\ t_{01}^{xx}(2),\ \ldots$	t_0	$t_{02}^{xx}(1),\ t_{02}^{xx}(2),\ \ldots$		t_0	$t_{012}^{xx}(1),\ t_{012}^{xx}(2),\ \ldots$
	t_f	$t_{f1}^{xx}(1),\ t_{f1}^{xx}(2),\ldots$	t_f	$t_{f2}^{xx}(1),\ t_{f2}^{xx}(2),\ldots$		t_f	$t_{f12}^{xx}(1),\ t_{f12}^{xx}(2),\ldots$
	λ_{t_0}	$\lambda_{t_0 1}^{xx}(1),\ \lambda_{t_0 1}^{xx}(2),\ldots$	λ_{t_0}	$\lambda_{t_0 2}^{xx}(1),\ \lambda_{t_0 2}^{xx}(2),\ldots$		λ_{t_0}	$\lambda_{t_0 12}^{xx}(1),\ \lambda_{t_0 12}^{xx}(2),\ldots$
	m_f	$m_{f1}^{xx}(1),\ m_{f1}^{xx}(2),\ldots$	m_f	$m_{f2}^{xx}(1),\ m_{f2}^{xx}(2),\ldots$		m_f	$m_{f12}^{xx}(1),\ m_{f12}^{xx}(2),\ldots$

Table Assembly

Asteroids	Station						
	1		2		…	12	
1	Asteroid ID	xx	Asteroid ID	xx	…	Asteroid ID	xx
	t_0	$t_{01}^{xx}(1)$, $t_{01}^{xx}(2)$, …	t_0	$t_{02}^{xx}(1)$, $t_{02}^{xx}(2)$, …		t_0	$t_{012}^{xx}(1)$, $t_{012}^{xx}(2)$, …
	t_f	$t_{f1}^{xx}(1)$, $t_{f1}^{xx}(2)$,…	t_f	$t_{f2}^{xx}(1)$, $t_{f2}^{xx}(2)$,…		t_f	$t_{f12}^{xx}(1)$, $t_{f12}^{xx}(2)$,…
	λ_{t_0}	$\lambda_{t_0 1}^{xx}(1)$, $\lambda_{t_0 1}^{xx}(2)$,…	λ_{t_0}	$\lambda_{t_0 2}^{xx}(1)$, $\lambda_{t_0 2}^{xx}(2)$,…		λ_{t_0}	$\lambda_{t_0 12}^{xx}(1)$, $\lambda_{t_0 12}^{xx}(2)$,…
	m_f	$m_{f1}^{xx}(1)$, $m_{f1}^{xx}(2)$,…	m_f	$m_{f2}^{xx}(1)$, $m_{f2}^{xx}(2)$,…		m_f	$m_{f12}^{xx}(1)$, $m_{f12}^{xx}(2)$,…
2	Asteroid ID	yy	Asteroid ID	yy	…	Asteroid ID	yy
	t_0	$t_{01}^{yy}(1)$, $t_{01}^{yy}(2)$, …	t_0	$t_{02}^{yy}(1)$, $t_{02}^{yy}(2)$, …		t_0	$t_{012}^{yy}(1)$, $t_{012}^{yy}(2)$, …
	t_f	$t_{f1}^{yy}(1)$, $t_{f1}^{yy}(2)$,…	t_f	$t_{f2}^{yy}(1)$, $t_{f2}^{yy}(2)$,…		t_f	$t_{f12}^{yy}(1)$, $t_{f12}^{yy}(2)$,…
	λ_{t_0}	$\lambda_{t_0 1}^{yy}(1)$, $\lambda_{t_0 1}^{yy}(2)$,…	λ_{t_0}	$\lambda_{t_0 2}^{yy}(1)$, $\lambda_{t_0 2}^{yy}(2)$,…		λ_{t_0}	$\lambda_{t_0 12}^{yy}(1)$, $\lambda_{t_0 12}^{yy}(2)$,…
	m_f	$m_{f1}^{yy}(1)$, $m_{f1}^{yy}(2)$,…	m_f	$m_{f2}^{yy}(1)$, $m_{f2}^{yy}(2)$,…		m_f	$m_{f12}^{yy}(1)$, $m_{f12}^{yy}(2)$,…
…	…		…		…	…	

Computational time: ≈ 12 m
(313 asteroid sequence, C++)

The Dispatcher

The Dispatcher

Objective function

- Inputs: asteroids arrival time and mass to each station
- Optimisation variables:
  - station sequence (x12)
  - number of asteroids (x12)
  - min delta time (x1)

1. First allocation
2. Distribution
3. Trimming of Motherships

- Output:
  $J = f(m_{min}, \Delta v)$ GTOC11 based

The Dispatcher

Objective function

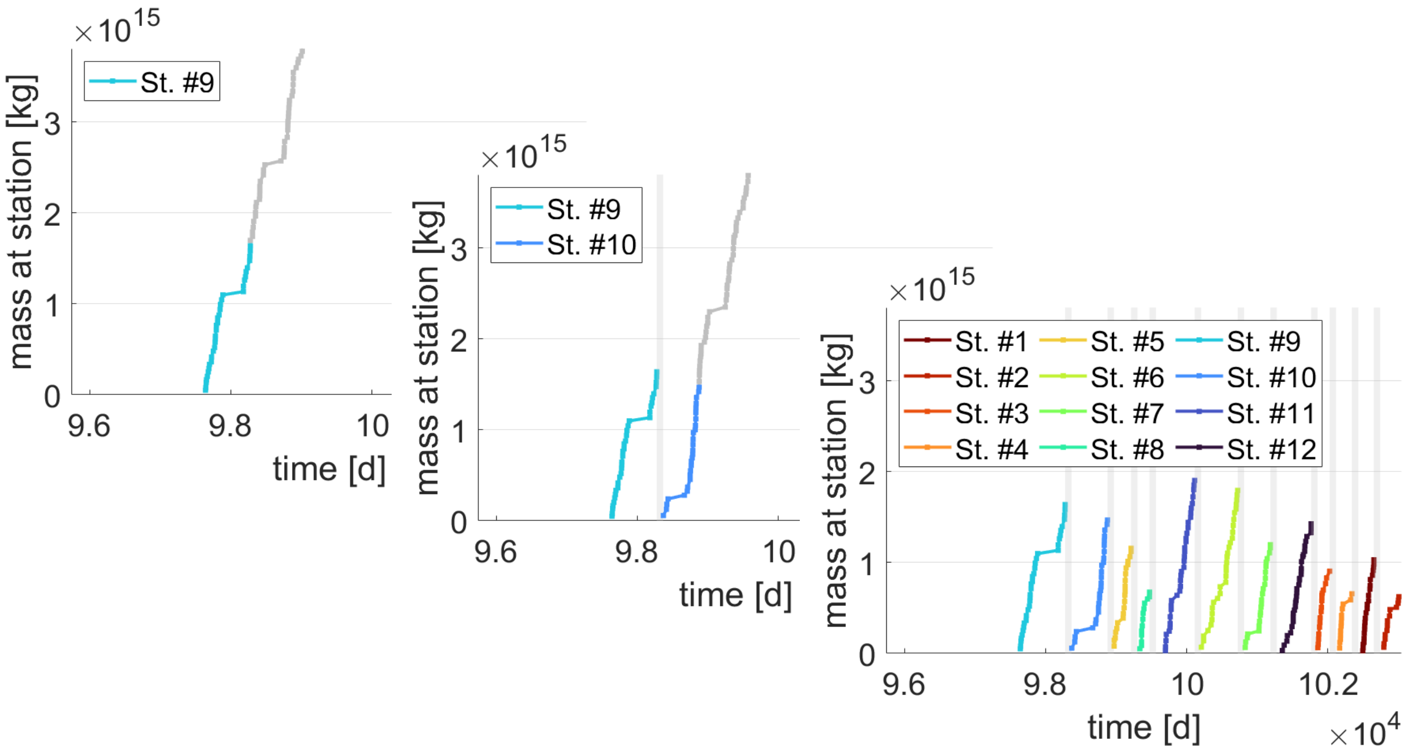
- Inputs: asteroids arrival time and mass to each station
- Optimisation variables:
  - station sequence (x12)
  - number of asteroids (x12)
  - min delta time (x1)

1. **First allocation**
2. Distribution refinement
3. Trimming of Motherships

- Output:
  J = f(m_{min},Δv) GTOC11 based

First allocation

a) Select the station
b) Ordering asteroids in arrival time, compatibly with arrival time constraints
c) Select the first n asteroids
d) Repeat a) to c) until all stations are filled



The Dispatcher

Objective function

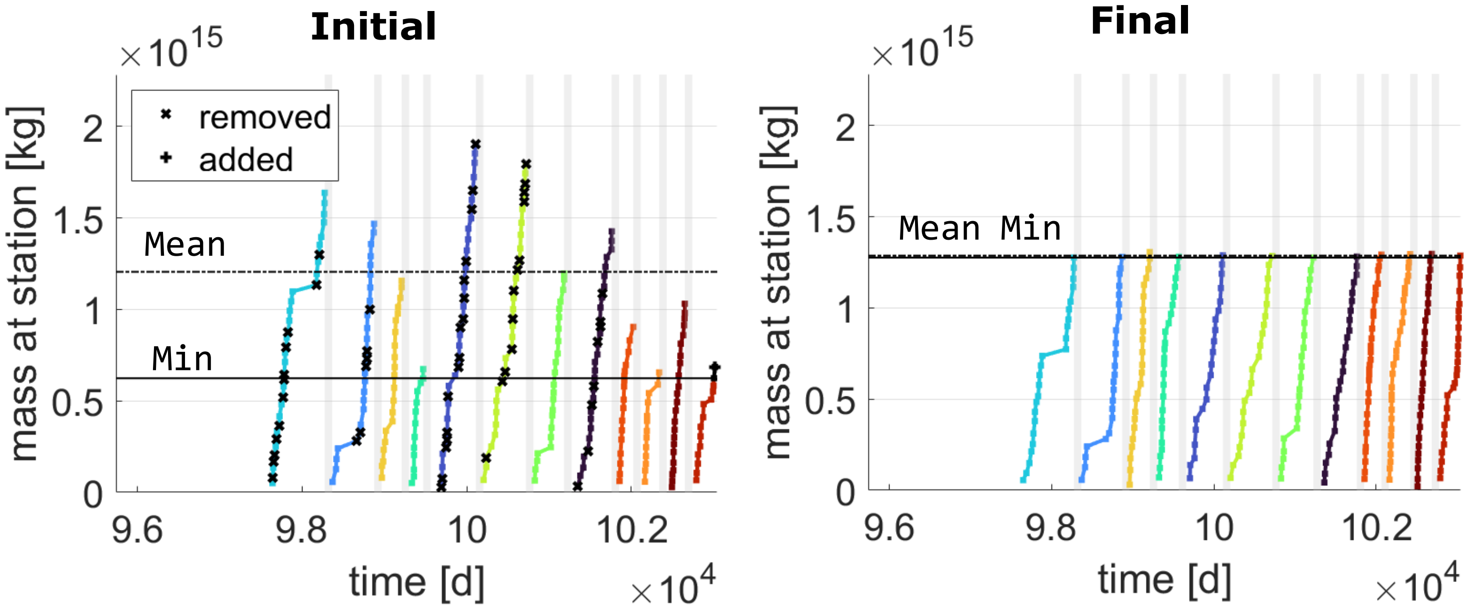
- Inputs: asteroids arrival time and mass to each station
- Optimisation variables:
  - station sequence (x12)
  - number of asteroids (x12)
  - min delta time (x12)

1. First allocation
2. **Distribution refinement**
3. Trimming of Motherships

- Output:
  $J = f(m_{min}, \Delta v)$ GTOC11 based

Distribution refinement

a) Identification of stations to be reduced ($m > m_{mean}$)
b) Remove asteroids with minimum mass first
c) Identification of station with minimum mass ($m = m_{min}$)
d) Allocation of asteroid with earliest arrival time
e) Repeat a) to d) until no asteroids can be allocated



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The Dispatcher

Objective function

- Inputs: asteroids arrival time and mass to each station
- Optimisation variables:
  - station sequence (x12)
  - number of asteroids (x12)
  - min delta time (x1)

1. First allocation
2. Distribution refinement
3. **Trimming of Motherships**

- Output:
  $J = f(m_{min},\Delta v)$ GTOC11 based

Trimming of Motherships

a) Identification of not used asteroids at the end of the MS chain
b) Remove asteroids from the MS and update Δv

Mothership No	No of Asteroid Removed	Δv gain (km/s)
1	1	0.27
2	1	0.25
4	2	0.55
6	4	3.47
7	2	0.11
8	2	0.98
9	5	1.80
10	2	1.06
Total	19	8.48

The Dispatcher

Optimisation approach

- Variable bound
  - number of asteroids 12-36
  - min delta time 90.25-95
- First assessments: 1-2 GA runs
- Detailed assessment: 5 PSO runs on HPC
  - Population 4000
  - Stall iterations 100
  - HybridFunction fmincon

HPC

- University of Surrey's Eureka High Performance Computing (HPC) Cluster
- Access for Dispatch: ≈ 48 hours prior to deadline
- 24 workers per task
- Computational time: ≈ 2 hours per task

Ring (AU)	# Asteroids Before	Objective	Ring (AU)	# Asteroids Before	Objective
1.000	313	5684	1.065	300	5822
1.040	290	5795	1.070	300	5872
1.045	300	5797	1.080	300	5863
1.050	313	5975	1.100	313	5849
1.055	300	5855	1.190	300	5641
1.057	300	5856	1.200	300	5661
1.060	313	5978	1.300	300	5666

	Specification
OS	Centos 7
Fabric	Intel Omni-Path (**OP**) and Infiniband (**IB**)
1 x Login Node	eureka.surrey.ac.uk
16 x CPU node (OP)	Intel Xeon E5-2660 v4 @ 2.0 GHz / 128 GB RAM
33 x CPU node (OP)	Intel Xeon Gold 5120 @ 2.20 GHz / 192 GB RAM
13 x CPU node (OP)	Intel Xeon E5-2680-v2 @ 2.80 GHz / 128 GB RAM
2 x CPU node (OP)	Intel Xeon Gold 5120 @ 2.20 GHz / 375 GB RAM
8 x CPU node (IB)	Intel Xeon E5-2470-0 @ 2.30 GHz / 64 GB RAM
12 x CPU node (IB)	Intel Xeon E5-2670 @ 2.60 GHz / 64+ GB RAM
2 x CPU node (IB)	Intel Xeon E5-2670-v2 @ 2.50 GHz / 128 GB RAM
6 x CPU node (IB)	Intel Xeon E5-2697-v2 @ 2.70 GHz / 128 GB RAM
1 x High Mem node (IB)	Intel Xeon E5-2670 @ 2.60 GHz / 256 GB RAM
6 x High Mem node (IB)	Intel Xeon E5-2670 v2 @ 2.50GHz / 256 GB RAM
3 x GPU node (IB)	Intel Xeon E5-2670 v2 @ 2.50GHz / 256 GB RAM + NVIDIA Tesla K20m
Parallel Storage	BeeGFS @ 56 TB
Standard storage	NFS @ ~5 TB
Queue	Slurm

The Refinement

From Tables to Rendezvous

<u>Inputs from interpolation & Dispatcher:</u>
- Launch time
- Target station
- Time of flight
- Initial costate guesses

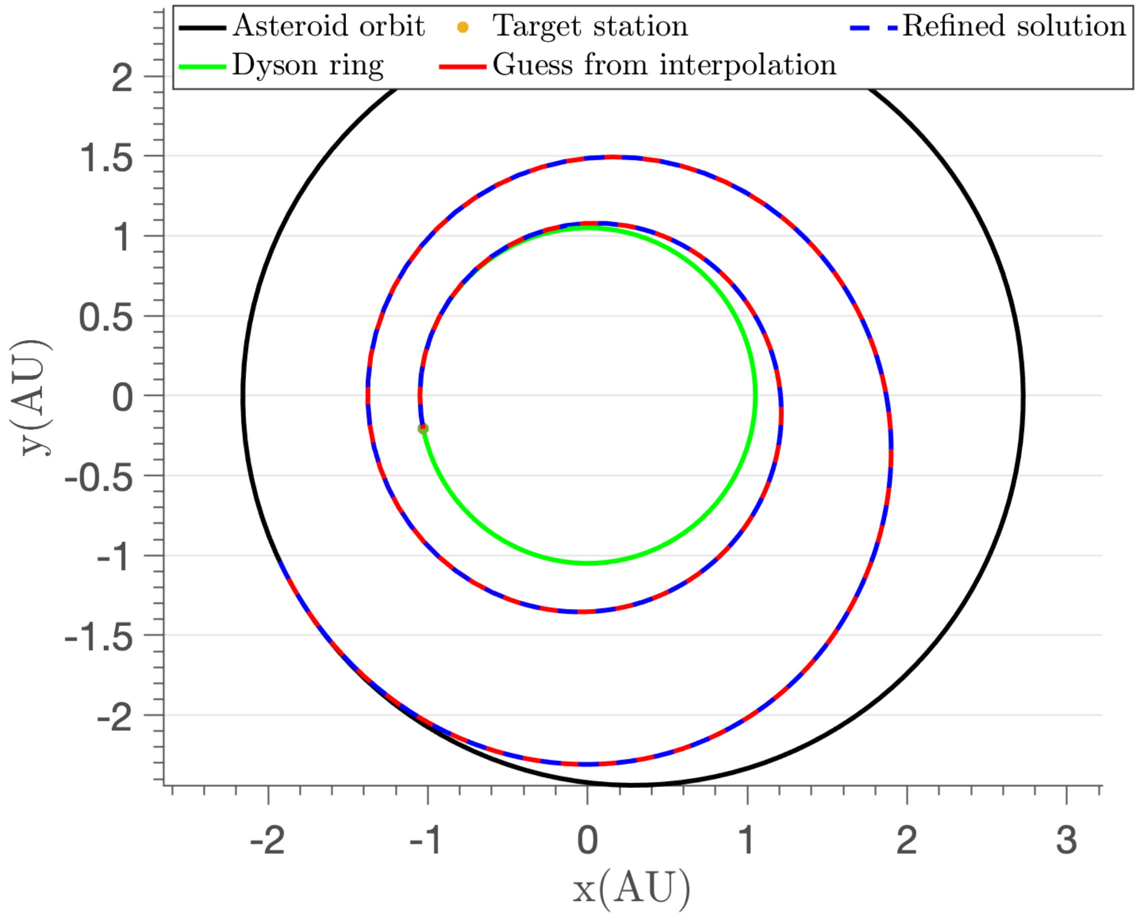
↓

Time optimal rendezvous problem

↓

<u>Output:</u>
- Refined time optimal rendezvous solution

Computational time: ≈ 10 m
(293 asteroids sequence, MATLAB)



Deep Space Manoeuvres (DSM)

Input:
- Trimmed chains from the Dispatcher

↓

- A single DSM in each Lambert's arc for fixed rendezvous epochs
- Solve a local parametric optimization problem

↓

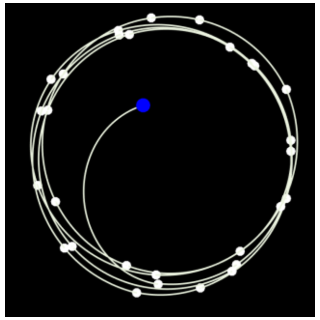
Output:
- Refined chains Δv

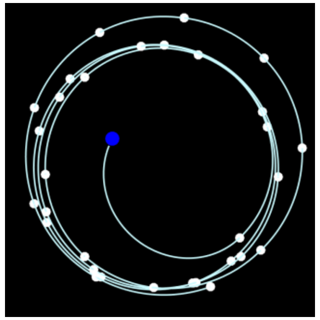
Before DSM J = 5975
After DSM J = 5992

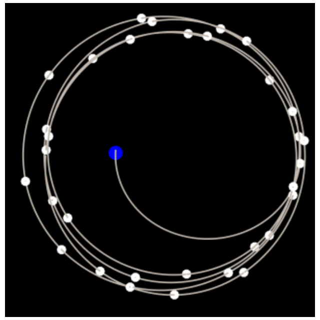
Computational time: ≈15 m
(293 asteroid sequence), MATLAB

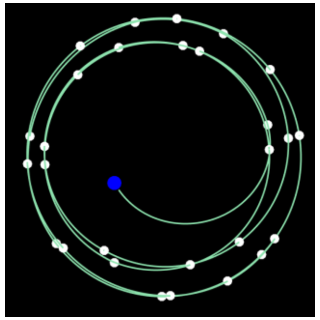
Mothership No	No of Asteroid Removed	No of DSM	Δv_{DSM} (m/s)	Δv gain (m/s)
1	0	21	2.09e-8	191.89
2	0	22	2.30e-8	175.36
3	0	12	7.32e-8	58.94
4	0	0	0.00	0.00
5	0	18	1.48e-8	42.48
6	0	14	1.15e-8	27.79
7	0	16	1.13e-8	20.04
8	0	17	1.47e-8	3.23
9	1	14	175.50	482.02
10	0	0.00	0.00	0.00
Total Δv			155.50	1001.74

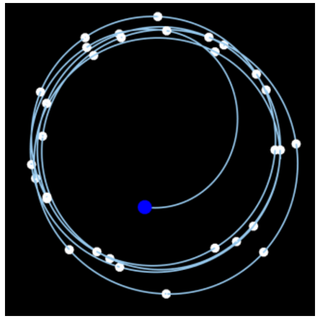
Final Solution - Mothership Chains

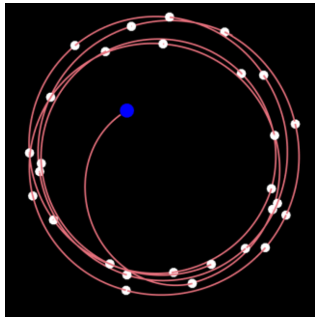

Mothership 1

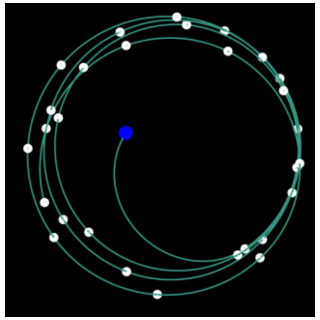

Mothership 2

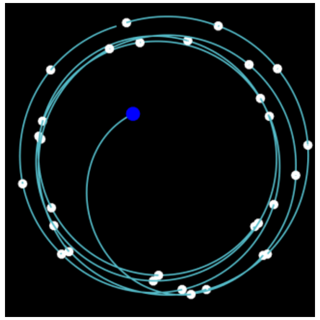

Mothership 3

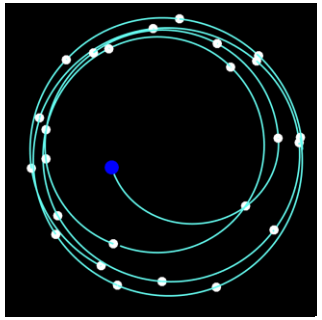

Mothership 4

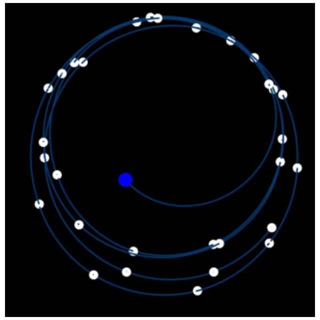

Mothership 5


Mothership 6


Mothership 7

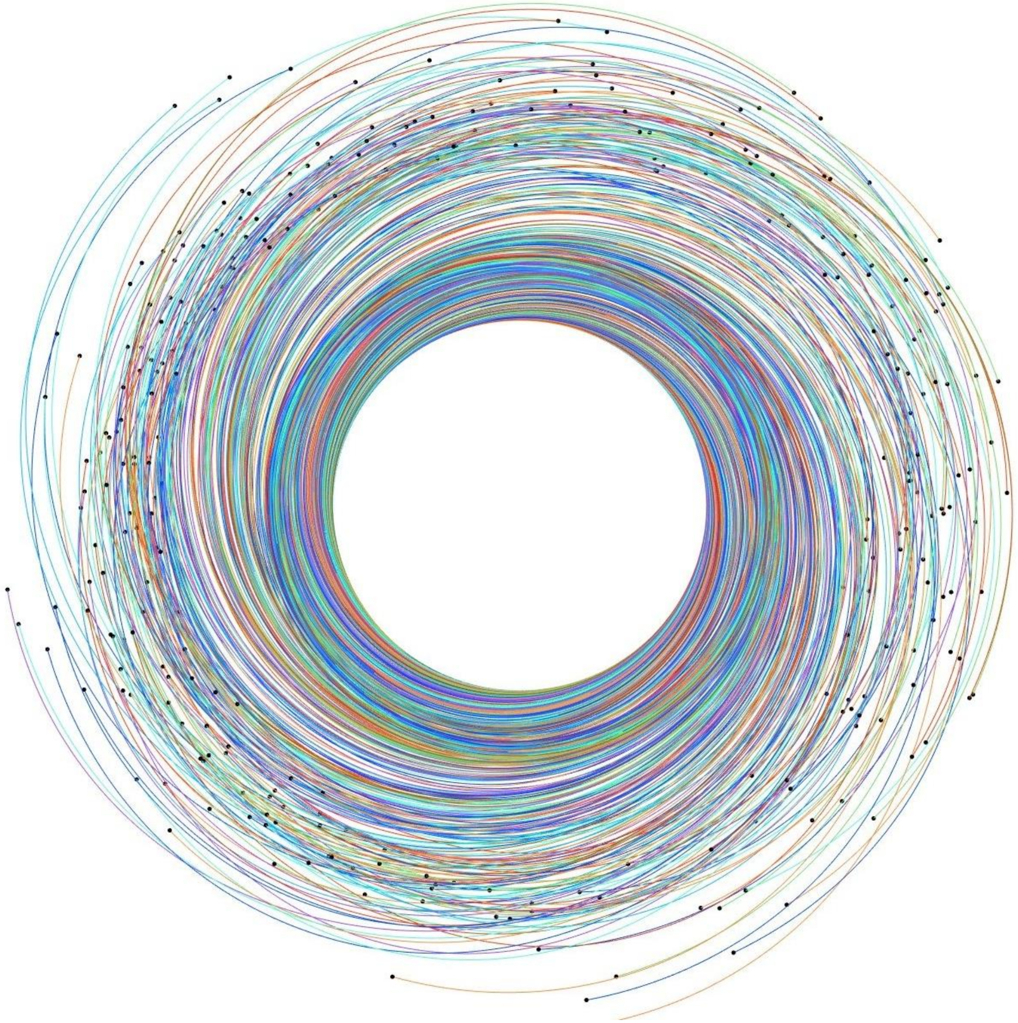

Mothership 8


Mothership 9


Mothership 10

Mother ship No	No of Asteroids Visited	Δv_{total} (km/s)
1	29	18.96
2	31	17.04
3	32	20.62
4	28	18.84
5	31	21.25
6	28	18.75
7	29	18.46
8	30	22.27
9	25	17.98
10	30	20.69
Total Δv		194.90

Final Solution - Dyson Ring

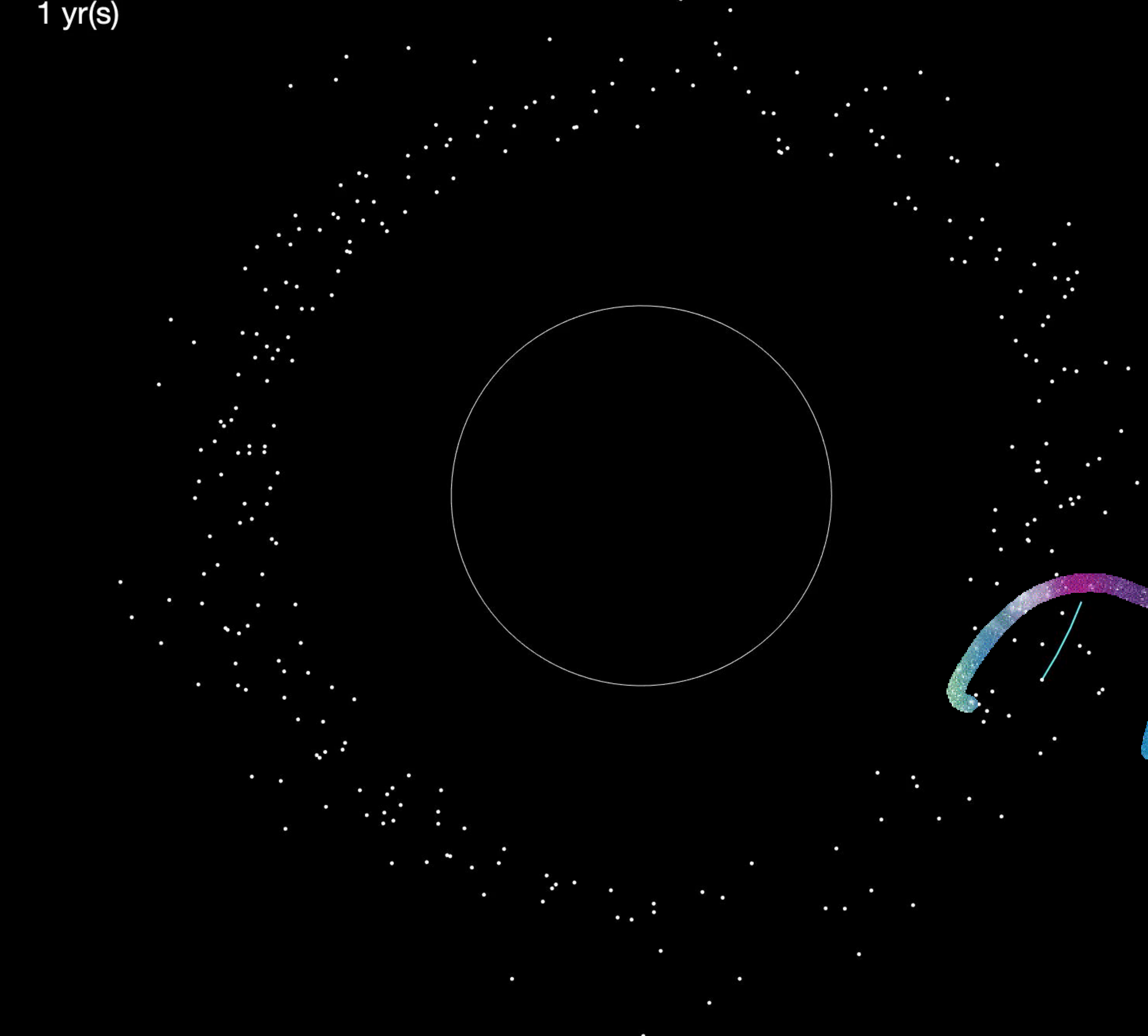


a (AU)	1.05
Inc (deg)	0.8517
RAAN (deg)	107.5743
Phase (deg)	0

J = 5992.3

Time of submission :
Nov 07, 2021 8:14 AM UTC

Station no	No of Asteroids	Total mass
1	27	1.2967E+15
2	26	1.2874E+15
3	29	1.2928E+15
4	28	1.2954E+15
5	24	1.3068E+15
6	21	1.2860E+15
7	22	1.2768E+15
8	24	1.2815E+15
9	24	1.2767E+15
10	23	1.2808E+15
11	21	1.2890E+15
12	24	1.2819E+15



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SUPAERO

Thank you!