清華大學航天航空学院
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航天动力学与控制实验室
Laboratory of Astrodynamics

GTOC 11: Results from Tsinghua University and Shanghai Institute of Satellite Engineering

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GTOC11 Workshop
12/18/2021

$$J = B \cdot \frac{10^{-10} \cdot M_{\min}}{a_{\text{Dyson}}^2 \sum_{k=1}^{10} \left(1 + \Delta v_k^{\text{Total}} / 50\right)^2}$$

Outline

- **Preliminary Analysis**
  - Asteroids Distribution and Selection
  - Optimal Asteroid-to-Asteroid Impulses Number
  - Optimal Radius of Dyson Ring
- **Optimization Methods**
  - The Trajectory of Motherships Design
  - The Dyson Ring Power Stations Assignment of Asteroids
  - Local optimization
- **Results**
  - Final Result Analysis
  - Results of Top 10 Teams

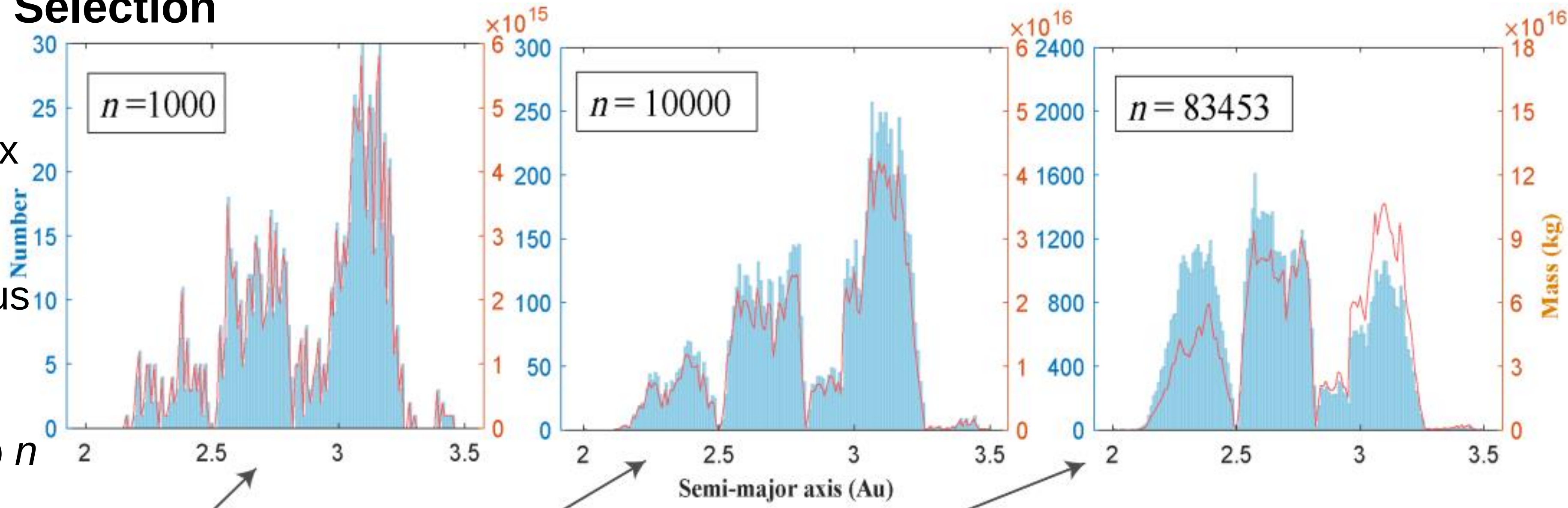
➢ Asteroids Distribution and Selection

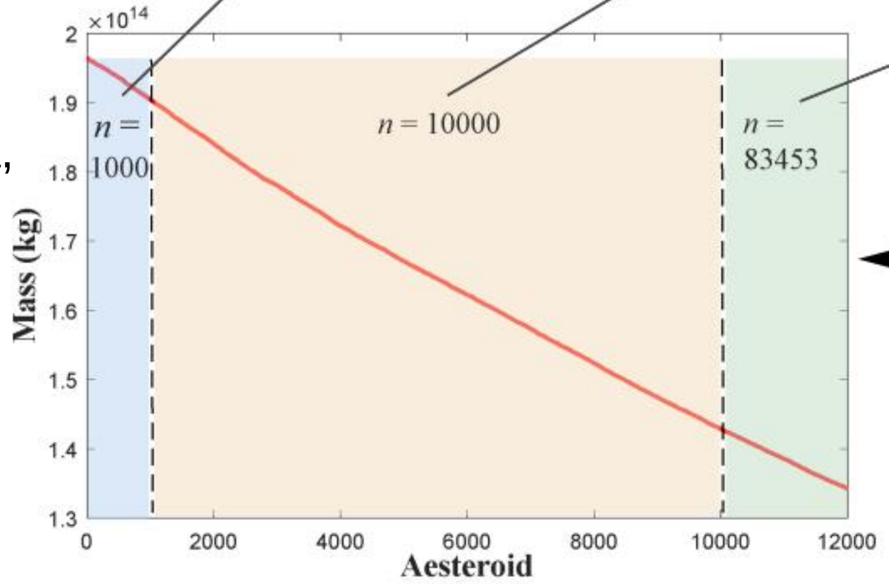
- Mass of asteroids was directly related to the performance index
- The inclination and eccentricity were considered, but not obvious
- The asteroids were sorted by mass and analyzed with the top *n* asteroids:

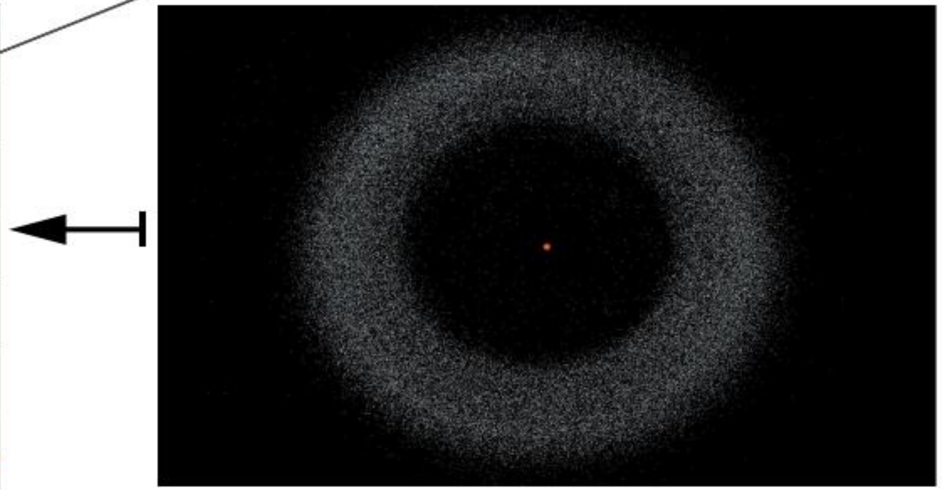
The close blue and red curves indicate asteroids are close in mass, like *n* = 1,000; vice versa, like *n* = 83,453.

- Asteroids Selection:

1,000 for quick tests;
10,000 for global optimization;
remaining potential asteroids filled by local optimization







Optimal Asteroid-to-Asteroid Impulses Number

- After 1, 000 tests, **two-impulse transfer were optimal** in 97% of cases. In the remaining 3% of cases, ΔV was too large to adopt. The asteroid-to-asteroid transfer is solved directly based on Lambert problem.

Test Scenario

Two asteroids were randomly selected from all the 83,453 asteroids, and the departure and arrival epochs were randomly specified (the time interval was within 1 year). Then, optimized successively the total velocity increment ΔV of 2, 3 and 4 impulses. Finally, collected ΔV of various situations.

Optimal Radius of Dyson Ring

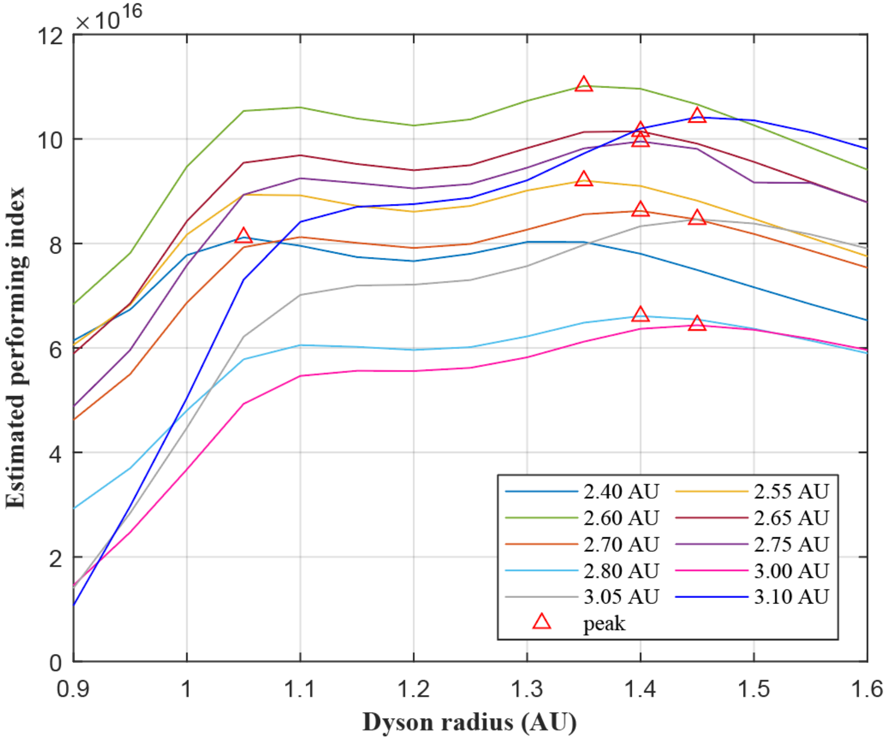
$$J = B \cdot \frac{10^{-10} \cdot M_{\min}}{a_{\mathrm{Dyson}}^2 \sum_{k=1}^{10} \left(1 + \Delta v_k^{\mathrm{Total}} / 50\right)^2}$$

- Define estimated performance index

$$J^* = \frac{1 - \alpha \Delta t}{a_{\mathrm{Dyson}}^2} M$$

M: The total mass of the asteroids around the departure orbit

- With 0.05 AU as the step size, respectively calculate the minimum transfer time from the asteroid orbits at 1.0-3.5 AU (circular orbits with an inclination angle of 0) to arrive at 0.65-3.0 AU orbits (phase-angle-free)



- The optimal Dyson Ring radius is at **1.0-1.5 AU**.

- The globally optimal trajectory design problem is further divided into two sub-problems:

1. the trajectory of motherships design

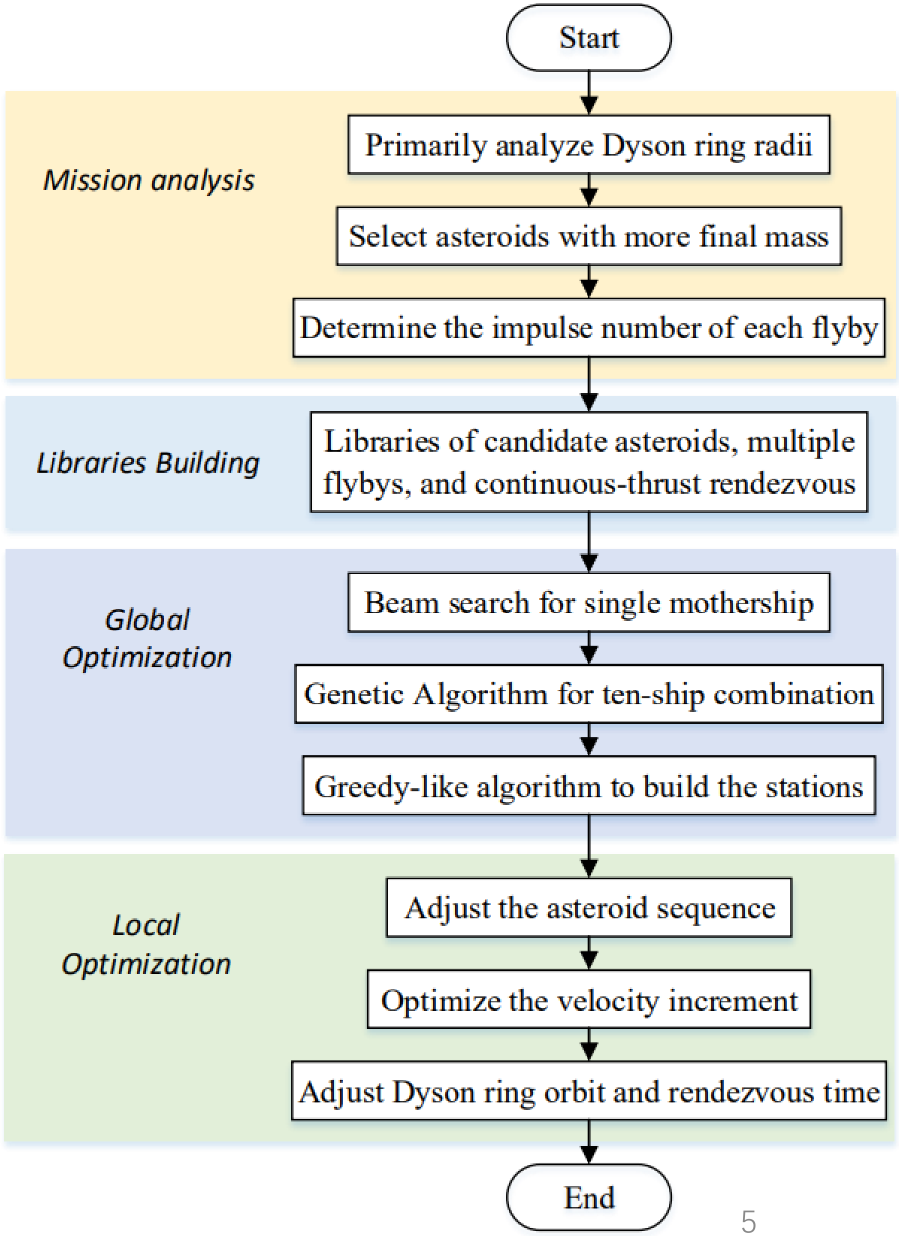
2. the Dyson Ring power stations assignment of asteroids.

- The first problem only considers optimizing the asteroids-flyby sequence of the mother ships and corresponding flyby epochs.

- The second problem optimizes the orbital elements of the Dyson Ring, construction time window of each station, and assignment of asteroids, based on the visited asteroids obtained.

- The two problems are studied completely independently, and they are coupled as the competition progresses:

  In the first problem, the performance index are gradually replaced with more accurate estimated M_{min} to ensure the global optimization.



➢ the trajectory of motherships design

- Relative speed is smaller than 2 km/s at the flyby time
- Handling flyby constraints:

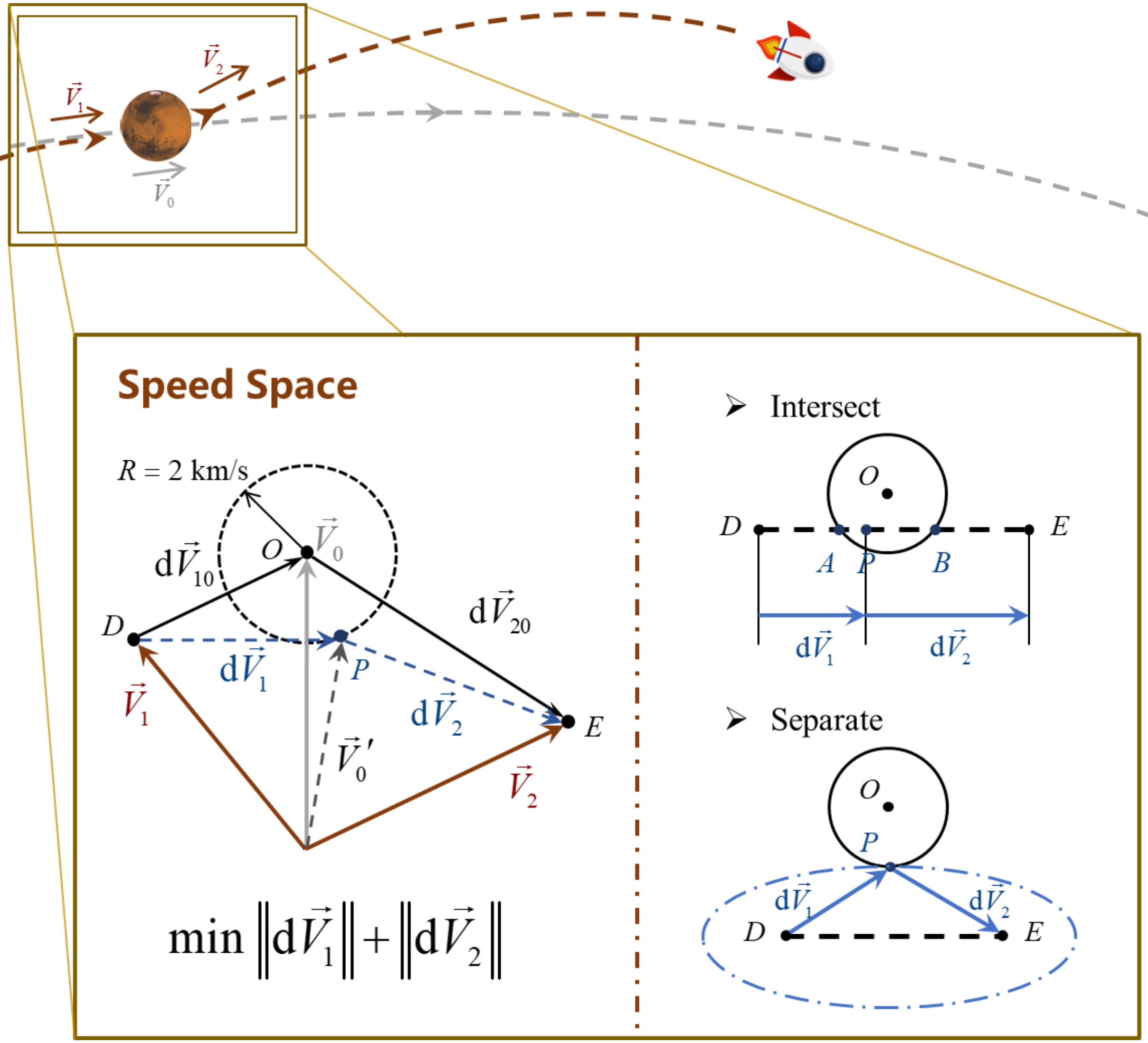
1.Directly solve the rendezvous problem;
2.According to a geometric method, use the flyby constraint to correct the velocity increment.

- Geometric method (correct total ΔV):

$$\min \left\|d\vec{V}_1\right\| + \left\|d\vec{V}_2\right\|$$

In complex geometric situations:

Knowing the two focal points of the ellipse, find the **minimum semi-major axis** of the ellipse, so that the ellipse is tangent to another given circle.



- the trajectory of motherships design

■ **Build Library**

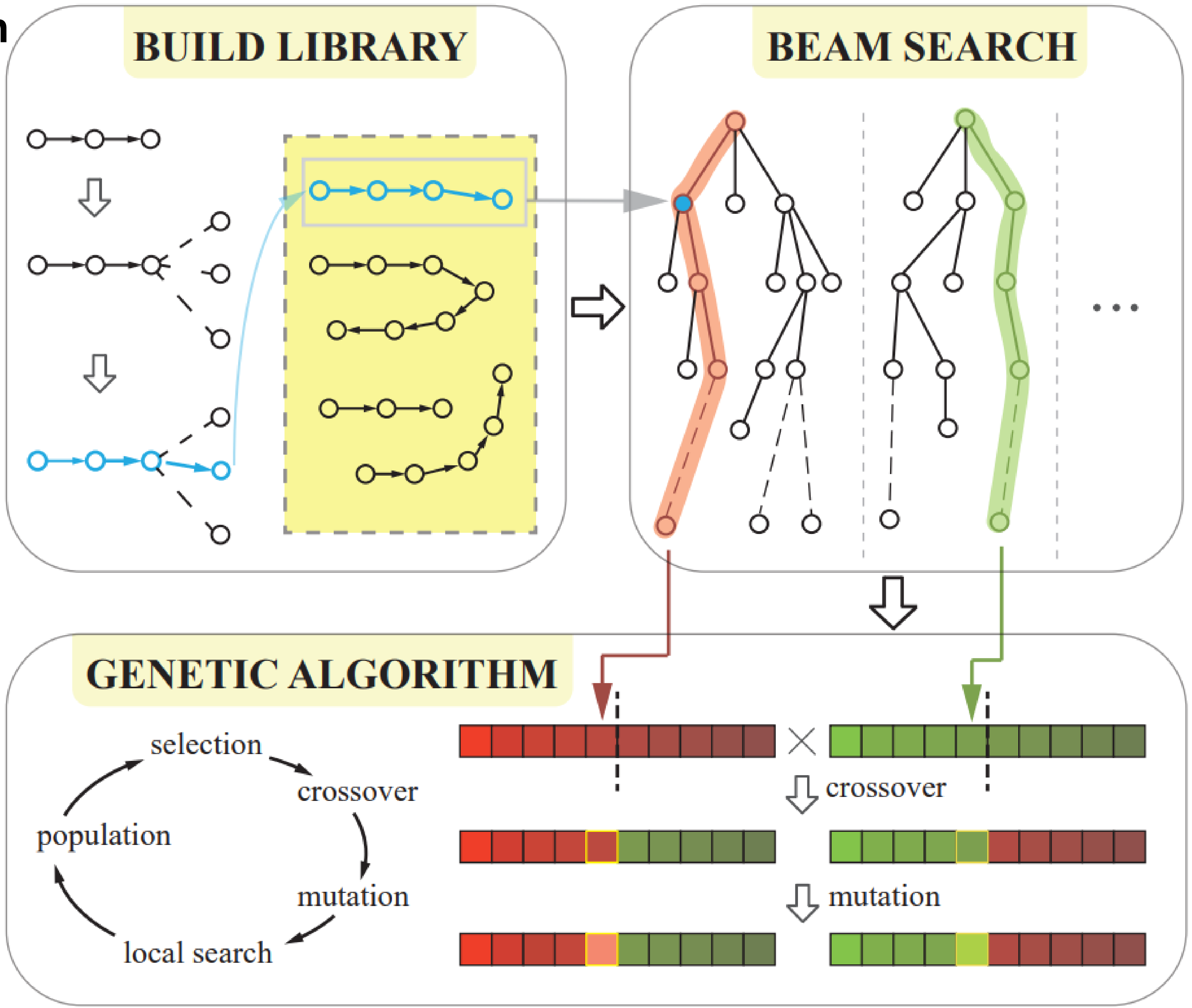
Build “asteroid-chain” library. Each "asteroid-chain" contains the sequence and flyby epochs of 3-8 asteroids, and the average velocity increment of asteroid-to-asteroid is about 0.4 km/s

■ **Beam Search**

Connect multiple "asteroid-chain“ s to generate a single ship trajectory (number of flying by asteroids is about 40). Starting from the different first asteroids, a large number of single-ship trajectories are generated to be used in GA

■ **Genetic Algorithm**

A genetic algorithm with the local search is adopted to obtain the final trajectory of 10 ships. In the GA, a gene of the chromosome is represented as the single-ship trajectory obtained by the beam search.



➢ Dyson Ring power stations assignment of asteroids.

- **Low-thrust trajectory optimization**: indirect method (equinoctial elements) [1]

  Some techniques:
  1. normalize the initial costate variables to enhance the convergence rate
  2. predict transfer time range to improve the optimality

$$J = \lambda_0 \int_{t_0}^{t_f} 1\mathrm{d}t \qquad \dot{\boldsymbol{x}} = a\boldsymbol{M}\boldsymbol{\beta} + \boldsymbol{D} \qquad \begin{matrix} \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \\ \boldsymbol{x}(t_f) = \boldsymbol{x}_f \end{matrix}$$

$$\boldsymbol{H} = \boldsymbol{\lambda}_x \cdot \dot{\boldsymbol{x}} + \lambda_0 = a\boldsymbol{\lambda}_x^{\mathrm{T}} \boldsymbol{M}\boldsymbol{\beta} + \boldsymbol{\lambda}_x^{\mathrm{T}} \boldsymbol{D} + \lambda_0$$

$$\boldsymbol{\beta} = -\frac{\boldsymbol{M}^T \boldsymbol{\lambda}_x}{\left\| \boldsymbol{M}^T \boldsymbol{\lambda}_x \right\|} \qquad \dot{\boldsymbol{\lambda}}_x = -\frac{\partial \boldsymbol{H}}{\partial \boldsymbol{x}} = -a\frac{\partial\left(\boldsymbol{\lambda}_x^{\mathrm{T}} \boldsymbol{M}\boldsymbol{\beta}\right)}{\partial \boldsymbol{x}} - \frac{\partial\left(\boldsymbol{\lambda}_x^{\mathrm{T}} \boldsymbol{D}\right)}{\partial \boldsymbol{x}}$$

$$\Phi\left(\boldsymbol{\lambda}(t_0); t_f\right) = \begin{bmatrix} \boldsymbol{x}(t_f) - \boldsymbol{x}_f \\ H(t_f) - \boldsymbol{\lambda}_x(t_f) \cdot \dot{\boldsymbol{x}}_f \\ \left\| \boldsymbol{\lambda}(t_0) \right\| - 1 \end{bmatrix} = \boldsymbol{0}$$

- **Build database**: (indirect method needs 0.1 s, not efficiency to global optimization)

  Asteroids (83453 options); departure true anomaly, 0 to 345 degrees, steps of 15 degrees (24 options); Dyson Ring radius, 0.8 to 2.0 AU, steps of 0.05 AU (25 options); station phase angle, 0 to 345 degrees, steps of 15 degrees (24 options). Use “int” to storage, and the final database size is about 4.5 Gb.

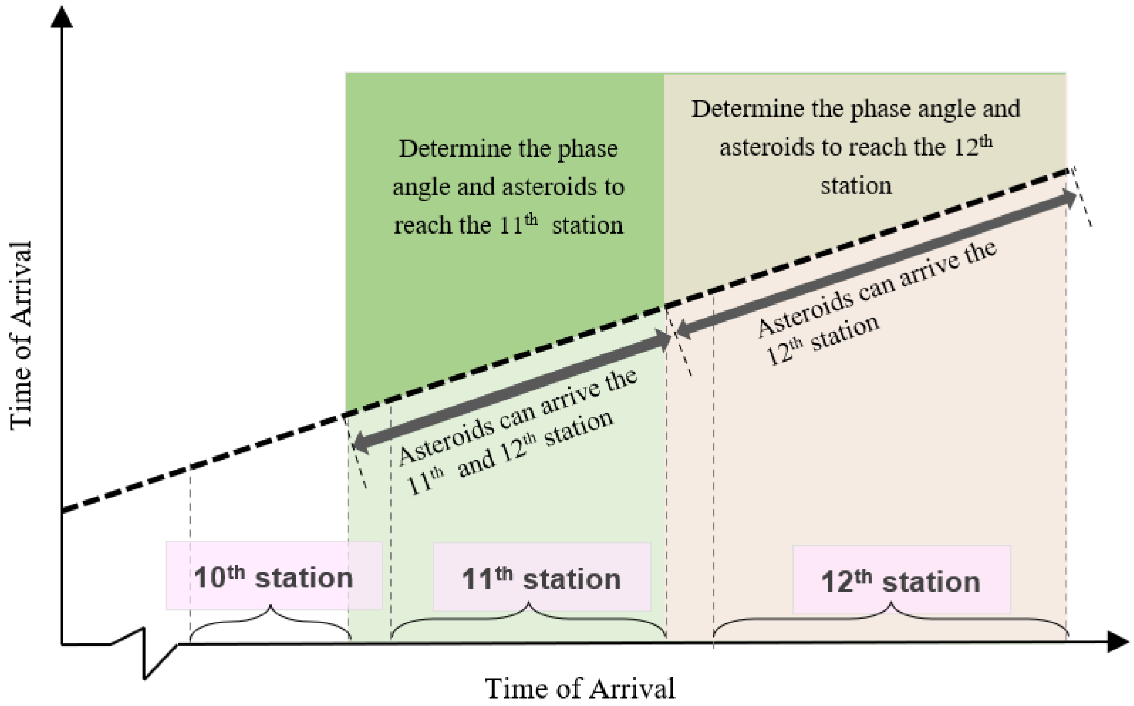
[1] F. Jiang, H. Baoyin, and J. Li, “Practical techniques for low-thrust trajectory optimization with homotopic approach,” J. Guid. Control. Dyn., vol. 35, no. 1, pp. 245–258, 2012.

Dyson Ring power stations assignment of asteroids.

- Database gets the transfer time very quickly. A greedy-like algorithm is adopted to solve the assignment problem.

- The algorithm is aimed to determine the phase angle of stations from the late-arrival asteroids and adopt subsequent local search to complete the asteroid assignment

Input: Orbital elements of the Dyson Ring, constructing time window of each station
Output: Phase angle of each station, assignment of asteroids, estimated arrival time

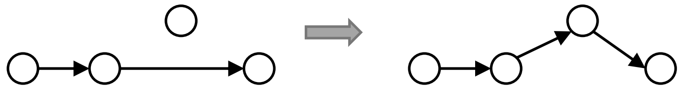



```
Initialize stations construction interval and the radius of Dyson Ring
Sort the visited asteroids according to the time of arrival epoch (phase-angle-free)
Group asteroids (1-12) in chronological order
for the i th station from the last to the first station
    for feasible phase angle
        for all the last asteroids in group i to 12
            Calculate the optimal transfer time
            If reachable, add the asteroid arrival mass
        end for
        Record the mass of this station
    end for
    Select the phase angle corresponding to the maximum mass station
    Determine the phase angle and asteroids to reach the i th station
end for
Local search: Enum all the allocated asteroids in other stations to move to the
minimum mass station, and select the asteroid with the largest total mass of the
station. Repeat the process until mass of the minimum mass station no longer
increases
```

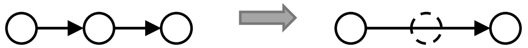
➢ Local optimization

- **Flyby time optimization**

  Local optimizer: NLOPT [1]



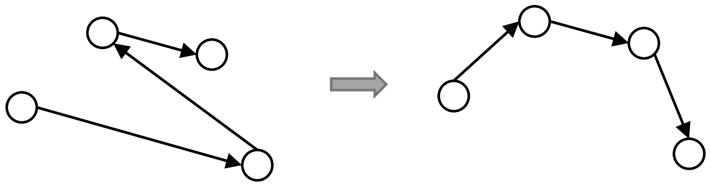
(a) add



(b) delete

- **Adjustment the asteroid- flyby sequence:**

  Add, delete, swap



(c) swap

- **Adjustment Dyson Ring orbit:**

Update database: the inclination and RAAN are determined by the average orbital angular momentum of the visited asteroids, and the step size of the semi-major axis is reduced

PSO: optimize the stations construction time window

- **Adjustment rendezvous time:**

Calculate multiple times and take the maximum arrival mass

[1] S. G. Johnson, "The NLopt nonlinear-optimization package." 2010. Available: http://github.com/stevengj/nlopt

■ Mission analysis

Deepen the understanding of the problem
Reduce the search space

■ Libraries building

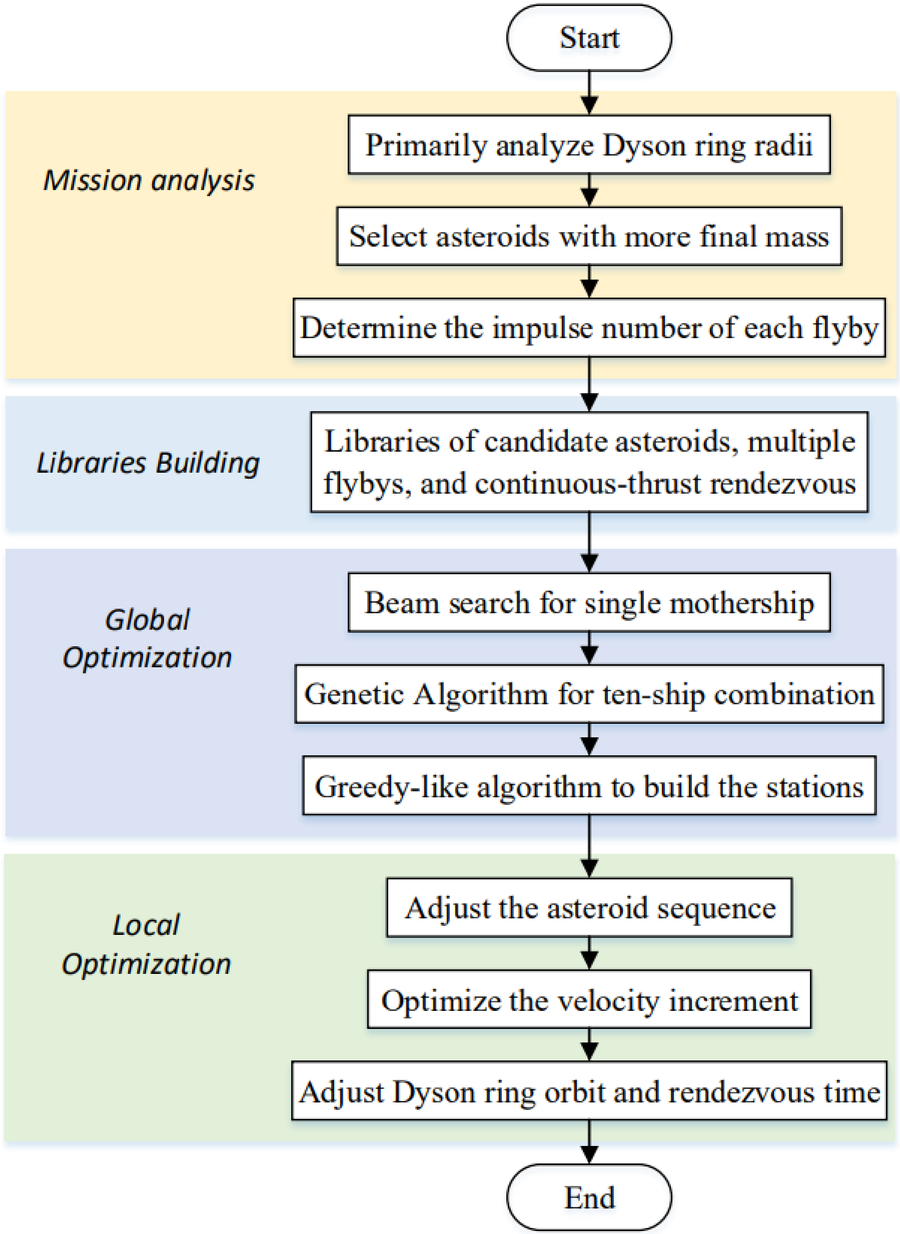
Make full use of computation resources to ensure global optimization efficiency

■ Global optimization

Use mature code and strategies, and make partial improvements based on the problem

■ Local optimization

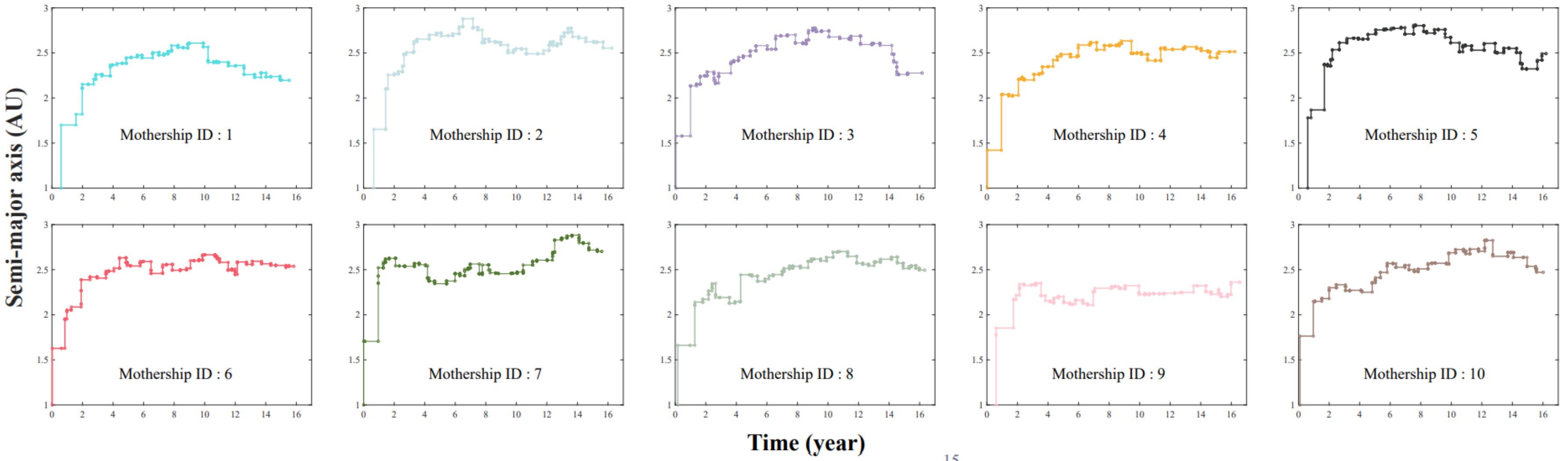
Further improve the results obtained by global optimization

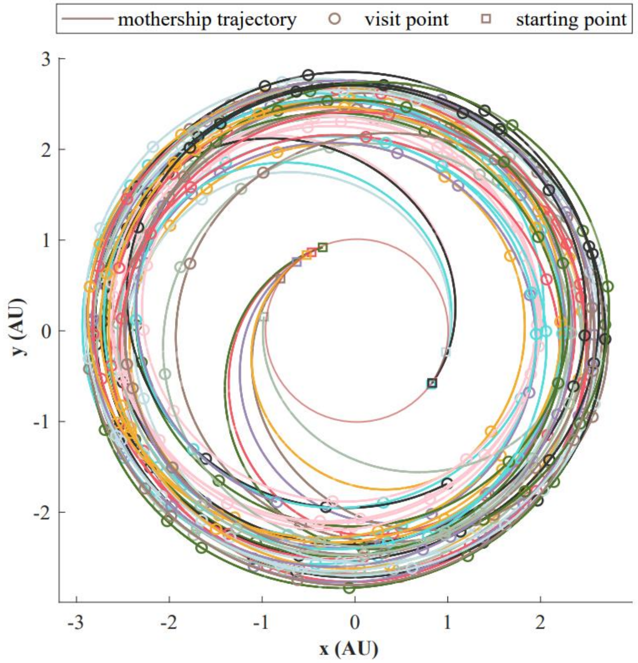


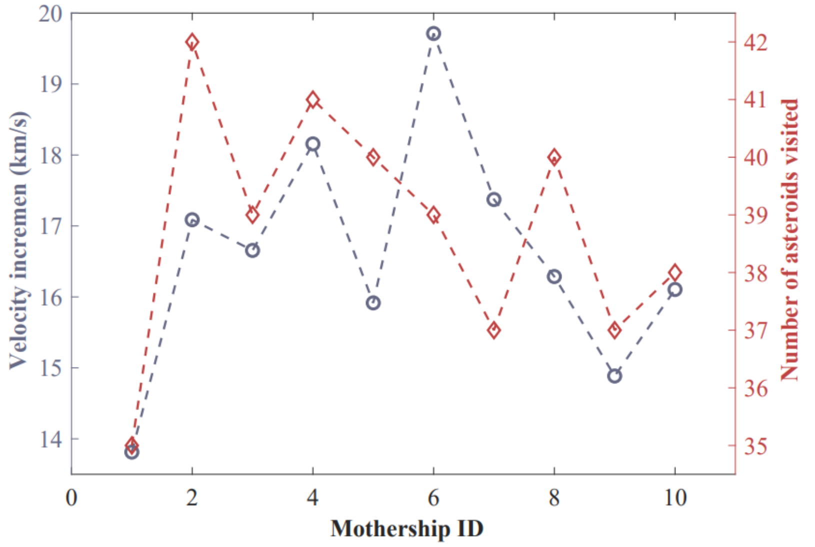
GTOC 11 Workshop: Results analysis

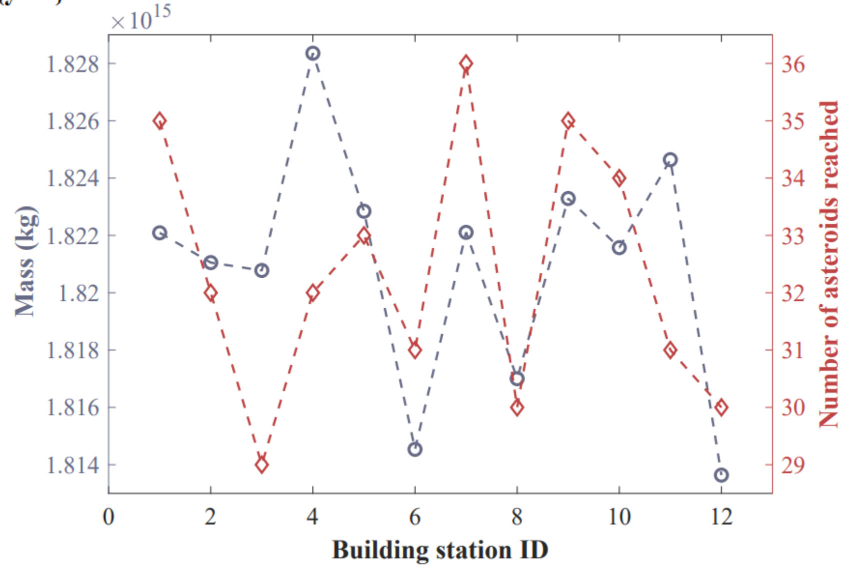
■ Final Result Analysis

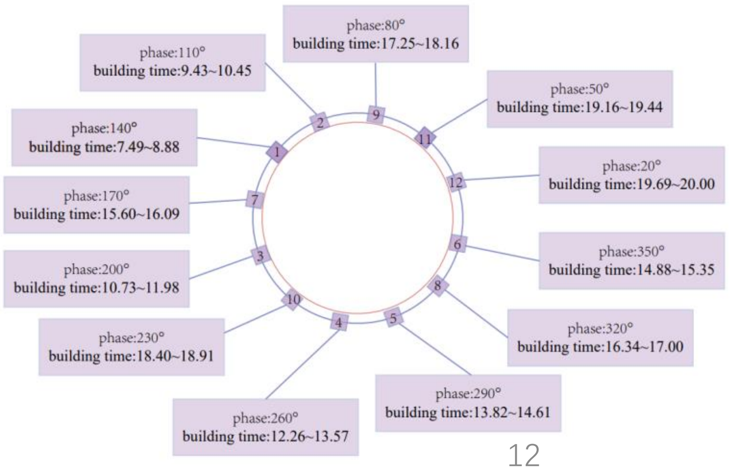
N: 388 dV_{aver}: 16.6 km/s M_{min}: 1.814*10^{15} kg a_{Dyson}: 1.10 AU











Team	TsinghuaLAD &509	ACT& Friends	the Antipodes	UT Austin	ASRL	The Eccentric Anomalies	HIT	GHWZZ	ASTL-NUAA	Team_BIT &ITNS
Solution Detail										
Dyson Ring semi-major axis a (AU)	1.10	1.32	1.05	1.10	1.09	1.30	1.10	1.40	1.10	1.00
Dyson Ring inclination i (deg)	0.72	1.43	0.85	0.00	0.00	0.00	0.00	0.00	3.52	8.00
Total number of the transferred asteroids N	388	301	293	235	209	346	250	294	213	199
Average impulses per mothership Δv (km/s)	16.6	17.4	19.5	13.1	14.7	21.4	15.6	13.2	25.4	25.2
Maximal mass M_{max} (10^{15}kg)	1.828	2.044	1.307	1.141	1.139	1.916	1.132	1.595	1.063	0.874
Minimal mass M_{min} (10^{15}kg)	1.814	2.013	1.277	1.133	1.100	1.892	1.085	1.502	1.031	0.800
*Theoretical upper bound of M_{min} (10^{15}kg)	1.929	2.220	1.450	1.281	1.248	2.302	1.325	1.920	1.216	0.997
*Optimization efficiency of dividing station η	0.940	0.907	0.881	0.884	0.881	0.822	0.819	0.782	0.848	0.802
Solution Score										
*Estimated Score	8211	6263	6188	5761	5294	6026	5725	5241	4010	3860
Best Score (from Leaderboard)	**8443**	**6359**	**5992**	**5885**	**5525**	**5487**	**5208**	**4794**	**3735**	**3532**

* Calculated by evaluation procedure from TsinghuaLAD&509

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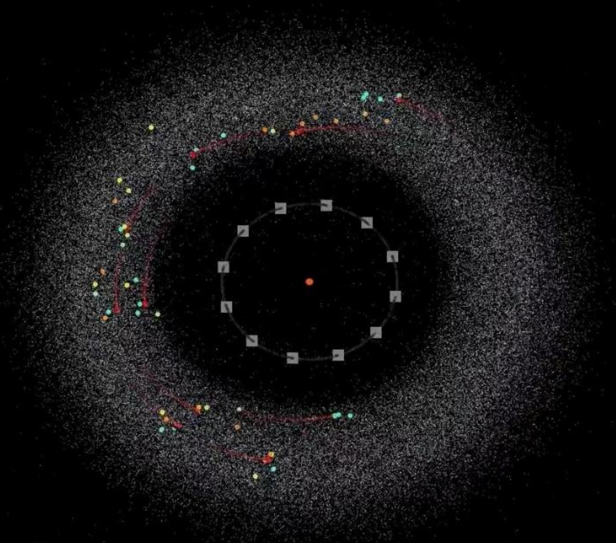
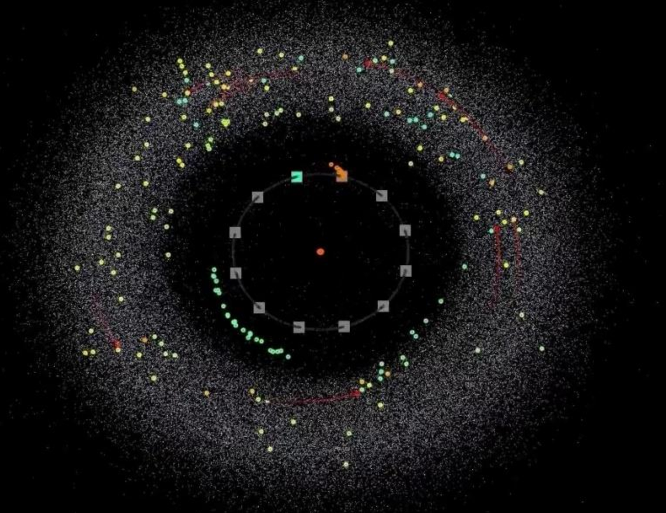
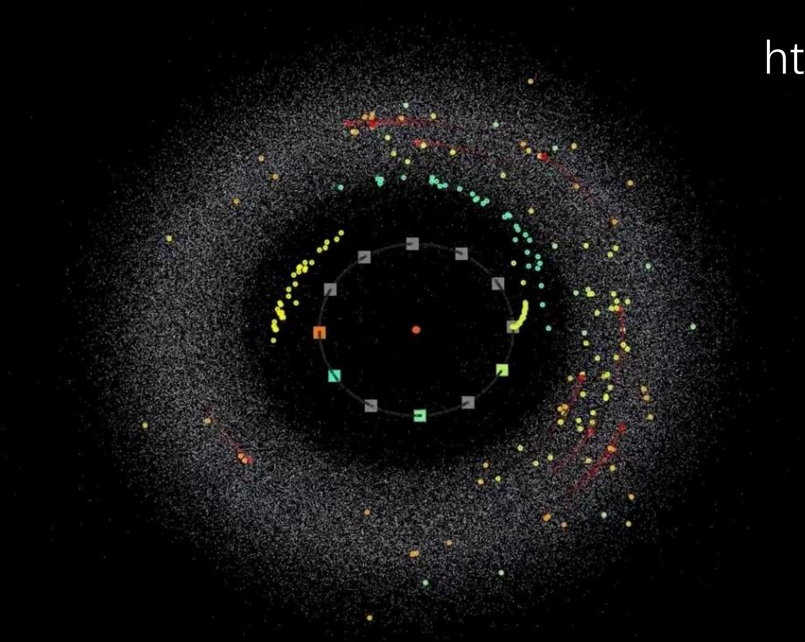
GTOC11 - Dyson Sphere

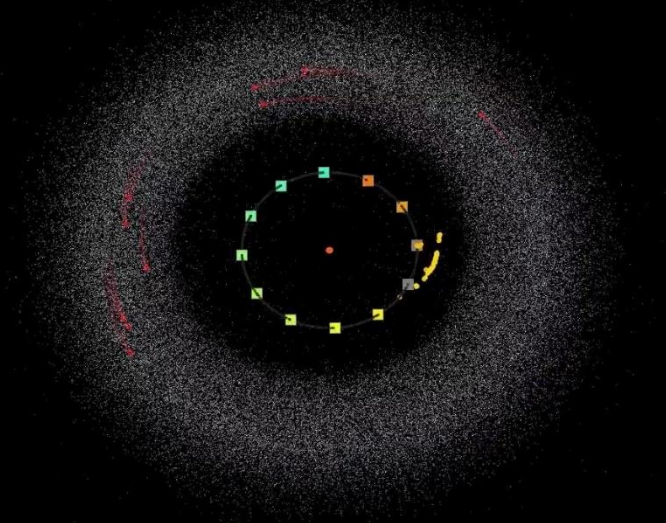
Solution from TsinghuaLAD&509





Video available:
https://www.bilibili.com/video/BV1yS4y197pP





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Thank you!

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