Problem Description for the 4th Global Trajectory Optimisation Competition

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Background

The Global Trajectory Optimisation Competition was initiated in 2005 by the Advanced Concepts Team of the European Space Agency. The Outer Planets Mission Analysis Group of the Jet Propulsion Laboratory, winner of GTOC1, organised the GTOC2 competition in 2006. Then, the Aerospace Propulsion Group of the Dipartimento di Energetica of the Politecnico di Torino, winner of the GTOC2, organised the GTOC3 in 2008. Finally, the Interplanetary Mission Analysis team of the Centre National d'Etudes Spatiales de Toulouse, winner of the GTOC3, is very pleased to organise the GTOC4 this year.

The aim of this document is to reveal the problem to be solved for GTOC4.

Introduction

The GTOC problems are traditionally global optimisation problems, that is to say complex optimisation problems characterised by a large number of local optima. Such problems can be solved either by means of local or global optimisation methods.

When using local optimisers, the engineer's experience plays a crucial role in finding a good initial guess for the optimiser. Indeed, this experience helps in determining the region of the design space that potentially contains the global optimum. At the opposite, global optimisers have to be used in conjunction with pruning methods in order to reduce the computing time by avoiding a full exploration of the whole search domain.

This year again, the proposed problem is a global optimisation problem and aims at fulfilling the following important criteria:

- the design space is large and leads to an important number of local optima,
- the problem is complex but in any case it can be solved within the 4-weeks period allowed for the competition,
- its formulation is simple enough so that it can be solved by researchers not experienced in astrodynamics,
- even if some registered teams have already developed their own optimisation tools for interplanetary missions, the problem specificities make it new to all the teams.

Problem Description

Generalities

The mission proposed this year may be entitled: "How to maximise the relevance of a rendezvous mission to a given NEA by visiting the largest set of intermediate asteroids".

More precisely, let us assume that a spacecraft is launched from the Earth. This spacecraft has first to visit (flyby) a maximum number of asteroids (from a given list of NEAs). Finally, it must rendezvous with a last asteroid of that same list within ten years from departure.

The performance index to be maximised is the number of visited asteroids, but when two solutions are associated with the same number of visited NEAs, a secondary performance index has to be maximised: the final mass of the spacecraft.

Moreover, we assume that the spacecraft is equipped with an electric propulsion system and that gravity assists are not allowed during the mission.

The use of electric propulsion yields an optimal control formulation for the GTOC4 problem once a sequence of asteroids has been chosen. The huge number of such feasible asteroids sequences leads to a large number of local optima for the problem.

Spacecraft and Trajectory Constraints

The spacecraft is launched from the Earth, with hyperbolic excess velocity vector $\mathbf{v}_\infty$ of up to 4.0 km/s in modulus and of unconstrained direction. The year of launch must lie in the range 2015 to 2025, inclusive.

After launch, the spacecraft must first flyby a maximum number of NEAs, taken from the list of the ASCII file `ast_ephem_gtoc4.txt`, and then must rendezvous with an asteroid of the same list. The choice of asteroids is part of the optimisation process. In addition, **each asteroid must be visited only once during the mission and the asteroid chosen for the final rendezvous must not have been visited before**.

The flight time, τ, measured from launch to the date of the final rendezvous **must not exceed 10 years**. Gravity assists from any planets **are not permitted**. The spacecraft has a fixed initial mass, i.e. wet mass, m_i of 1500 kg (that is not affected by the launch $\mathbf{v}_\infty$). We assume here that the **spacecraft dry mass m_d is equal to 500 kg**, thereby **the propellant mass m_p is 1000 kg**, i.e. $m_i = m_d + m_p$.

The spacecraft has a constant specific impulse *Isp* of 3000 s and its thrust level T is bounded by 0.135 N (this maximum value is constant and so does not depend on the distance between the spacecraft and the Sun). The thrust level can be modulated at will, that means that T can take any value between 0 and 0.135 N. In addition, there is no constraint on the thrust direction. The spacecraft mass only varies during thrusting periods and is constant when the engine is off (coast periods).

The required models for flybys and rendezvous are given in the Appendix. A rendezvous requires the spacecraft position and velocity to be the same as those of the target asteroid, while a flyby prescribes that only the spacecraft position must be equal to the position of the visited asteroid at the time of the flyby.

Performance index

As said in the previous section, the flight time τ measured from launch to the date of the final rendezvous must not exceed 10 years. Objective of the optimisation is to maximise the total number of asteroids visited before the rendezvous (each asteroid being visited at most one time, and the last asteroid chosen for the rendezvous having not been visited before). This quantity may be written under the following form:

$$J = \sum_{j=1}^{n} \alpha_j$$

where n is the total number of NEAs in the list of the ASCII file `ast_ephem_gtoc4.txt` and where $\alpha_j \in \{0, 1\}$ denotes the number of times that asteroid number j, ($j = 1,\ldots,n$) has been visited during the mission. So, only two integer values, 0 and 1, are allowed for the α_j ($j = 1,\ldots,n$). In addition, the asteroid chosen for the final rendezvous (the index of which is denoted by j_f) must be associated with the value $\alpha_{j_f} = 0$. As a consequence, performance index J may not exceed the value $(n - 1)$.

As said before, when two solutions yield the same value of J, we consider that the best one is the solution that maximises the following secondary performance index:

$$K = m_f$$

where m_f denotes the spacecraft final mass that has to satisfy the following important constraint:

$$m_f \geq m_d = 500\,\text{kg}$$

Dynamical model

The Earth and asteroids are assumed to follow Keplerian orbits around the Sun. The only forces acting on the spacecraft are the Sun's gravity and the thrust produced by the engine (when this last one is on). The Earth's Keplerian orbital parameters are given in Table 1 below. The asteroids' Keplerian orbital parameters are provided in the file `ast_ephem_gtoc4.txt` that gives:

1) Asteroid name, 2) epoch in modified Julian date (MJD), 3) semi major axis in AU, 4) eccentricity, 5) inclination in degrees, 6) longitude of the ascending node in degrees, 7) argument of periapsis in degrees, 8) mean anomaly at epoch in degrees.

Moreover, Earth's and asteroids' orbital elements are given in the J2000 heliocentric ecliptic frame. Other required constants are given in Table 2.

Table 1: Earth's orbital elements in the J2000 heliocentric ecliptic reference frame.

Semimajor axis a, AU	0.999988049532578
Eccentricity e	$1.671681163160. 10^{-2}$
Inclination i, deg	$0.8854353079654. 10^{-3}$
Longitude of Ascending Node (LAN) Ω,deg	175.40647696473
Argument of periapsis ω, deg	287.61577546182
Mean anomaly at epoch M, deg	257.60683707535
Epoch t, MJD	54000

Table 2: Constants and conversion.

Sun's gravitational parameter μ_S, km^3/s^2	$1.32712440018. 10^{11}$
Astronomical Unit AU, km	$1.49597870691. 10^{8}$
Standard acceleration due to gravity, g_0, m/s^2	9.80665
Day, s	86400
Year, days	365.25
00:00 01 January 2015, MJD	57023
24:00 31 December 2025, MJD	61041

Solution format

Each team should return its best solution by email to regis.bertrand@cnes.fr on or before March 30, 2009.

Two files must be returned. The first one should contain:

- a short description of the method used,
- a summary of the best solution found, at least: GTOC4 names of the visited asteroids, GTOC4 name of the final asteroid chosen for rendezvous, launch date, launch v_∞, date and spacecraft mass at each flyby, date of the final rendezvous, thrust durations, total flight time τ, value of the performance index J, value of the final mass m_f,
- a visual representation of the trajectory, such as a projection of the trajectory onto the ecliptic plane.

This file should preferably be in PDF or PS format but Microsoft Word format should also be acceptable.

The second file will be used to verify the solution returned. It must follow the format and units provided in the ASCII template file `solution.txt`. As can be seen in the file, trajectory data have to be provided at one-day increments for each inter-body phase of the trajectory. In addition, trajectory data have also to be provided at each time corresponding either to a flyby or to the final rendezvous. So, when a flyby (or the final rendezvous) does not fall exactly on a one-day increment, the last time point for the phase should be reported using a partial-day increment from the previous time point. Moreover, the coordinate frame should be the J2000 heliocentric ecliptic frame.

Appendix

This appendix provides the equations describing the dynamics of the problem along with other background information.

Nomenclature

Orbital elements and related quantities

a: semi major axis

e: eccentricity

i: inclination

Ω: longitude of the ascending node

ω: argument of periapsis

M: mean anomaly at epoch

θ: true anomaly

E: eccentric anomaly

r: distance from the Sun

γ: flight path angle

μ_S: Sun's gravitational parameter

Position and velocity

r: position vector in J2000 heliocentric ecliptic frame

v: velocity vector in J2000 heliocentric ecliptic frame

x, y, z: position components

v_x, v_y, v_z: velocity components

Departure

$\mathbf{v}_\infty$: hyperbolic excess velocity vector

$v_\infty = |\mathbf{v}_\infty|$: hyperbolic excess velocity magnitude

Other quantities

t: time

τ: overall mission duration

m: mass

m_d: dry mass

m_p: propellant mass

Isp: specific impulse of the engine

T: thrust magnitude of the engine

g_0: standard acceleration due to gravity at Earth's surface

J: primary performance index

K: secondary performance index

Subscripts and superscripts

$()_0$: at epoch

$()_i$: initial value

$()_{Aj}$: value at asteroid number j ($j = 1,\ldots,n$)

$()_f$: final value

$()_E$: value at the Earth

$()_S$: value at the Sun

$()_{max}$: maximum value

$()_{min}$: minimum value

$()'$: time derivative

Problem dynamics and conversion between elements

The motion of the Earth and asteroids around the Sun is governed by the following equations:

$$x'' = -\mu_S \frac{x}{r^3} \quad y'' = -\mu_S \frac{y}{r^3} \quad z'' = -\mu_S \frac{z}{r^3}$$

where

$$r = \sqrt{x^2 + y^2 + z^2} = \frac{a(1-e^2)}{1 + e\cos(\theta)}$$

The motion of the spacecraft around the Sun is governed by the same equations but with introduction of the acceleration due to the thrust and equation for the mass:

$$x'' = -\mu_S \frac{x}{r^3} + \frac{T_x}{m} \quad y'' = -\mu_S \frac{y}{r^3} + \frac{T_y}{m} \quad z'' = -\mu_S \frac{z}{r^3} + \frac{T_z}{m} \quad m' = -\frac{T}{g_0 Isp}$$

The thrust magnitude is bounded:

$$0 \le T = \sqrt{T_x^2 + T_y^2 + T_z^2} \le 0.135 \text{ N}$$

Conversion from orbit elements to Cartesian quantities is as follows:

$$\begin{aligned}
x &= r[\cos(\theta+\omega)\cos\Omega - \sin(\theta+\omega)\cos i \sin\Omega] \\
y &= r[\cos(\theta+\omega)\sin\Omega + \sin(\theta+\omega)\cos i \cos\Omega] \\
z &= r[\sin(\theta+\omega)\sin i] \\
v_x &= v[-\sin(\theta+\omega-\gamma)\cos\Omega - \cos(\theta+\omega-\gamma)\cos i \sin\Omega] \\
v_y &= v[-\sin(\theta+\omega-\gamma)\sin\Omega + \cos(\theta+\omega-\gamma)\cos i \cos\Omega] \\
v_z &= v[\cos(\theta+\omega-\gamma)\sin i]
\end{aligned}$$

where the velocity magnitude v and the flight path angle γ are given by:

$$v = \sqrt{\frac{2\mu_S}{r} - \frac{\mu_S}{a}} \quad \tan\gamma = \frac{e\sin\theta}{1+e\cos\theta}$$

For an elliptic orbit, the true anomaly is related to the eccentric anomaly by:

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}$$

and the eccentric anomaly is related to the mean anomaly by Kepler's equation,

$$M = E - e\sin E$$

while the mean anomaly is related to time and to the mean anomaly M_0 at epoch t_0 by

$$M = M_0 + \sqrt{\frac{\mu_S}{a^3}}(t - t_0)$$

Thus, based on the provided orbital parameters, the Cartesian positions and velocities of the Earth and asteroids may be computed as functions of time with only the minor problem of having to solve Kepler's equation for E by some iterative algorithm. Then, for the Earth, the asteroids and a non-thrusting spacecraft, the equations of motion do not need to be numerically integrated to find position and velocity at some given time.

Launch, flyby and rendezvous

Launch occurs at time t_i with hyperbolic excess velocity vector $\mathbf{v}_{\infty,\mathrm{i}}$. The spacecraft position and velocity are given by:

$$\mathbf{r}_\mathrm{i} = \mathbf{r}_\mathrm{E}(t_i) \quad \mathbf{v}_\mathrm{i} = \mathbf{v}_\mathrm{E}(t_i) + \mathbf{v}_{\infty,\mathrm{i}}$$

with initial mass m_i = 1500 kg and the following constraints:

$$|\mathbf{v}_{\infty,\mathrm{i}}| \le 4.0 \text{ km/s} \quad 57023 \text{ MJD} \le t_i \le 61041 \text{ MJD}$$

Each asteroid flyby occurs at time t_j (for asteroid number j, j = 1,…,n) when the spacecraft matches the position of the asteroid:

$$\mathbf{r}_j = \mathbf{r}_{A_j}(t_j)$$

The final rendezvous occurs at time t_f when the spacecraft matches the position **and the velocity** of asteroid A_{j_f}

$$\mathbf{r}_f = \mathbf{r}_{A_{j_f}}(t_f) \quad \mathbf{v}_f = \mathbf{v}_{A_{j_f}}(t_f)$$

The final spacecraft mass has to fulfil the following constraint:

$$m_f \geq m_d = 500\,\text{kg}$$

The overall mission duration τ is constrained:

$$\tau = t_f - t_i \leq 10 \text{ years}$$

Finally, the constraints on position and velocity must be satisfied with accuracy of at least 1000 km and 1 m/s respectively (these values are in terms of the Euclidian norm of the vector differences).

Glossary

- Modified Julian Date (MJD) is defined as the number of days past from a defined date in the past, namely 00:00 18 November 1858.

- Flyby: meeting a body such as an asteroid by matching its position (this body is considered as a moving point is space).

- Rendezvous: meeting a body such as an asteroid by matching its position **and its velocity** (this body is considered again as a moving point is space).