Problem Description for the 7th Global Trajectory Optimisation Competition

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Background

The Global Trajectory Optimisation Competition was inaugurated in 2005 by Dario Izzo of the Advanced Concepts Team, European Space Agency. After the first edition, following competitions were organized by the winning team of the preceding GTOC edition: the Outer Planet Mission Analysis Group of the Jet Propulsion Laboratory (GTOC2); the Aerospace Propulsion Group of the Dipartimento di Energetica of the Politecnico di Torino (GTOC3); the Interplanetary Mission Analysis Group of the Centre National d'Etudes Spatiales de Toulouse (GTOC4); the Faculty of Mechanics and Mathematics of Lomonosov Moscow State University (GTOC5) and, for the second time, the Outer Planet Mission Analysis Group of the Jet Propulsion Laboratory (GTOC6). This tradition is maintained, and the joint team Politecnico di Torino - Università di Roma "Sapienza" is pleased to organize the seventh edition of the competition, GTOC7. This document reveals the problem that is to be solved for GTOC7.

Introduction

Global optimization consists in finding the global optimum of a given performance index in a large domain, typically characterized by the presence of a large number of local optima. The existing methods to solve such problems in trajectory optimization, as shown by the results of previous GTOC editions, are

- local optimization methods on selected subsets of the whole domain;
- global optimization methods that scan the whole domain.

The latter are obviously preferable when the whole domain can actually be explored completely, efficiently and with sufficient accuracy; otherwise methods to prune the less promising solutions must be adopted. On the other hand, when the domain is large, methods to define the subsets to be explored by local optimizer must be sought by using sorts of "global" exploration procedures. Under this point of view, the juxtaposition between global and local methods seem to become more subtle and even vanish.

The problem proposed for this year competition aims at fulfilling the following criteria:

- the design space is large and a large number of local optima exist;
- the problem is complex but not overwhelming, and can be analyzed within the prescribed 4-week time frame;
- its mathematical formulation is sufficiently simple so that it should also be solved by researchers not experienced in astrodynamics;
- even though registered teams may have developed tools for the analysis of the proposed kind of mission, the problem peculiarities should make it new to all the teams.

As in GTOC1 through GTOC5 a low-thrust heliocentric mission is considered, but impulsive maneuver are also introduced in the analysis of a mission with multiple cooperating spacecraft.

Problem Description

Generalities

The proposed mission is a multiple-ship mission to Main Belt asteroids. A mother ship launches form Earth and releases, at proper times, exploration probes, which must rendezvous with one or more asteroids and then return to and rendezvous with the mother ship. This problem may have an interest *per se* as far as asteroid missions are concerned, but has also similarities with geocentric missions for satellite refurbishment or debris removal.

The mother ship employs high-thrust nuclear propulsion. The probes have autonomous electric propulsion systems. Gravity assists are excluded. The primary performance index to be maximized is the overall number of asteroids reached by the probes. The sum of probe masses at end of mission is the problem secondary performance index in case of tie between teams.

Spacecraft and Trajectory Constraints

A mother ship is launched from the Earth, with hyperbolic excess velocity v_∞ between 0 and 6 km/s, with unconstrained direction. The escape mass is 24000 kg and is assumed to be independent of v_∞. The launch can happen on any date between January 1, 2021 00:00 UT (modified Julian date MJD 59215) and December 31, 2030 24:00 UT (MJD 62867), inclusive.

The mother-ship departure mass is composed of: 6000 kg of fixed masses (structures, propulsion system, etc.) 12000 kg of propellant for the mother ship propulsion system, three probes weighing 2000 kg each. Each probe is released from the mother ship at any time after departure, performs rendezvous with one or more asteroids and returns to the mother ship for rendezvous. The probes must remain at each reached asteroid for a minimum time of 30 days. Rendezvous prescribes that spacecraft position and velocity are the same as those of the target body; note that rendezvous between mother ship and probe can be obtained by maneuvering with either (mother-ship impulse and probe thruster off or no impulse and probe thruster on) or with both spacecraft (impulse and probe thruster on). The probe thruster is off during the stay at the asteroids, before release, and after rendezvous with the mother ship. For each probe, the time from release to rendezvous with the mother ship (capture) must be at most 6 years. The time from mother ship departure from Earth to capture of the last probe must be at most 12 years.

The mother ship has a propulsion system based on a nuclear thermal rocket (NTR). Since the NTR thrust is large, mother ship maneuvers must be modeled as impulses, which instantaneously change the mother ship velocity (and mass). The propellant for each impulse is evaluated according to Tsiolkowsky's rocket equation. A maximum number of 10 impulses is admitted, provided there is sufficient propellant. The specific impulse of the NTR rocket is assumed to be $(I_{sp})_{NTR} = 900$ s.

Each probe leaves the mother ship with zero relative velocity and has an electric propulsion (EP) system, with specific impulse $(I_{sp})_{EP}$ of 3000 s and a maximum thrust level T_{max} of 0.3 N. There is no constraint on the thrust direction. The spacecraft mass only varies because of the propellant consumption during thrusting and is otherwise constant (no mass dumping or collecting is allowed). The 2000 kg of probe mass are divided in 800 kg of fixed masses (structure, payload, propulsion system, etc.) and 1200 kg of propellant for the EP system.

Performance index

Objective of the optimization is to maximize the number of visited asteroid. One point is assigned for each asteroid that:

- (at least) one probe has had rendezvous with;
- the same probe has stayed in rendezvous conditions with it for at least 30 days;
- the same probe has returned to and has performed rendezvous with the mother ship.

The latter condition is mandatory and no point is assigned for the asteroids reached by a probe that fails to return to the mother ship. Multiple rendezvous of the same asteroid (by the same or another probe) do not provide additional points.

The performance index is therefore

$$J = \sum_{i=1}^{N_{ast}} \alpha_i \tag{1}$$

where the summation is extended to all the N_{ast} asteroids of the database `GTOC7ASTEROIDS.TXT` and α_i is 1 if at least one probe had rendezvous with asteroid i, stayed with it for 30 days and then returned to the mother ship, and 0 otherwise. Mother-ship rendezvous with an asteroid does not assign any point.

Particular situations may arise if probe and mother ship are both in rendezvous conditions with an asteroid; two cases could be relevant for the competition: first, the mother ship performs rendezvous with an asteroid and then releases the probe; second, the probe is in rendezvous conditions with an asteroid and is picked up by the mother ship for the final rendezvous. For these situations the logic is to replicate a traditional maneuver, assuming that the probes must be released from the mother ship in order to operate at the asteroid during the 30-day stay.

In the first case, the probe release time, which is relevant for the probe trip-time constraint and the stay-time constraint, can be either equal to (probe immediately released) or larger than (later release) the mother-ship asteroid rendezvous time. The asteroid counts as first rendezvous of the probe and one point can be assigned (unless the asteroid has already been visited), provided all the constraints are satisfied.

In the second case, the capture time, which is relevant for the probe trip-time constraint and stay-time constraint, must be again either equal to (probe immediately captured) or larger than (later capture) the mother-ship rendezvous time with asteroid and probe. If the last probe is concerned, the capture time is also relevant for the overall mission time constraint.

Note that each probe cannot be released by the mother ship more than once, and after it has returned to the mother ship no other action by the probe is allowed. It is therefore not permitted to use the mother ship to move probes between asteroids.

The secondary performance index, in the case two or more teams have the same score J, is the sum of the final (i.e., after rendezvous with the mother ship) masses of the probes

$$J' = \sum_{j=1}^{3} \beta_j (m_f)_{Pj} \tag{2}$$

where $P1, P2, P3$ refer to the three probes and subscript f corresponds to the final time; β_j is 1 if the probe has returned to the mother ship (or has not been released) and 0 otherwise. If a probe is not released by the mother ship it obviously does not contribute to increase J but the full 2000 kg contribute to J'.

Dynamical Models

The Earth and asteroids are assumed to follow Keplerian (conic) orbits around the Sun. The only forces acting on the spacecraft are the Sun's gravity and, when on, the thrust from the propulsion system. The Earth's Keplerian orbital parameters are provided in Table 1.

Table 1: Earth's orbital elements in the J2000 heliocentric ecliptic reference frame.

semimajor axis a, AU	0.999988049532578
eccentricity e	$1.671681163160 \cdot 10^{-2}$
inclination i, deg.	$0.8854353079654 \cdot 10^{-3}$
longitude of ascending node Ω, deg.	175.40647696473
Argument of periapsis ω, deg.	287.61577546182
Mean anomaly at epoch M, deg.	257.60683707535
Epoch t, MJD	54000

The asteroids' Keplerian orbital parameters are provided in the ASCII file `GTOC7ASTEROIDS.TXT`, which provides 1) asteroid GTOC7 identification number, 2) epoch, in modified Julian date, 3) semimajor axis in AU, 4) eccentricity, 5) inclination in degrees, 6) argument of periapsis in degrees, 7) longitude of the ascending node in degrees, 7) mean anomaly at epoch in degrees, 9) asteroid name. Earth's and asteroids' orbital elements are expressed in the J2000 heliocentric ecliptic frame. The elements are taken from the public, small-body database maintained by JPL and accessible at http://ssd.jpl.nasa.gov. Since orbital elements are periodically checked and modified, the official asteroid elements for this problem are those provided in the file `GTOC7ASTEROIDS.TXT`. Other required constants are shown in Table 2.

Table 2: Constants and conversion.

Sun's gravitational parameter μ_S, $\mathrm{km^3/s^2}$	$1.32712440018 \cdot 10^{11}$
Astronomical Unit AU, km	$1.49597870691 \cdot 10^{8}$
Standard acceleration due to gravity, g_0, $\mathrm{m/s^2}$	9.80665
Day, s	86400
Year, days	365.25
01 January 2021 00:00 ET, MJD	59215
31 December 2030 24:00 ET, MJD	62867

Solution Format

Each team should return its best solution by email to lorenzo.casalino@polito.it on or before June 17, 2014, 18:00 UT (20:00 CET). Five files must be returned. The first file (PDF format is preferred) should contain:

- a brief description of the methods used,
- a summary of the best trajectory found with, at least: mother ship launch date, launch v_∞, dates of probe release, GTOC7 numbers of the asteroids visited by each probe, dates of probe final rendezvous with the mother ship, probe and mother ship final masses, and value of the performance indexes (primary and secondary).
- a visual representation of the trajectories, such as a projection of the trajectory onto the ecliptic plane.

The other files, which will be used to verify the solution, must provide the trajectories of mother ship and probes and should be named `M.TXT`, `P1.TXT`, `P2.TXT`, and `P3.TXT`. They must follow the format and units provided in the ASCII template files `Mexample.TXT` and `P1example.TXT`. The coordinate frame must be the J2000 heliocentric ecliptic frame.

The mother ship follows Keplerian dynamics and its trajectory can be easily reconstructed. The file should provide position, velocity and mass at times when specific events (departure from Earth, impulses, probe release and rendezvous) occur.

Probe trajectory data are required at one-day increments for each inter-body phase of the trajectory, providing position, velocity, mass and thrust components. The first time point for each phase should correspond with body (mother ship or asteroid) departure; the second time point should be one day thence, and so on. If arrival at an asteroid or mother ship does not fall on a one-day increment, then the last time point for the phase should be reported using a partial-day increment from the previous time point. Partial day increments must also be used to highlight times when the engine is switched on or off.

Appendix

This appendix provides a set of equations describing the dynamics of this problem along with other background information.

Nomenclature

Orbital elements and related quantities

a	= semimajor axis
e	= eccentricity
i	= inclination
Ω	= longitude of ascending node
ω	= argument of periapsis
M	= mean anomaly at epoch
θ	= true anomaly
E	= eccentric anomaly
r	= distance from the Sun
γ	= flight path angle
μ_S	= Sun's gravitational parameter

Position and velocity

$\boldsymbol{r}$	= position vector
$\boldsymbol{v}$	= velocity vector
x, y, z	= position components in J2000 heliocentric ecliptic frame
v_x, v_y, v_z	= velocity components in J2000 heliocentric ecliptic frame

Departure

$\boldsymbol{v}_\infty$	= hyperbolic excess velocity vector
v_∞	= hyperbolic excess velocity magnitude

Other quantities

t	= time
m	= mass
I_{sp}	= specific impulse
T	= thrust
g_0	= standard acceleration due to gravity at Earth's surface
J	= primary performance index
J'	= secondary performance index

Subscripts and superscripts

0	= at epoch
i	= initial value
$-$	= just before event
$+$	= just after event
f	= final value
E	= Earth
M	= mother ship
Ai	= i-th asteroid
$P1$, $P2$, $P3$	= first, second and third probe
max	= maximum value
min	= minimum value
$\dot{()}$	= time derivative

Problem dynamics and conversion between elements

The motion of Earth, asteroids and mother ship around Sun is governed by the two-body problem equations:

$$\ddot{x} = -\mu_S \frac{x}{r^3} \qquad \ddot{y} = -\mu_S \frac{y}{r^3} \qquad \ddot{z} = -\mu_S \frac{z}{r^3}$$

where

$$r = \sqrt{x^2 + y^2 + z^2} = \frac{a(1-e^2)}{1 + e\cos\theta}$$

The motion of the spacecraft around the Sun is governed by the same formulas but with the addition of the x, y, z components of the thrust acceleration and an equation for the mass:

$$\ddot{x} = -\mu_S \frac{x}{r^3} + \frac{T_x}{m} \qquad \ddot{y} = -\mu_S \frac{y}{r^3} + \frac{T_y}{m} \qquad \ddot{z} = -\mu_S \frac{z}{r^3} + \frac{T_z}{m} \qquad \dot{m} = -\frac{T}{g_0 (I_{sp})_{EP}}$$

The thrust magnitude is constrained

$$0 \le T = \sqrt{T_x^2 + T_y^2 + T_z^2} \le 0.3\,\mathrm{N}$$

Conversion from orbit elements to Cartesian quantities is as follows

$$x = r[\cos(\theta+\omega)\cos\Omega - \sin(\theta+\omega)\cos i \sin\Omega]$$

$$y = r[\cos(\theta+\omega)\sin\Omega + \sin(\theta+\omega)\cos i \cos\Omega]$$

$$z = r[\sin(\theta+\omega)\sin i]$$

$$v_x = v[-\sin(\theta+\omega-\gamma)\cos\Omega - \cos(\theta+\omega-\gamma)\cos i \sin\Omega]$$

$$v_y = v[-\sin(\theta+\omega-\gamma)\sin\Omega + \cos(\theta+\omega-\gamma)\cos i \cos\Omega]$$

$$v_z = v[\cos(\theta+\omega-\gamma)\sin i]$$

where the velocity magnitude v and the flight path angle γ are

$$v = \sqrt{\frac{2\mu_S}{r} - \frac{\mu_S}{a}} \qquad\qquad \tan\gamma = \frac{e\sin\theta}{1+e\cos\theta}$$

For an elliptic orbit the true anomaly is related to the eccentric anomaly by

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}$$

and the eccentric anomaly is related to the mean anomaly by Kepler's equation,

$$M = E - e\sin E,$$

while the mean anomaly is related to time and the mean anomaly M_0 at epoch t_0 by

$$M - M_0 = \sqrt{\frac{\mu_S}{a^3}}(t-t_0)$$

Thus, based on the provided orbital parameters, the Cartesian positions and velocities of the Earth and asteroids may be computed as a function of time with only the minor nuisance of having to solve Kepler's equation for E by some iterative procedure. The same is true for the mother ship motion between impulses. This means that for the Earth, the asteroids, and a non-thrusting spacecraft, the equations of motion do not need to be numerically integrated to find position and velocity at some given time.

Earth departure

Mother ship launch occurs at time t_i with hyperbolic excess velocity $\boldsymbol{v}_\infty$. The spacecraft position, velocity and mass are

$$(\boldsymbol{r}_i)_M = \boldsymbol{r}_E(t_i) \qquad (\boldsymbol{v}_i)_M = \boldsymbol{v}_E(t_i) + \boldsymbol{v}_\infty \qquad (m_i)_M = 24000\,\text{kg}$$

with the constraints

$$|\boldsymbol{v}_\infty| \le 6\,\text{km/s} \qquad 59215\,\text{MJD} \le t_i \le 62867\,\text{MJD}$$

Probe release and rendezvous

At each instantaneous probe release and rendezvous the mother ship and probe position and velocity are the same. Position and velocity do not change but the mother ship mass is discontinuous: at release of probe Pj ($j = 1, \ldots 3$)

$$(m_+)_M - (m_-)_M = -2000\,\text{kg}$$

$$(m_i)_{Pj} = 2000\,\text{kg}$$

and at rendezvous between probe Pj and mother ship

$$(m_+)_M - (m_-)_M = (m_f)_{Pj}$$

Impulses

At impulses, position is constant. The required propellant mass is expressed as a function of the magnitude of the velocity change through Tsiolkowsky's equation

$$\Delta V = |(\boldsymbol{V}_+)_M - (\boldsymbol{V}_-)_M|$$

$$(m_+)_M = (m_-)_M \ \exp\left[-\frac{\Delta V}{g_0 (I_{sp})_{NTR}}\right]$$

Asteroid rendezvous and departure

Rendezvous occurs at time t_{rv} when the probe Pj matches the position and velocity of the asteroid Ai

$$\boldsymbol{r}_{Pj} = \boldsymbol{r}_{Ai}(t_{rv}) \qquad\qquad \boldsymbol{v}_{Pj} = \boldsymbol{v}_{Ai}(t_{rv})$$

Departure from asteroid Ai occurs at time t_{de} with the same conditions

$$\boldsymbol{r}_{Pj} = \boldsymbol{r}_{Ai}(t_{de}) \qquad\qquad \boldsymbol{v}_{Pj} = \boldsymbol{v}_{Ai}(t_{de})$$

The stay time constraint is expressed as

$$t_{de} - t_{rv} \geq 30\,\text{days}$$

Time and mass constraints

The overall mission time-length is constrained

$$\max(t_f)_{Pj} - t_i \leq 12\,\text{years}$$

The trip time of each probe is also constrained

$$(t_f)_{Pj} - (t_i)_{Pj} \leq 6\,\text{years}$$

Final masses are constrained

$$(m_f)_M - \sum_{j=1}^{3} (m_f)_{Pj} \geq 6000\,\text{kg}$$

$$(m_f)_{Pj} \geq 800\,\text{kg}$$

The constraints on position and velocity must be satisfied with accuracy of at least 1000 km and 1 m/s, respectively (these numbers are for the euclidean norm of the vector differences).