Solution Description for the 7th Global Trajectory Optimisation Competition

Team 30: Politecnico di Milano
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Attached files: Team30-M.txt, Team30-P1.txt, Team30-P2.txt, Team30-P3.txt
1. Summary of the Solution

In the following subsections all relevant data related to the mission are presented.

1.1 Objective functions

- Primary performance index: $J = 31$
- Secondary performance index: $J' = 2462.92$ kg
- Number of asteroids rendezvous: $N_{ast} = 35$
- Asteroids visited: #531, #656, #1402, #1891, #1906, #3016, #3326, #3412, #3418, #3749, #3899, #4393, #4828, #4998, #5046, #5711, #5727, #6575, #8149, #8442, #8472, #9645, #9646, #10772, #10943, #11037, #12527, #12543, #12614, #16080, #16193

1.2 Mother ship

Total mission duration: 9.882 years

Departure epoch from Earth: MJD 62093.5
Arrival epoch at asteroid #3326: MJD 62568.2
Epoch of release of Probe 1: MJD 62606.0
Epoch of release of Probe 2: MJD 63350.0
Epoch of release of Probe 3: MJD 63653.0
Departure epoch from asteroid #3326: MJD 63653.0
Arrival epoch at asteroid #3412: MJD 64345.2
Epoch of rendezvous with Probe 1: MJD 64757.0
Epoch of rendezvous with Probe 2: MJD 65480.0
Epoch of rendezvous with Probe 3: MJD 65703.0

Earth departure $v_{\infty}$: 6.000 km/s
Earth departure $\Delta v$ (exceeding $v_{\infty}$): 0.217 km/s
Asteroid #3326 arrival $\Delta v$: 4.942 km/s
Asteroid #3326 departure $\Delta v$: 0.395 km/s
Asteroid #3412 arrival $\Delta v$: 0.512 km/s

Earth departure propellant: 583.447 kg
Asteroid #3326 arrival propellant: 10040.692 kg
Asteroid #3326 departure propellant: 322.576 kg
Asteroid #3412 arrival propellant: 397.689 kg
Total propellant used: 11344.405 kg
Total propellant remaining: 655.595 kg
Final mother ship mass: 6655.595 kg
1.3 Probe 1

Release at asteroid: #3326
Collection at asteroid: #3412
Mission duration: 5.889 years
Final probe mass: 826.191 kg
Total points scored: 11

Stay at asteroid #3326: 31 days

**Segment 1:**
Departure from asteroid #3326: 62637 MJD
Arrival at asteroid #4828: 62882 MJD
Stay at asteroid #4828: 30 days

**Segment 2:**
Departure from asteroid #4828: 62912 MJD
Arrival at asteroid #3749: 63132 MJD
Stay at asteroid #3749: 30 days

**Segment 3:**
Departure from asteroid #3749: 63162 MJD
Arrival at asteroid #8149: 63327 MJD
Stay at asteroid #8149: 30 days

**Segment 4:**
Departure from asteroid #8149: 63357 MJD
Arrival at asteroid #10943: 63577 MJD
Stay at asteroid #10943: 30 days

**Segment 5:**
Departure from asteroid #10943: 63607 MJD
Arrival at asteroid #5711: 63787 MJD
Stay at asteroid #5711: 30 days

**Segment 6:**
Departure from asteroid #5711: 63817 MJD
Arrival at asteroid #9646: 64007 MJD
Stay at asteroid #9646: 30 days

**Segment 7:**
Departure from asteroid #9646: 64037 MJD
Arrival at asteroid #8442: 64207 MJD
Stay at asteroid #8442: 30 days

**Segment 8:**
Departure from asteroid #8442: 64237 MJD
Arrival at asteroid #12527: 64367 MJD
Stay at asteroid #12527: 30 days

**Segment 9:**
Departure from asteroid #12527: 64397 MJD
Arrival at asteroid #5727: 64547 MJD
Stay at asteroid #5727: 30 days

**Segment 10:**
Departure from asteroid #5727: 64577 MJD
Arrival at asteroid #3412: 64727 MJD
Stay at asteroid #3412: 30 days
1.4 Probe 2

Release at asteroid: #3326
Collection at asteroid: #3412
Mission duration: 5.831 years
Final probe mass: 815.924 kg
Total points scored: 10

Segment 1:
Departure from asteroid #3326: 63350 MJD
Arrival at asteroid #11037: 63600 MJD
Stay at asteroid #11037: 30 days

Segment 2:
Departure from asteroid #11037: 63630 MJD
Arrival at asteroid #3418: 63845 MJD
Stay at asteroid #3418: 30 days

Segment 3:
Departure from asteroid #3418: 63875 MJD
Arrival at asteroid #3899: 64050 MJD
Stay at asteroid #3899: 30 days

Segment 4:
Departure from asteroid #3899: 64080 MJD
Arrival at asteroid #9645: 64270 MJD
Stay at asteroid #9645: 30 days

Segment 5:
Departure from asteroid #9645: 64300 MJD
Arrival at asteroid #6575: 64465 MJD
Stay at asteroid #6575: 30 days

Segment 6:
Departure from asteroid #6575: 64495 MJD
Arrival at asteroid #1402: 64695 MJD
Stay at asteroid #1402: 30 days

Segment 7:
Departure from asteroid #1402: 64725 MJD
Arrival at asteroid #16193: 64810 MJD
Stay at asteroid #16193: 30 days

Segment 8:
Departure from asteroid #16193: 64840 MJD
Arrival at asteroid #8472: 65010 MJD
Stay at asteroid #8472: 30 days

Segment 9:
Departure from asteroid #8472: 65040 MJD
Arrival at asteroid #531: 65190 MJD
Stay at asteroid #531: 30 days

Segment 10:
Departure from asteroid #531: 65220 MJD
Arrival at asteroid #4393: 65330 MJD
Stay at asteroid #4393: 30 days

Segment 11:
Departure from asteroid #4393: 65360 MJD
Arrival at asteroid #3412: 65450 MJD
Stay at asteroid #3412: 30 days (no point scored)
1.5 Probe 3

Release at asteroid: #3326
Collection at asteroid: #3412
Mission duration: 5.612 years
Final probe mass: 820,800 kg
Total points scored: 10

Segment 1:
Departure from asteroid #3326: 63653 MJD
Arrival at asteroid #16080: 63833 MJD
Stay at asteroid #16080: 30 days

Segment 2:
Departure from asteroid #16080: 63863 MJD
Arrival at asteroid #4998: 64098 MJD
Stay at asteroid #4998: 30 days

Segment 3:
Departure from asteroid #4998: 64128 MJD
Arrival at asteroid #12543: 64293 MJD
Stay at asteroid #12543: 30 days

Segment 4:
Departure from asteroid #12543: 64323 MJD
Arrival at asteroid #1891: 64453 MJD
Stay at asteroid #1891: 30 days

Segment 5:
Departure from asteroid #1891: 64483 MJD
Arrival at asteroid #10772: 64653 MJD
Stay at asteroid #10772: 30 days

Segment 6:
Departure from asteroid #10772: 64683 MJD
Arrival at asteroid #656: 64788 MJD
Stay at asteroid #656: 30 days

Segment 7:
Departure from asteroid #656: 64818 MJD
Arrival at asteroid #3016: 64988 MJD
Stay at asteroid #3016: 30 days

Segment 8:
Departure from asteroid #3016: 65018 MJD
Arrival at asteroid #1906: 65133 MJD
Stay at asteroid #1906: 30 days

Segment 9:
Departure from asteroid #1906: 65163 MJD
Arrival at asteroid #5046: 65313 MJD
Stay at asteroid #5046: 30 days

Segment 10:
Departure from asteroid #5046: 65343 MJD
Arrival at asteroid #12614: 65493 MJD
Stay at asteroid #12614: 30 days

Segment 11:
Departure from asteroid #12614: 65523 MJD
Arrival at asteroid #3412: 65673 MJD
Stay at asteroid #3412: 30 days (no point scored)
2. Brief description of the optimisation strategy

The optimization strategy used in finding this solution was a mix of an exhaustive search in the sequence space combined with a genetic optimizer and subsequent manual optimization of the results.

2.1 Strategy

The mother ship leaves the Earth and rendezvous with an asteroid A. Once in rendezvous conditions with asteroid A, the mother ship releases the three probes at different times. One of the three probes stays at the asteroid for longer than 30 days, and therefore one point is scored. The three probes then perform their tours, each one of them leading to the same final asteroid B. Meanwhile, the mother ship travels from asteroid A to asteroid B where it meets with the probes. At asteroid B another point is scored as at least one probe stays in rendezvous conditions for at least 30 days before being collected by the mother ship.

2.2 Optimal sequences generation

Sequences compatible with the limitations of low thrust propulsion are constructed by repeatedly computing the cost of a Lambert transfer from the current asteroid to all other possible asteroids. The resulting Δv is weighted using a correction factor to account for low thrust propulsion limitations. The best feasible asteroids are selected for further investigation and the process is repeated for each new sequence until either time or fuel are exhausted. This generation of sequences is implemented in a massively parallelized algorithm using OpenCL to run on GPUs (NVidia Tesla K20 cards). This allows for an exhaustive search of the entire space of all possible sequences over all GTOC7 asteroids given a certain starting date and asteroid within a matter of seconds. The resulting optimal sequences are then post-processed and analysed for suitability in the mother ship trajectory optimization process and finally subjected to a local optimization process to solve for the low thrust trajectories.

2.3 Mother ship trajectory optimization

The mother ship trajectory design is carried out by running a genetic algorithm that optimizes the final mother ship mass taking into account the constraints on mission duration, total number of manoeuvres, propellant available, synchronization with probes release and collection. In the optimization process, the optimal sequence generator is called repeatedly for each individual of the generation to maximize the number of asteroids visited.

2.4 Low-thrust optimization

The low-thrust trajectory optimization problem can be phrased as an Optimal Control Problem (OCP). Using calculus of variation theory, the OCP is converted to a Two-Point Boundary Value Problem (TPBVP). In each segment, given the missing part of initial costates, the final states and costates can be obtained by integrating canonical equations and the implicit control law. The related shooting process consists of finding 7 unknown initial costates in order to satisfy 7 equality constraints. In order to cope with the convergence
problem of the fuel-optimal problem, a homotopy method is implemented by starting from the easier energy-optimal problem. In addition, a bisection method is used to accurately detect the switching points together with a fixed step 4th-order Runge-Kutta integrator. Starting from the near-feasible sequences computed in section 2.2, this allows solving for correct low thrust trajectories by manually varying the departure times at the asteroids to ensure convergence. Once all legs of the tours of the probes have converged, the final constraints due to the corrected sequences are used to adjust the mother ship trajectory as needed.
3. Visualizations of the Solution

Fig. 1: Transfer trajectory of probe 1

Fig. 2: Thrust and mass profile of probe 1

Fig. 3: Transfer trajectory of probe 2

Fig. 4: Thrust and mass profile of probe 2

Fig. 5: Transfer trajectory of probe 3

Fig. 6: Thrust and mass profile of probe 3
Fig. 7: Transfer trajectory of all 3 probes

Fig. 8: Transfer trajectory of the mother ship
4. Sequence of Events

The following table provides a chronological overview of the exact sequence of events of this mission.

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<th>Events</th>
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