Final Results of the
4th Global Trajectory Optimisation Competition

Régis Bertrand, Richard Epenoy and Benoît Meyssignac

Centre National d'Etudes Spatiales (CNES)
18, avenue Edouard Belin
31401 Toulouse Cedex 9, France
regis.bertrand@cnes.fr

Background

Forty seven teams registered for the 4th edition of the Global Trajectory Optimisation Competition, held from March 2 to March 30 2009. Twenty six solutions have been returned before the official closure of the competition. This document details the solutions found and the methods used by the different teams.

Problem description

The mission proposed this year may be entitled: "How to maximise the relevance of a rendezvous mission to a given NEA by visiting the largest set of intermediate asteroids".

More precisely, let us assume that a spacecraft is launched from the Earth. This spacecraft has first to visit (flyby) a maximum number of asteroids (from a given list of NEAs). Finally, it must rendezvous with a last asteroid of that same list within ten years from departure. The performance index to be maximised is the number of visited asteroids, but when two solutions are associated with the same number of visited NEAs, a secondary performance index has to be maximised: the final mass of the spacecraft.

Moreover, we assume that the spacecraft is equipped with an electric propulsion system and that gravity assists are not allowed during the mission. The use of electric propulsion yields an optimal control formulation for the GTOC4 problem once a sequence of asteroids has been chosen. The huge number of such feasible asteroids sequences leads to a large number of local optima for the problem.

Results

Twenty six solutions have been returned before the official closure of the competition. Twenty three were considered correct but for two of them major constraints violations make them be not acceptable. One solution was rejected because it has been received after the official closure of the competition. Some explanations concerning the constraints violations:

- The minor constraints violations mainly concern the Earth's ephemeris at the beginning of the mission. Indeed, some teams have used different models compared with the one given in the problem description. But these constraints violations have no impact on the results. This is the reason why the associated solutions were ranked.
- The major constraints violations are due to a misunderstanding of the problem or issues on the extrapolation model. These violations have consequences on the results. So the associated solutions were not ranked.

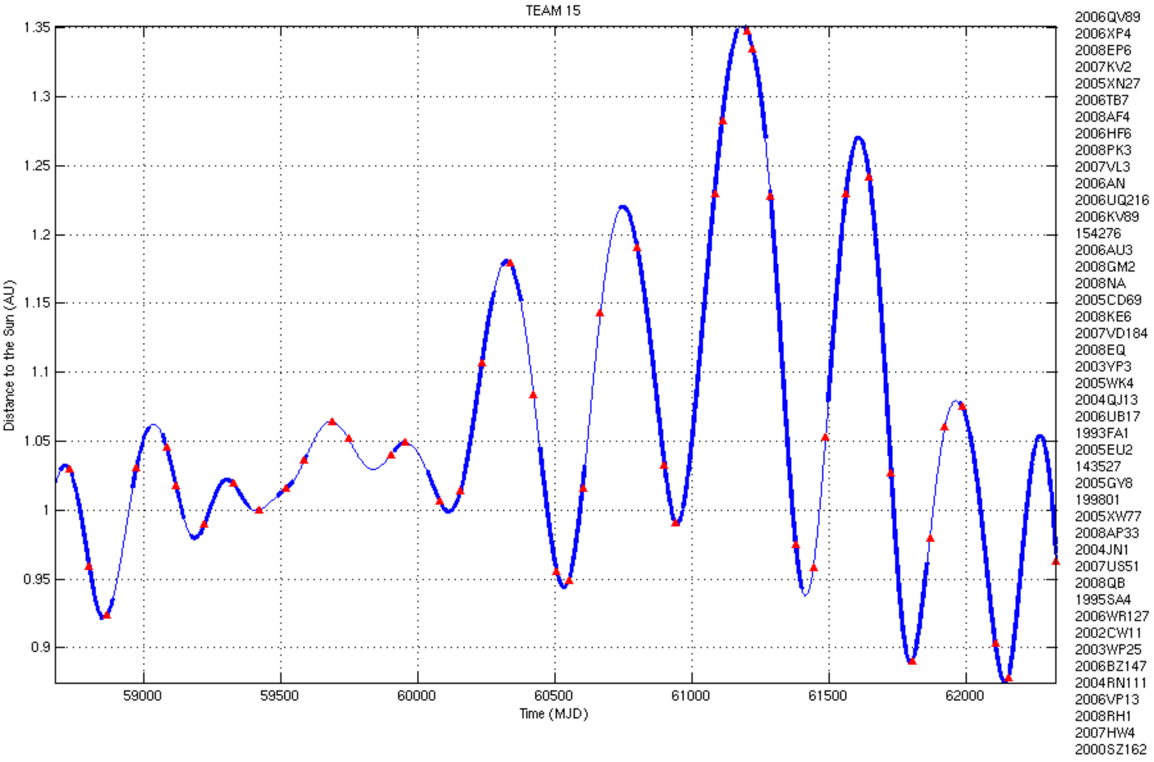
The ranking is summarised in the following table. The remaining sections describe briefly the solutions found and the methods used from the descriptions returned by the teams.

rank	**team #**	**team name**	***J***	**$K = m_f$ [kg]**	**duration [year]**	**rendezvous asteroid**
1	15	Moscow State University	**44**	**553.46**	10	2000SZ162
2	25	The Aerospace Corporation	**44**	**516.83**	10	2000SZ162
3	12	Advanced Concepts Team, ESA	**42**	**511.45**	10	2008UA202
4	20	DEIMOS Space	**39**	**605.44**	10	2006BZ147
5	41	GMV	**39**	**516.30**	10	2007YF
6	19	Jet Propulsion Laboratory	**38**	**515.87**	10	138911
7	8	Politecnico di Torino, Universita di Roma La Sapienza	**36**	**574.44**	10	2006QQ56
8	32	University of Texas at Austin, Odyssey Space Research, ERC Incorporated	**32**	**639.86**	9.69	2006UB17
9	34	University of Glasgow University of Strathclyde	**29**	**715.21**	9.98	2006QQ56
10	13	Thales Alenia Space	**27**	**533.25**	10	2006QQ56
11	10	University of Trento	**26**	**721.73**	9.73	2006UB17
12	46	University of Bremen, Politecnico di Milano	**26**	**577.97**	9.82	2008GM2
13	31	Moscow Aviation Institute, Research Institute of Applied Mechanics and Electrodynamics	**24**	**720.62**	10	2007YF
14	2	Georgia Institute of Technology	**24**	**500.27**	9.5	2008UA202
15	42	TOMLAB	**22**	**615.22**	9.65	2006XP4
16	6	VEGA	**20**	**653.07**	10	2008UA202
17	5	DLR German Space Operations Center Aachen University of Applied Sciences	**20**	**635.09**	10	2005BG28
18(a)	38	Team Astroshape	**20**	**524.48**	10	2006SV5
19	40	DLR Institute of Space Systems	**19**	**592.35**	10	138911
20	4	Tsinghua University	**18**	**539.98**	10	138911
21	11	University of Missouri	**15**	**836.06**	10	2005CD69
22	9	Beijing University of Aeronautics and Astronautics	**13**	**651.87**	9.98	2006RJ1
23(a)	35	Texas A&M University	**12**	**697.93**	10	2006UB17
-(b)	37	Nanjing University of Aeronautics and Astronautics	**54**	**836.53**	9.58	2005SN5
-(b)	23	CHOPIN Team	**24**	**1436.33**	10.12	2008UA202
-(c)	18	Chinese Academy of Sciences	**19**	**872.65**	9.68	2004XG

(a) minor constraints violation having negligible influence on the results
(b) major constraints violation, solution not ranked
(c) late solution

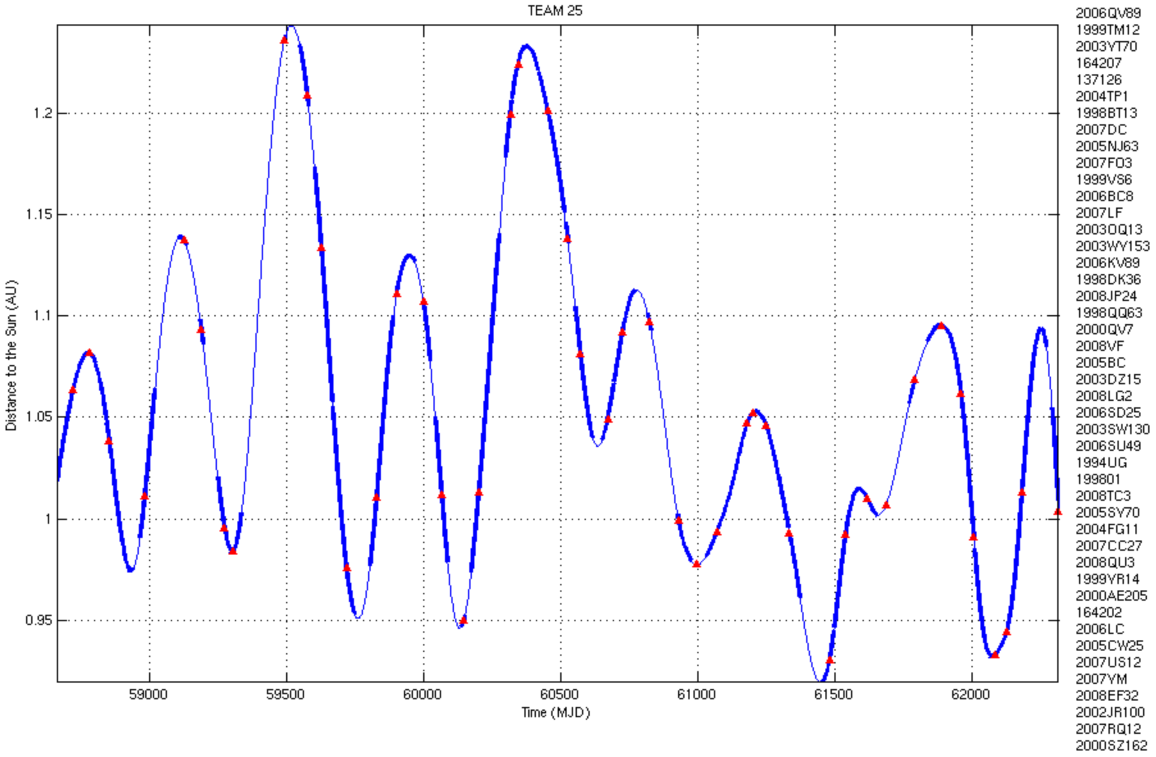
Team 15
Moscow State University (Russia)

The solution method is based on the analysis of the graph of perspective trajectories. In a first time, the Lambert's problems between the Earth and all the asteroids are solved and the solutions associated with a transfer time between 20 and 100 days and satisfying the departure infinite velocity constraint are kept. After that, each trajectory leg is computed in the same way by applying selection criterion based on the transfer time and propellant consumption. Progressively 2000, 1000 and 500 trajectories are analysed for each trajectory leg. Finally, the trajectories associated with low ΔV values are analysed in order to yield relevant low thrust optimal trajectories.



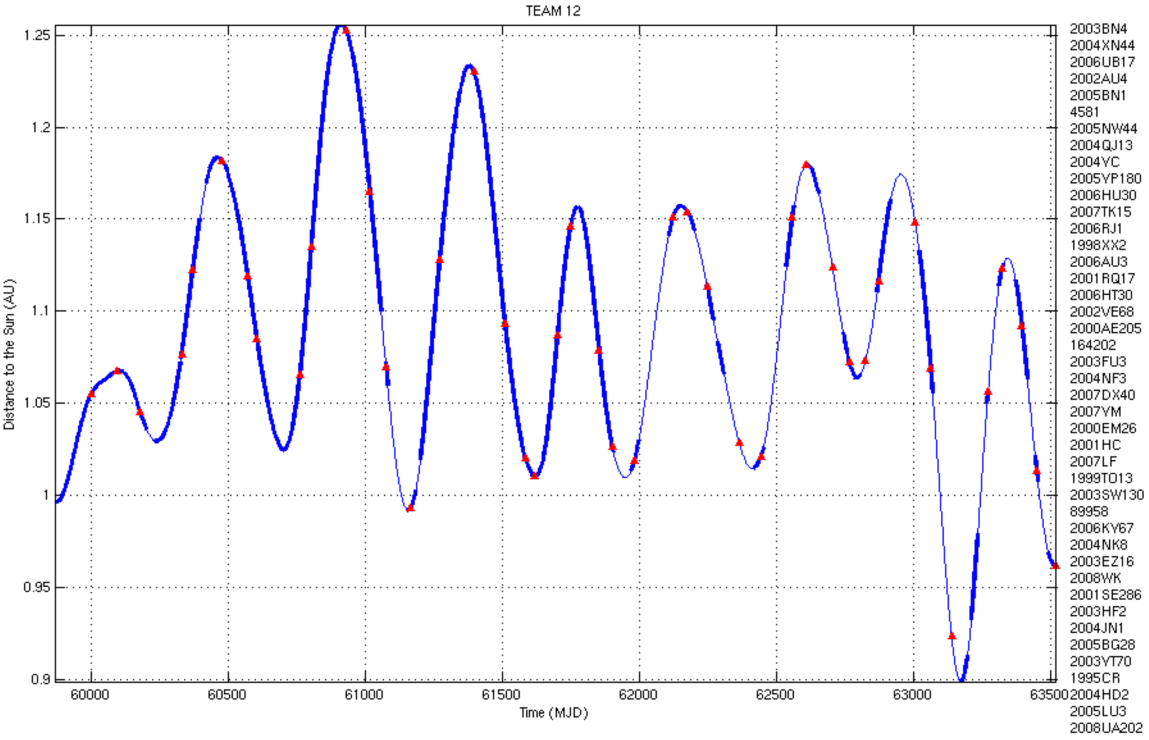
Team 25
The Aerospace Corporation (USA)

A first massive grid search is performed using a simplified dynamics model for the spacecraft and impulsive thrusts. The solutions are parameterised by the departure time at each leg and the duration of the transfer in increments of 15 days. A set of possible single leg transfers is computed using this approach. Then, these legs are patched together to form full missions. In a second step the missions coming from the above approach are locally optimised leg-by-leg by considering the low thrust characteristics of the engine. Finally, the software SOCS (Sparse Optimal Control Software Package) is used in order to optimise the full mission as a single optimisation problem starting from the initial guesses obtained above. The grid search results are used again at this step to determine if additional flybys can be added to the mission.



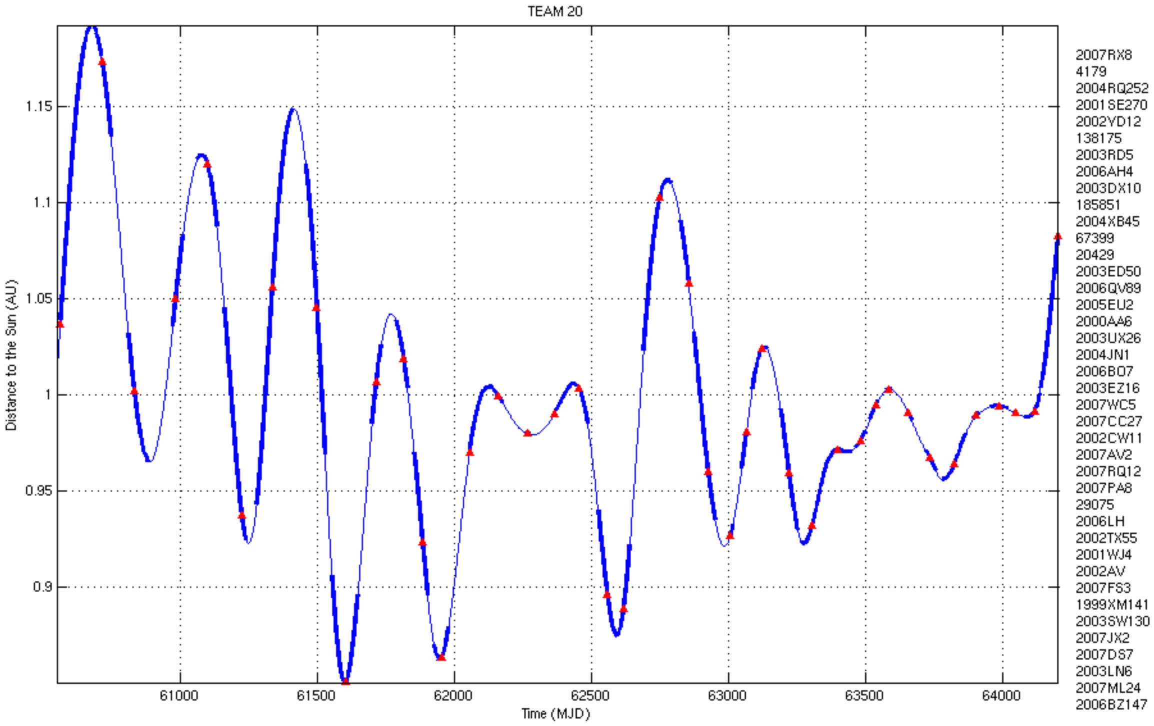
Team 12
Advanced Concepts Team, ESA (The Netherlands)

The proposed solution followed a 3 step process. First, a Branch and Prune (BP) method is used to look for chemically-powered optimal flyby trajectories: after selecting a list of rendezvous targets (10) using a criterion based on the minimum orbital distance (MOID) between asteroids, the resulting encounter trees are explored using the BP algorithm. An optimisation of the Lambert's arc joining the possible targets is done and the algorithm prune out those transfers that exceed the possible acceleration given by a continuous low thrust. Candidate asteroids sequences that are found and their corresponding encounter epochs are output for optimisation in the low thrust models. In a second step, a direct optimisation method based on an impulsive transcription (Sims-Flanagan) of the low thrust problem is used. The method is first used in 'single phase' mode to rank the thousands of asteroid sequences located by the branch and prune algorithm. The final output maximises the final mass of a candidate trajectory from the Branch and Prune algorithm. The solution found in this step is passed to an indirect method solver for further optimisation in an accurate model to improve the accuracy of the solution. In the last step, the solutions were used as initial guesses for solving the MPBVP arising from an indirect formulation. The MPBVP was solved using an interior point solver (IPOPT) and smoothing techniques.



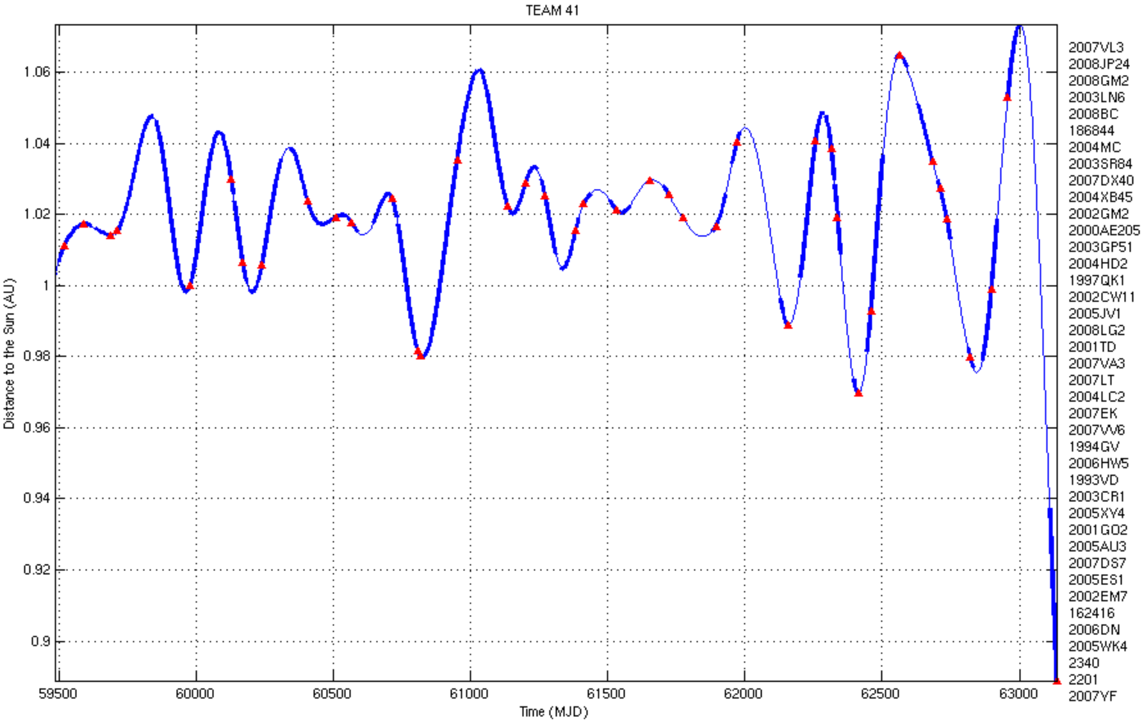
Team 20
DEIMOS Space (Spain)

In a first step a systematic search is performed by solving all the Lambert's problems between the pairs of asteroids given in the database. The solutions found are pruned by the maximum duration between asteroids and also the total ΔV. This approach leads to trajectories with up to 44 asteroids visited in 10 years. Then, the most promising solutions obtained at the first step are locally optimised by adjusting the departure date together with the flyby dates and velocities in order to increase the final mass. Finally, the Low Thrust Interplanetary Navigation Tool (LOTNAV) based on a direct method is used in order to maximise the final mass. This tool plays with the thrusting switches, the thrust law parameters but also with the departure and flyby dates and the departure v_∞.



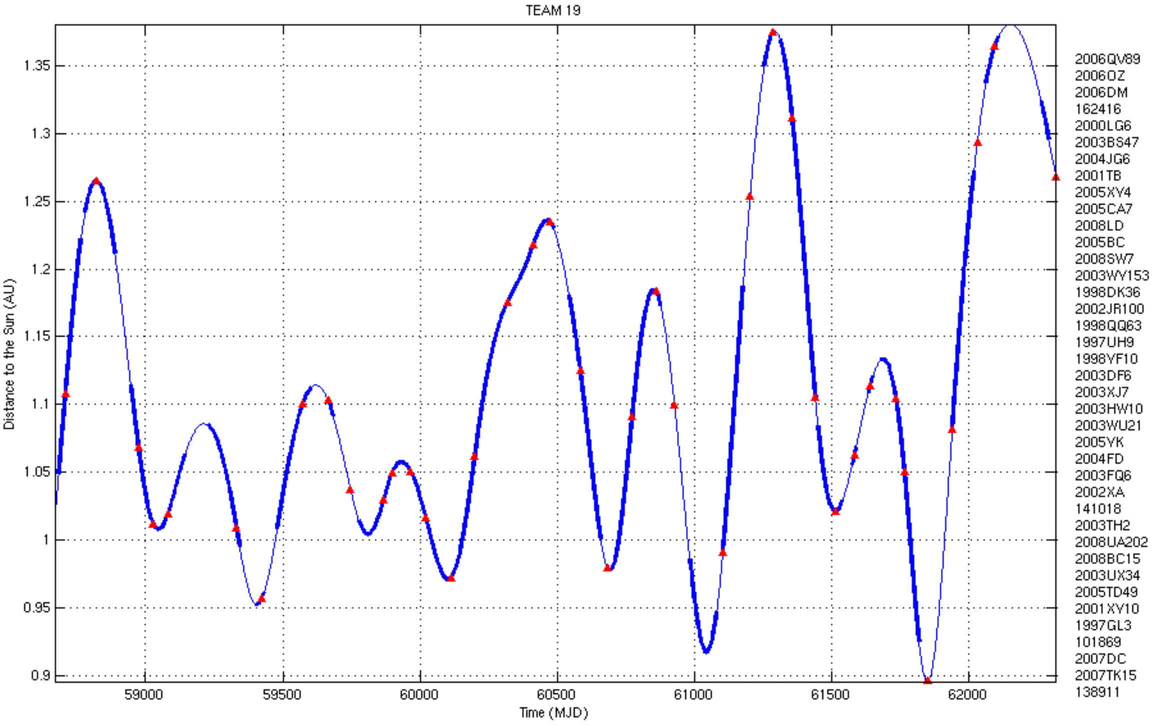
Team 41
GMV (Spain)

The solution method is based on five steps. The first step consists in finding an initial trajectory guess by optimising ballistic trajectories from the Earth to as many asteroids as possible within 10 years. Then, the impulsive trajectories (with impulsive manoeuvres at the asteroids fly-bys and the rendezvous) are deduced, a branch and pruning algorithm is used to select those asteroids that can be visited within the ΔV budget of the mission and a refinement is done thanks to a parameter optimisation of the departure, fly-by and rendezvous dates in order to minimise the total ΔV. The third step is dedicated to the computation of the relevant low thrust trajectories (for feasible trajectories) from the previous impulsive trajectories by means of a local optimiser. The fourth step is foreseen for the refinement of the feasible low thrust trajectories, i.e. to find opportunities to introduce additional asteroid fly-bys. Finally, an optimisation of the low thrust trajectory is performed by maximising the final mass.



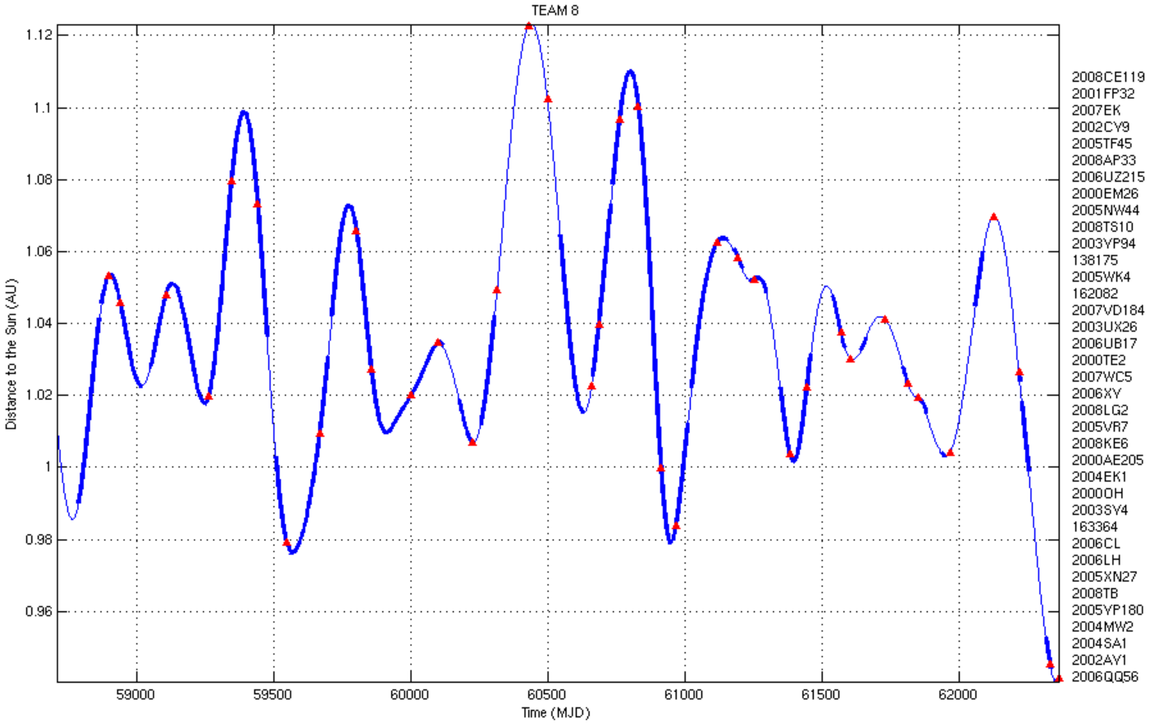
Team 19
Jet Propulsion Laboratory (USA)

In a first step a broad search is implemented by performing a combinatorial analysis of Lambert's problems. All Lambert's problems are solved between pairs of asteroids given in the database, subject to v_∞ and flight-time constraints. These combinations are pruned by different criteria such as the total ΔV and spacecraft orbital elements. In a second step the initial guesses from the broad search are optimised with the local optimiser MAILTO. This last one models a low thrust by a succession of small impulses whose direction and magnitude are to be optimised using the nonlinear SNOPT solver. Finally, the most promising solutions are refined by hand.



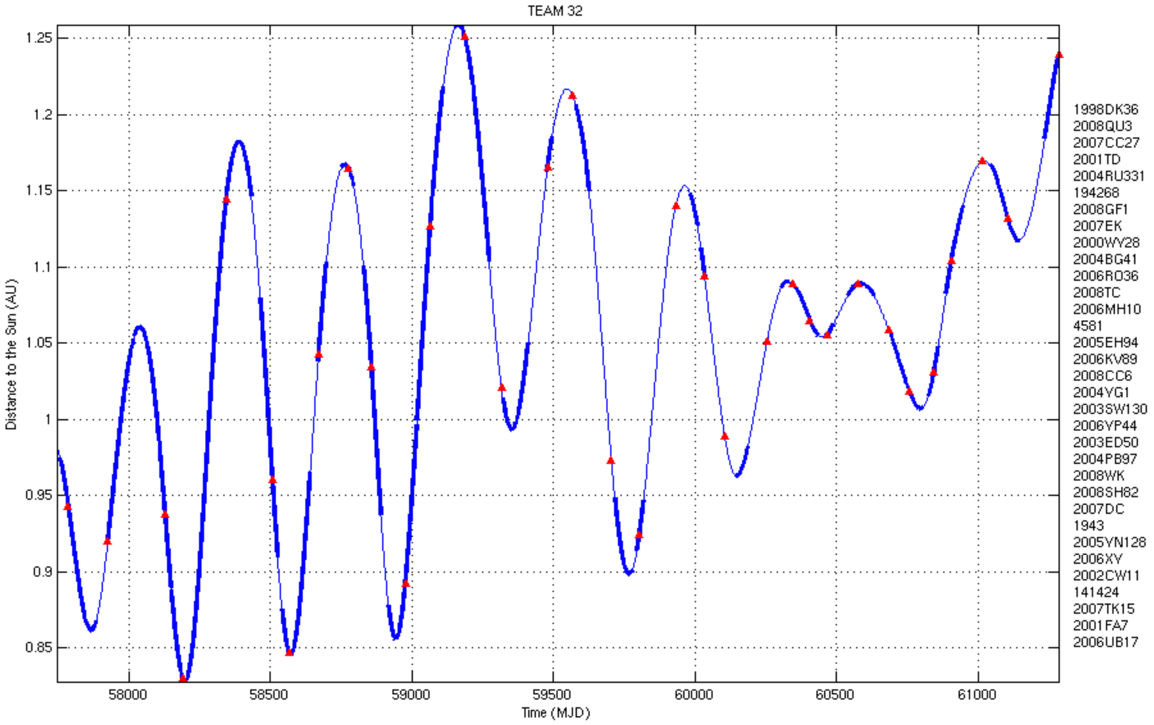
Team 8
Politecnico di Torino, Universita di Roma La Sapienza (Italy)

First some analytical considerations allowed to consider 2006QQ56 as the most interesting target for rendezvous because of its low eccentricity and inclination, together with its semi major axis close to 1 AU. Then, a preliminary analysis allowed to find a large number of minimum time trajectories from the Earth to a nodal intercept of asteroids chosen as first ones (departure legs) and from a nodal flyby of asteroids, chosen as last ones, to 2006QQ56 rendezvous (arrival legs) by means of an indirect method. These arcs were respectively propagated forward and backward to intercept additional asteroids. This automatic procedure produced trajectories with up to 40 intercepts within the ten-year trip time, but appeared at a large extent inaccurate. Therefore, a local optimisation of the asteroid-asteroid arcs was tried. The most promising missions were then analysed in details. This procedure provided a 28-intercept mission that was further analysed. Careful selection of asteroids, by taking the spacecraft actual position and orbit into account, allowed for asteroid replacements and additions that eventually produced the final trajectory with 36 intercepts.



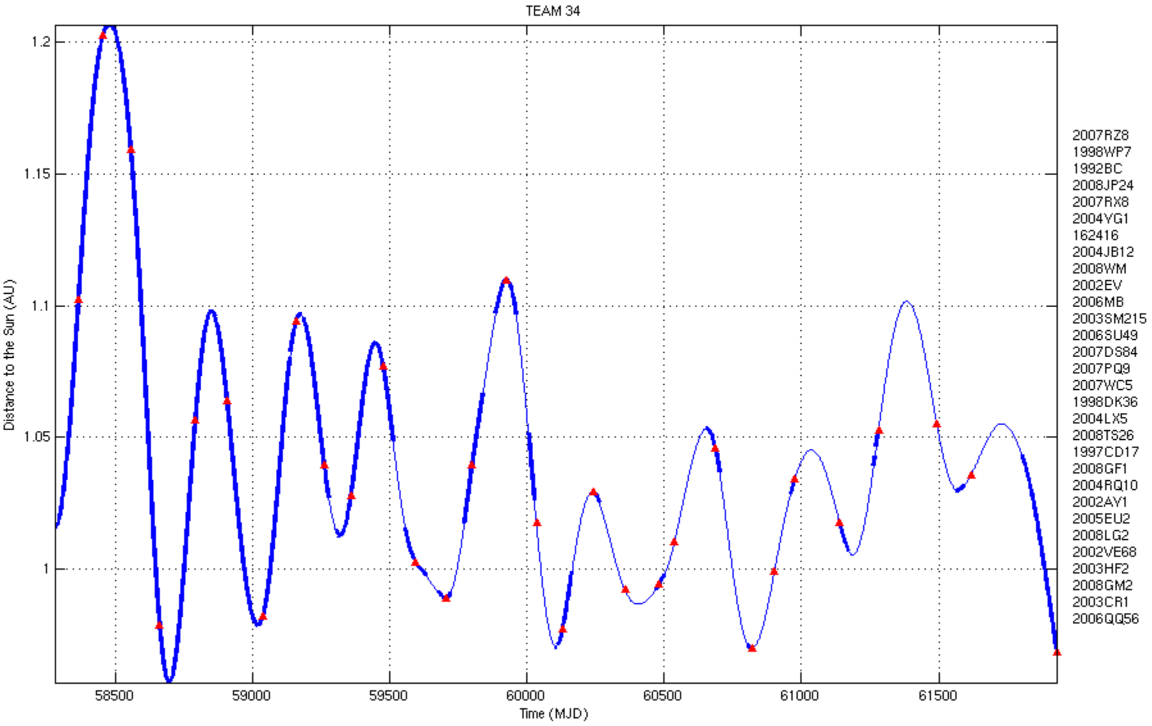
Team 32
University of Texas at Austin, Odyssey Space Research, ERC Incorporated (USA)

The solution method is based on four steps. The first one consists in computing impulsive suboptimal solutions within a global search tree. Then, the selected solutions are optimised thanks to an SQP algorithm in order to yield optimised impulsive trajectories. The third step is dedicated to the conversion of the trajectories into relevant finite burn ones and their optimisation by an SQP algorithm. Finally, the last step consists in a verification of the obtained trajectories by means of the Copernicus system. This last one allows to reconstruct the entire trajectory by satisfying the intermediate asteroid flyby constraints and the final rendezvous constraints.



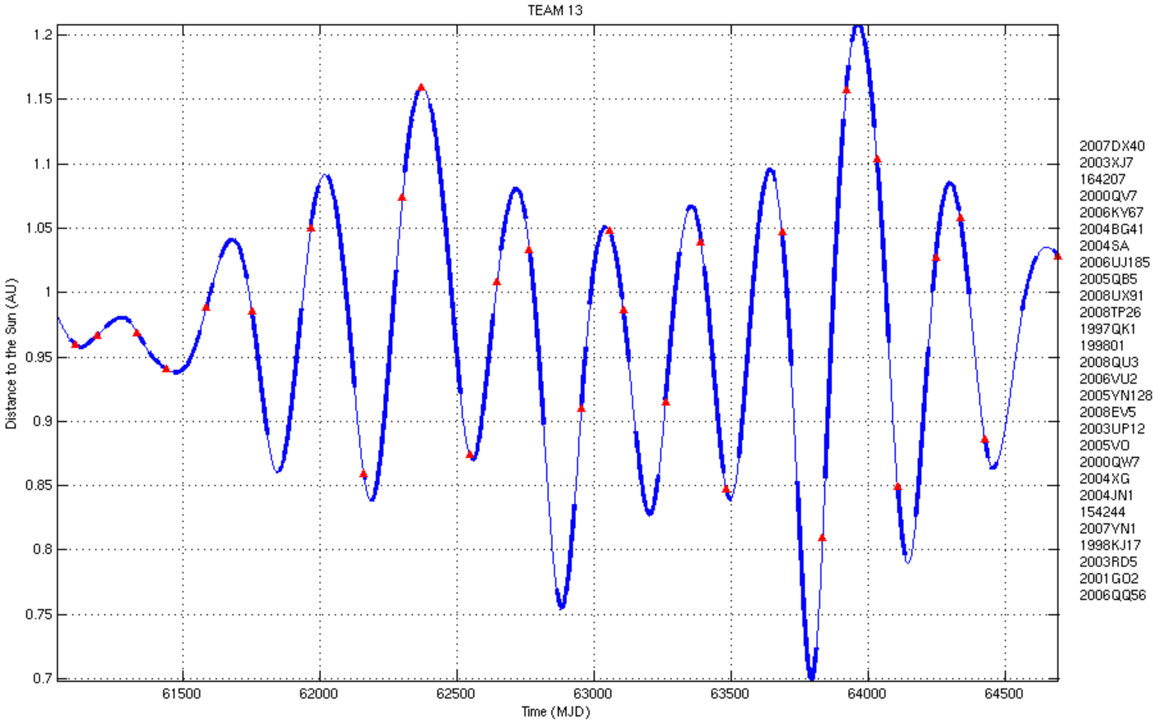
Team 34
University of Glasgow, University of Strathclyde (United Kingdom)

In a first step candidates for the flybys are selected in the following way: first the intersections between the asteroids trajectories and the ecliptic plane for the years 2015 to 2035 are computed. Then, Lambert's problems from any asteroid to any asteroid intersecting the ecliptic plane are solved and classified according to the total ΔV. In addition, minimum and maximum allowed durations are considered for each leg reducing the number of solutions. The last asteroid is considered in a list of 18 obtained by choosing in the database asteroids with a semi major axis between 0.7 AU and 1.0 AU, an eccentricity lower than 0.2 and an inclination between -5 degrees and 5 degrees. In a second step, once a sequence of flybys is determined, a genetic algorithm is used in order to minimise the sum of the changes in velocity at the departure of each asteroid plus the difference in velocity at the final asteroid. Finally, a direct transcription method (DITAN) is used in order to solve the optimal control problem taking into account the low thrust characteristics of the engine.



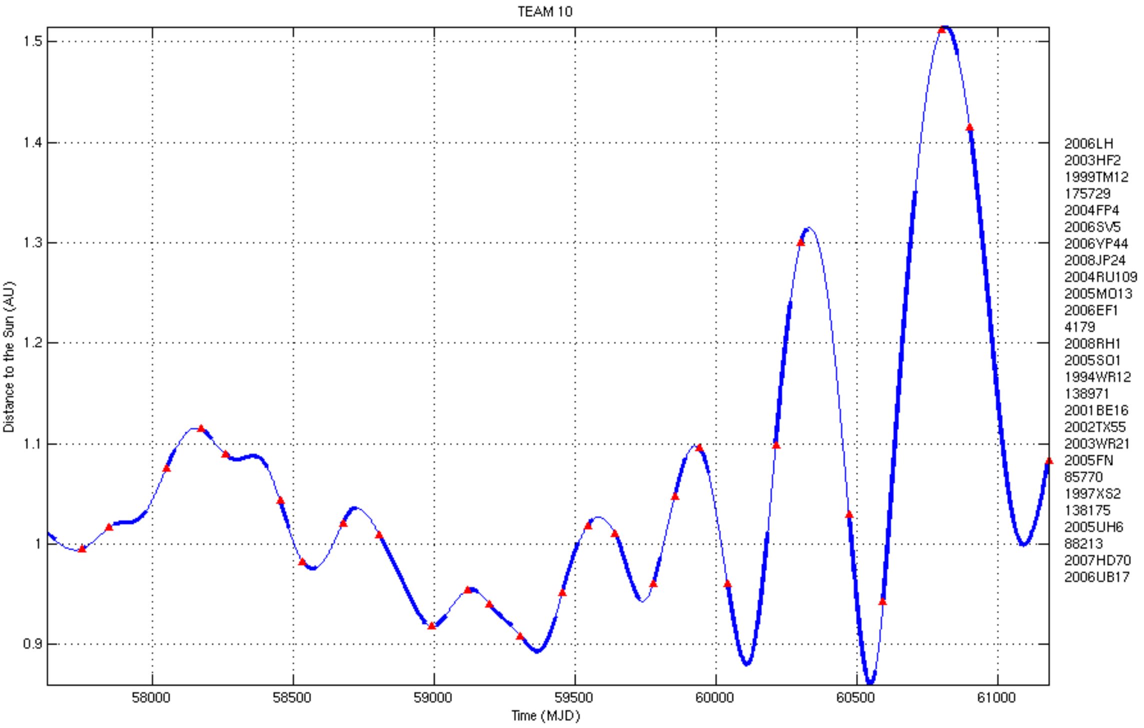
Team 13
Thales Alenia Space (France)

In a first step sequences of asteroids are obtained thanks to a branch and bound algorithm. More precisely, starting from the Earth and using a two weeks grid for the time departure, all minimum time trajectories are computed from the Earth to the asteroids in the database. Selecting a reduced set of trajectories corresponding to the smallest transfer durations, minimum time trajectories are computed again from the selected asteroids to all the remaining ones. This process is repeated until a ten-years-travel duration is reached. Then, selecting again a reduced set of solutions, a final asteroid is chosen for the rendezvous. In a second step, for each sequence of asteroids determined above, the fuel consumption is minimised and the flybys and departure dates are also tuned. All the calculations are done with T_3D software based on Pontryagin's Maximum Principle and shooting methods.



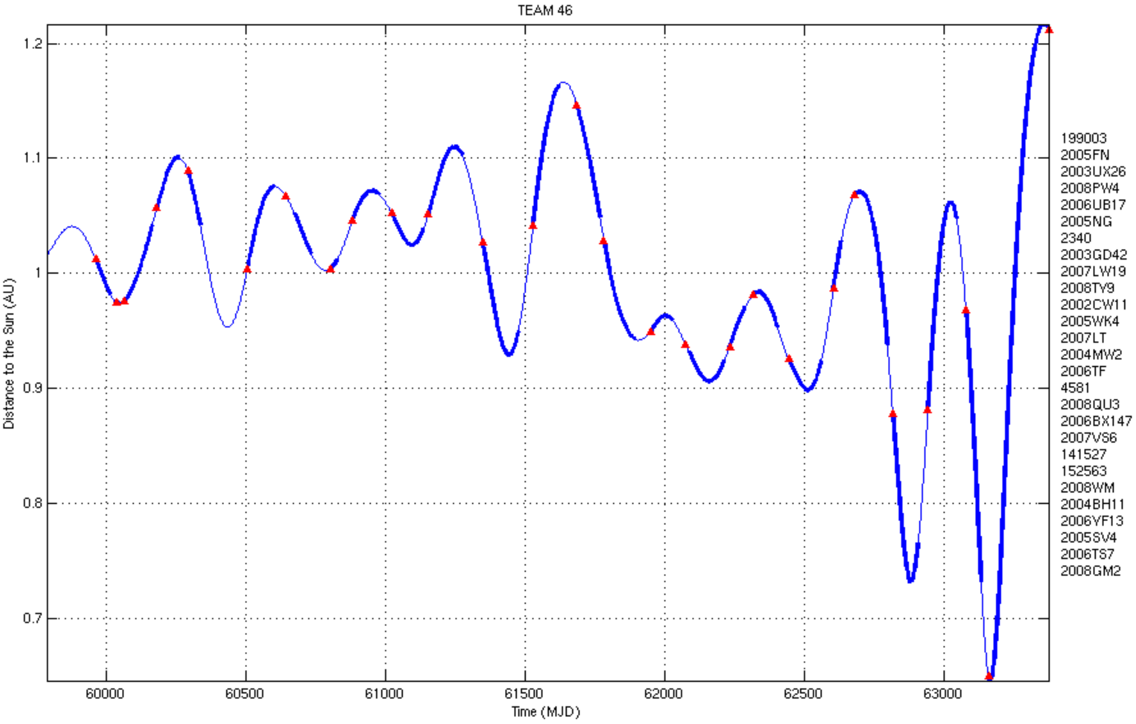
Team 10
University of Trento (Italy)

The problem has been formulated as a sum of sub-problems. Each sub-problem consists in the trajectory planning between each pair of selected asteroids or, initially, between the Earth and the first asteroid selected. Starting from the Earth a solution is computed for every asteroid chosen in the first time interval of 300 days. From these solutions only the best ten in terms of minimum time are selected for the next step. In the second step a new choice of asteroids is made starting from each of the ten final conditions of the previous step and for a new time interval of 300 days. Again only the best ten solutions in terms of overall minimum time are selected and so on. When the time, or the fuel consumption, exceeds a certain value, the solutions are also computed considering the rendezvous with the sequent asteroid. The best of these solutions in terms of minimum fuel consumption is saved as a possible final solution. The algorithm proceeds until the fuel is exhausted or until the overall flight time exceeds 10 years. The overall optimal control problem is formulated as a sum of two-point boundary value problems between the Earth and the first asteroid and between each pair of selected asteroids.



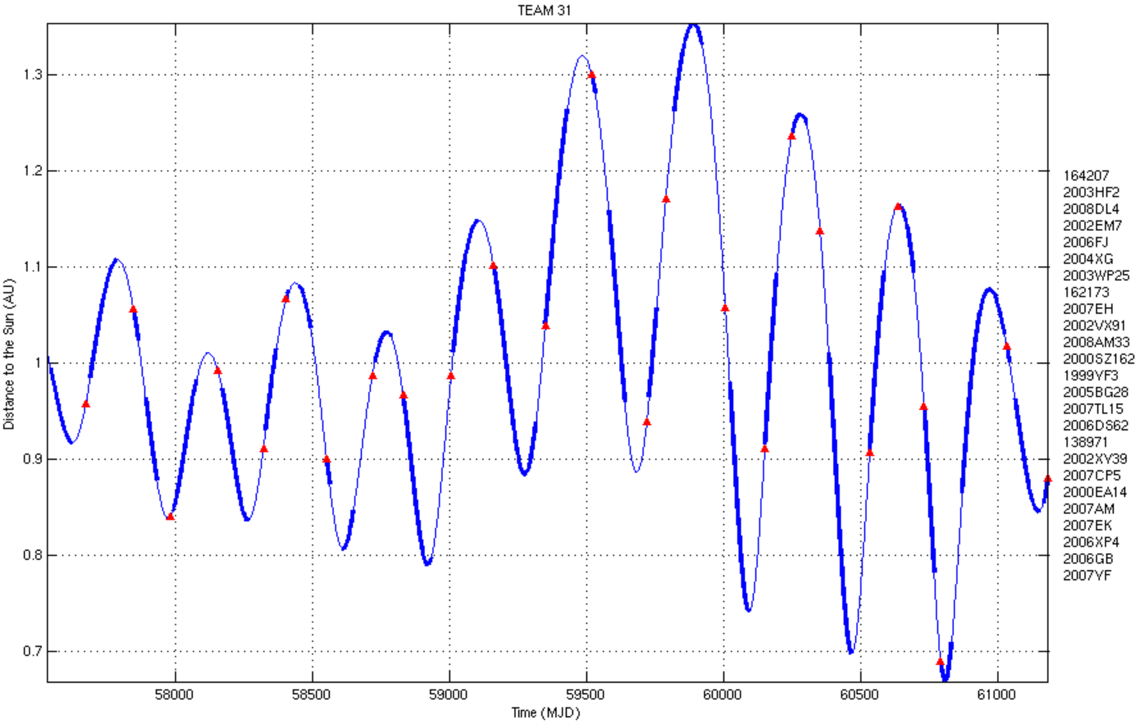
Team 46
University of Bremen (Germany), Politecnico di Milano (Italy)

The method used is based on sequential local optimisation techniques led by a new global strategy to define the sequence of the asteroids. Starting from the Earth with a given velocity at a given start date, it has been analysed which asteroids from the given set can be reached in a time interval of 200 days. A set of reachable points (a “ball”) in space is calculated assuming different fixed steering directions. The asteroid selected is the one that lies in the set of reachable points and can be reached first. Once the asteroid is selected, a local optimisation is performed to define the optimal trajectory that minimises the time of flight and the global energy. The rendezvous with the last asteroid is performed in a similar way. The optimal control problems are solved with the software library NUDOCCCS (reduction to NLP problems). This global algorithm has been run over different computers with different starting dates and randomly chosen initial velocities.



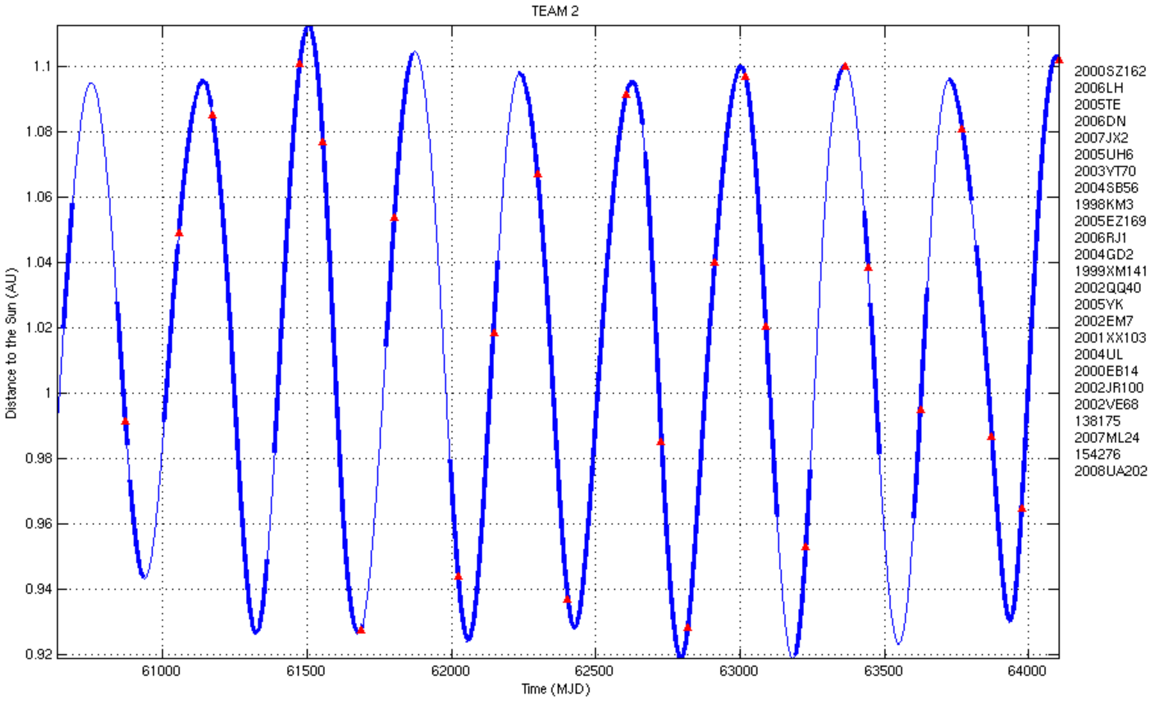
Team 31
Moscow Aviation Institute, Research Institute of Applied Mechanics and Electrodynamics (Russia)

The solution method is based on three main steps. First, the asteroids are sorted with respect to the inclination and the time of crossing the ecliptic plane. Then, a simplified 3-D global optimisation problem is solved with the following unknown variables: launch date, launch v_∞ and v_∞ direction in the ecliptic plane. The propagation of trajectories with constant thrust acceleration into the orbital reference frame allows to define the attainability domain. Power limited trajectories to the selected asteroids are then computed by means of the Pontryagin's maximum principle and homotopic method (EPOCH). The second step consists in applying a Lipschitzian global optimisation algorithm using a set of Lipschitz constants (DIRECT). Finally, the trajectories with constant ejection velocity are deduced from the power-limited trajectories by means of a homotopic algorithm.



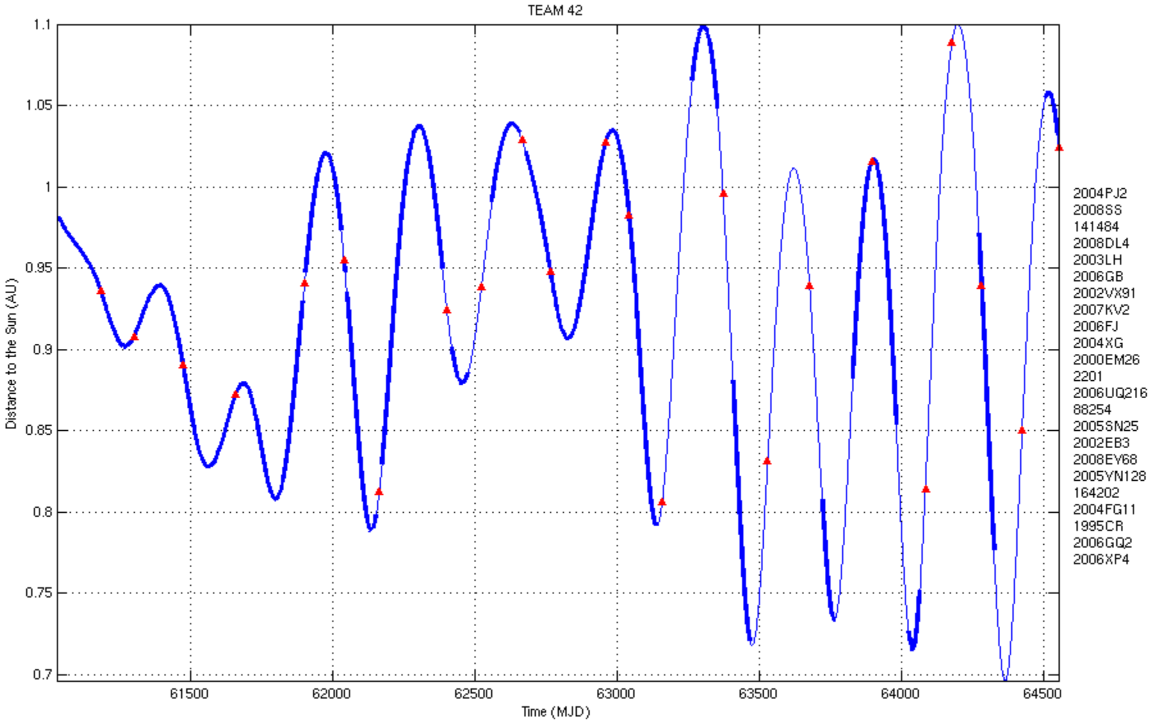
Team 2
Georgia Institute of Technology (USA)

First a pruning method based on a combinatorial scheme of ballistic Lambert's problems was used to determine, for each of the ten lowest ΔV rendezvous asteroids which was the best flyby sequence. Once a promising ballistic solution was identified for a given rendezvous asteroid, the low thrust phasing problem was considered with two different approaches. The ballistic solutions generated several thousand promising trajectories, which were used as initial guesses in a direct low thrust trajectory optimisation code. The best merit solution generated by the low thrust code was finally made feasible to the exact competition dynamics and optimised on a leg-by-leg basis using a robust local differential dynamic programming technique.



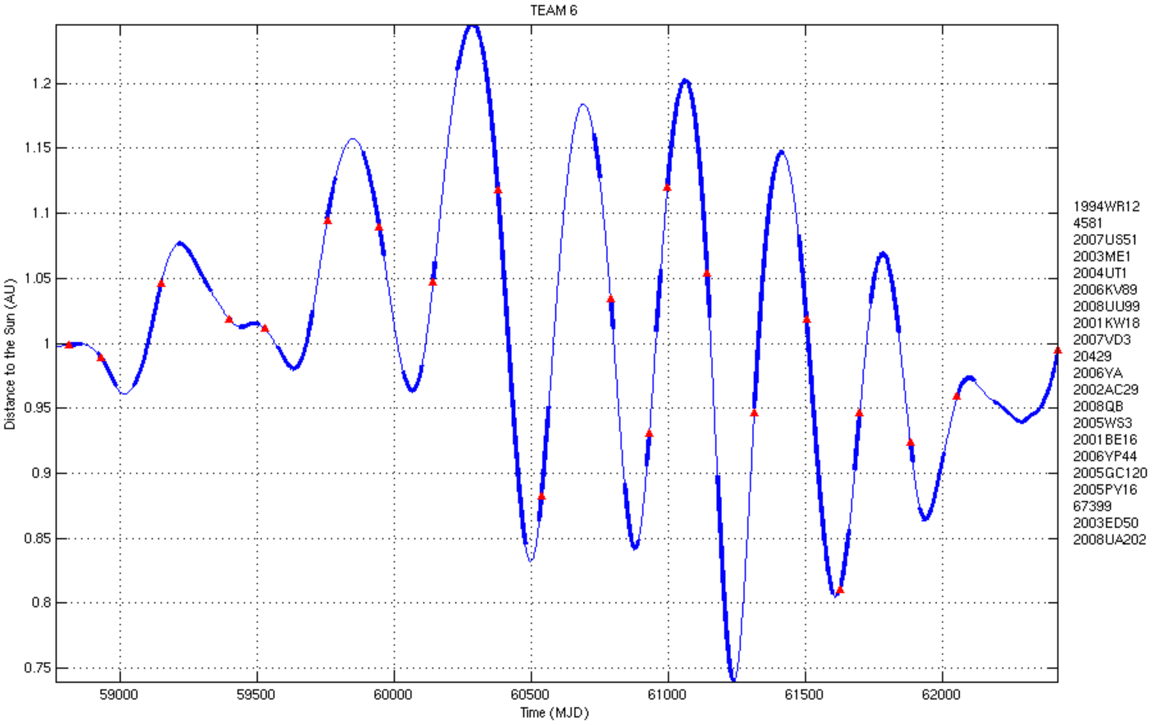
Team 42
TOMLAB (Sweden)

Solution method consists in: selecting a good target asteroid for rendezvous, computing an optimal trajectory to that asteroid (minimising the energy within 10 years and taking into account the 500 kg final mass limit), selecting an asteroid in the vicinity of the computed trajectory and re-computing the trajectory including the flyby (this is done many times), and finally optimising the trajectory by taking into account all the problem constraints and the objective function. For computing optimal trajectories, the PROPT (pseudospectral collocation transcription) and SNOPT (nonlinear programming problems solver) modules were used.



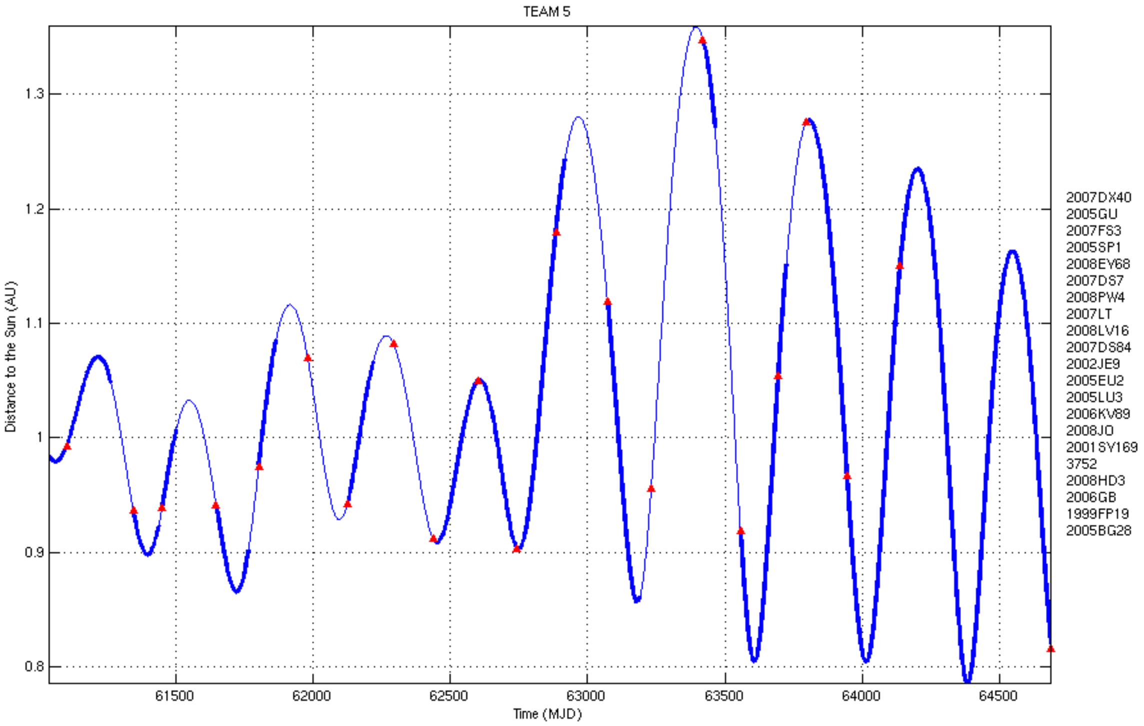
Team 6
VEGA Deutschland GmbH & Co. KG (Germany)

The derivation of the solution was divided in two stages: flyby sequence generation and determination of the low thrust trajectory. Flyby sequence generation was based on graph search algorithms. Several approaches were attempted in order to find an admissible low thrust trajectory, which was optimised using the Pontryagin's Maximum Principle. The flyby sequence generation used a directed acyclic graph in which vertices corresponded to prefixes of node sequences starting with launch node. Two vertices in the graph were connected by an edge when a flyby at the first vertex could be followed by the flyby at the second vertex without violating problem constraints on mission duration, fuel and maximum ΔV. Based on the Cartesian equations of motion, an optimal control problem was formulated imposing equality constraints on the final position and velocity, as provided by the pruning algorithm. To minimise the propellant mass consumption for the fixed time of flight, thrust magnitude and direction were optimised using the Pontryagin's Maximum principle.



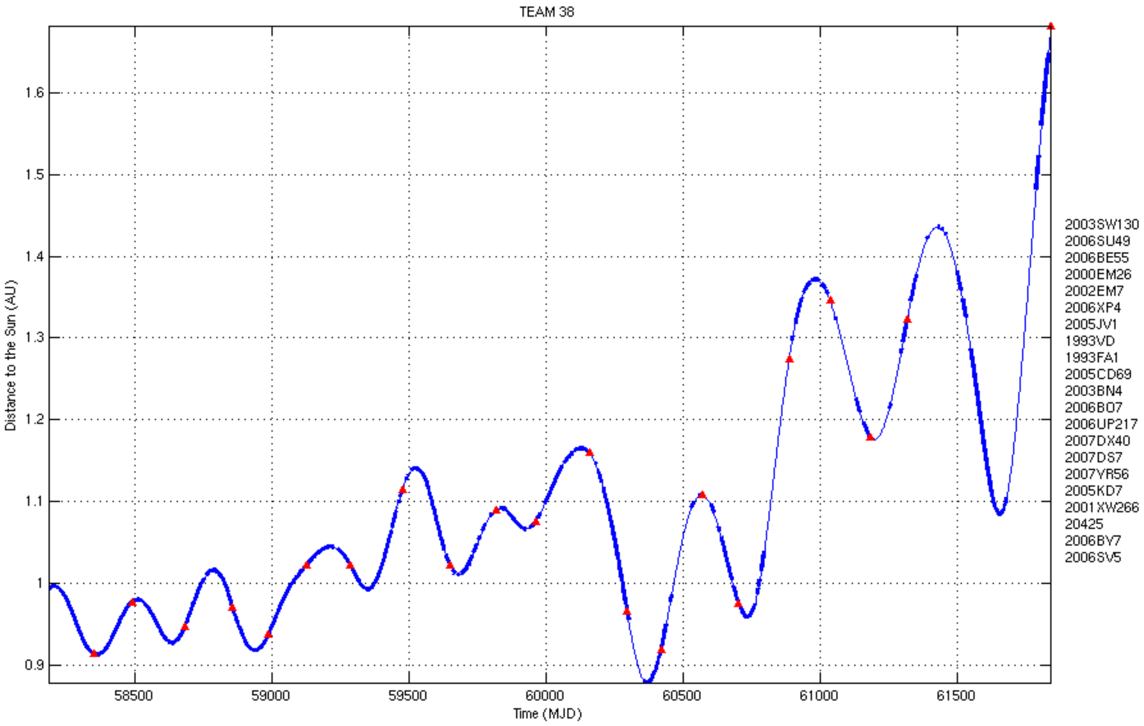
Team 5
DLR German Space Operation Center, Aachen University of Applied Sciences (Germany)

The solution was found using stepwise approach based on an artificial neural networks method (InTrance). This method was used in a first step to find promising sequences of flyby-bodies with maximum flyby-distances starting from 10^7 km down to 10^6 km, a time of flight of 10 years, ignoring the final rendezvous. For the most promising sequences found, the step 1 was repeated with a lower flight time of about one year and a maximum flyby-distance of 2.010^6 km, whereas InTrance had to minimise the sum of flyby-distances to the target bodies found. Then, InTrance was used to optimise a high accuracy trajectory to the first target body found from step 2. If this was not possible, the next body of the sequence was chosen for a flyby and so on. When the remaining flight time dropped below 1000 days, InTrance was also used to find a promising rendezvous body. The body, that fulfilled the rendezvous criteria the most got the highest fitness function value and was chosen as candidate for final rendezvous computation with high accuracy. Again, the final rendezvous leg to the body found after step 4 was optimised with InTrance with final accuracy settings.



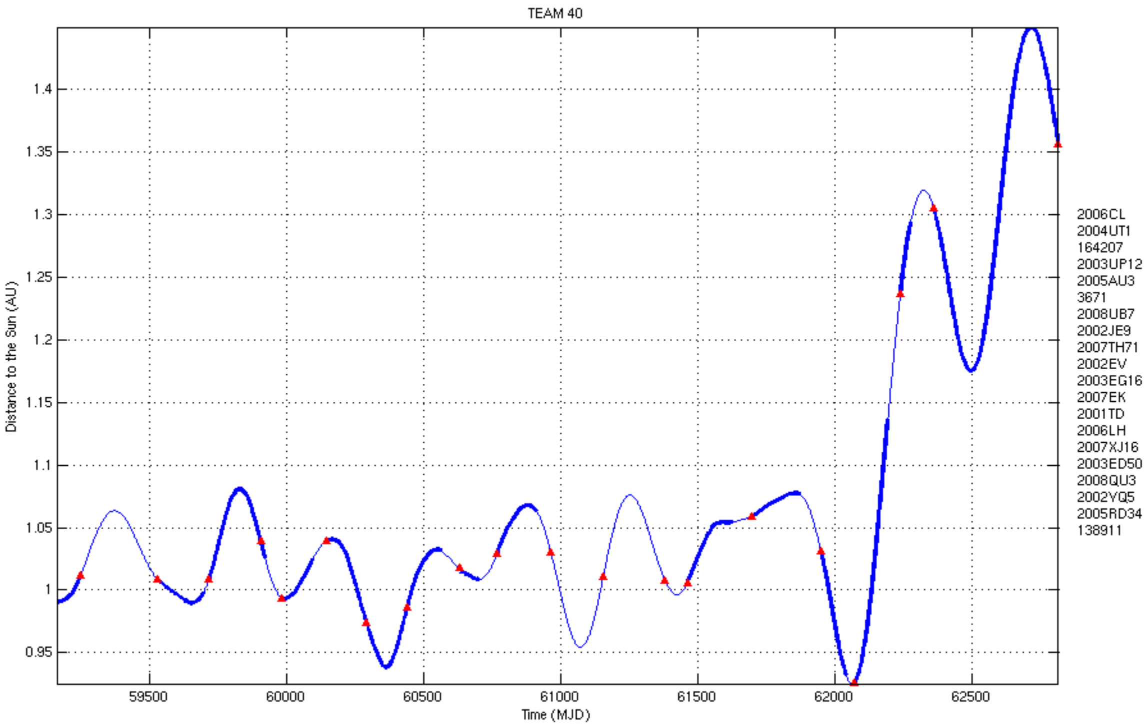
Team 38
Team Astroshape, University of Illinois, Embry-Riddle Aeronautical University (USA)

In a first step the launch date and v_∞ are chosen by a genetic algorithm. More precisely, the spacecraft trajectory is integrated forward in time assuming thrust at 50% of the maximum available in the direction of the velocity vector (spiral trajectory). At each step a region is computed around the spacecraft. If an asteroid of the database is found in this region the spacecraft is instantaneously moved to the location of the asteroid, introducing a discontinuity in position, with no change on the velocity vector. This process is repeated until ten years has elapsed. The genetic algorithm minimises a cost function in which appear the magnitudes of the differences in position between the spacecraft and the different asteroids. This step determines a sequence of asteroids. In a second step, once the sequence of asteroids is determined, the optimal control problem is solved thanks to a direct transcription method that converts it into a nonlinear programming problem.



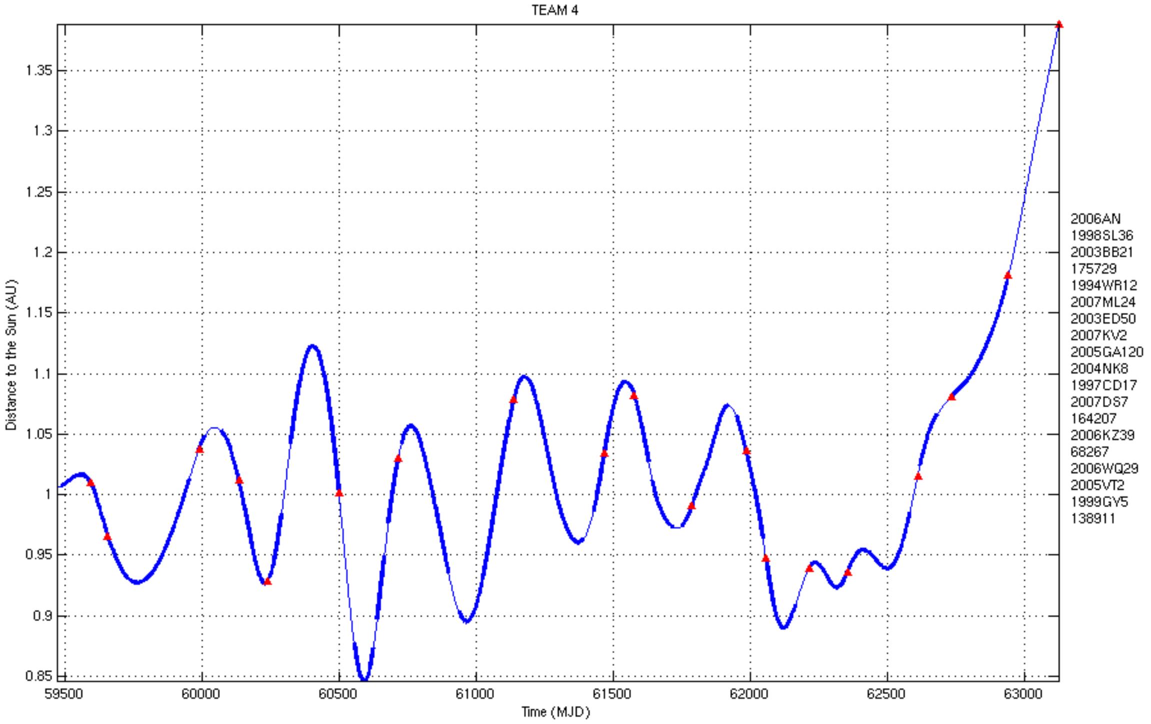
Team 40
DLR Institute of Space Systems (Germany)

The method used is purely based on domain knowledge. Analysing the formulas describing the variation of the orbital parameters gives a good estimate of the location to apply a force in order to change inclination, to go in or outbound. Several plots of the asteroids had been made to analyse their distribution. The other decision was to stay in a unique plane. It does not need to be the ecliptic one, but as a first guess the ecliptic plane was chosen. Four final candidates for rendezvous had been selected according to low eccentricity and inclination.



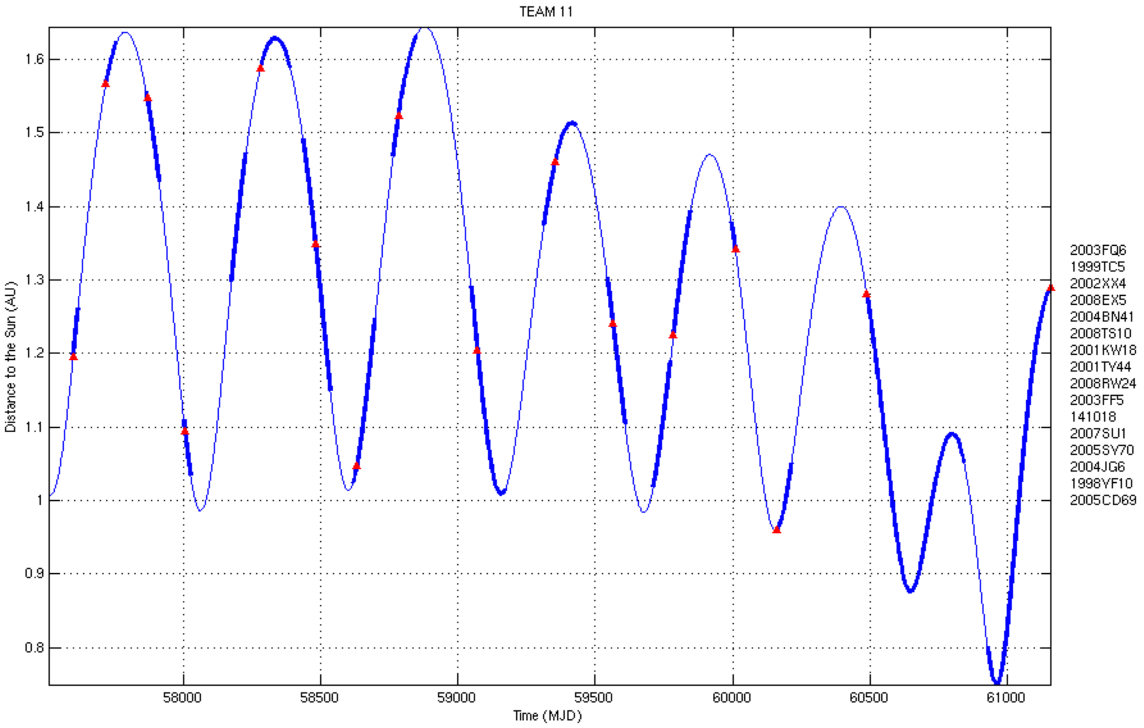
Team 4
Tsinghua University (China)

The method used is based on a two step approach. First, a global optimisation process based on a hybrid algorithm of PSO (Particle Swarm Optimisation) and DE (Differential Evolution) was used to find an optimised trajectory with an optimised flyby sequence. Since generally the solution given by this process did not satisfy the problem accuracy requirements, a second step based on a local optimisation method was used to improve the accuracy. The local optimisation algorithm is based on a direct method that optimises the fuel consumption of every subpart of each phase of the trajectory meanwhile it satisfies the flyby and rendezvous constraints.



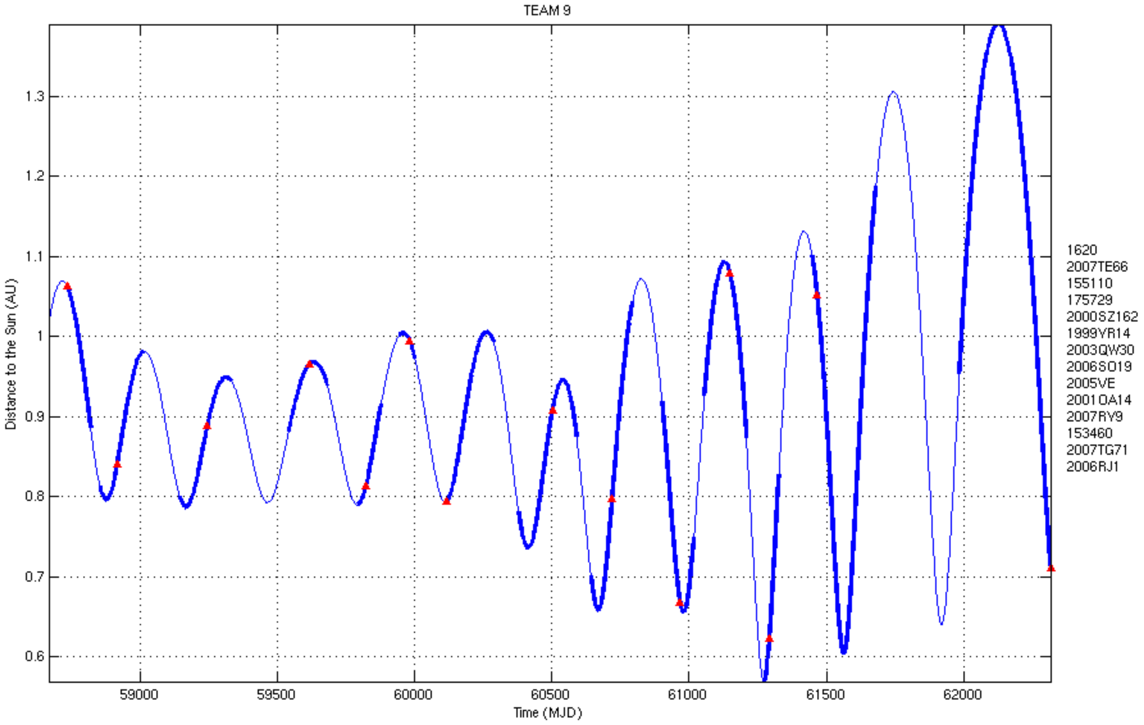
Team 11
University of Missouri (USA)

Considering that optimising the fuel consumption was less important than finding the best sequence of asteroids that can have a least average time of flight during a certain period of time the search was simplified into three sections of optimisation. The first section is the search of launch window and launch velocity, which will give the spacecraft a relatively good trajectory to "expose itself to the other asteroids". The second section is the search of flyby legs having the least average time of flight in a certain period of time. The third section is to search the rendezvous asteroids after a certain time of flight. In these three sub-problems, the possibility of turning a Lambert's result into a low thrust result is added as a constraint of picking sequence and time of flight. Besides this, at the critical points of the branching process, a modified shooting method is used to double check the feasibility of turning ballistic arc into a low thrust arc. By using this method, a Lambert's problem solver is effectively substituted by a low thrust local optimiser to find the local optimum for each flyby. At last, the good ranking results were fed to an indirect low thrust optimiser to get the final results.



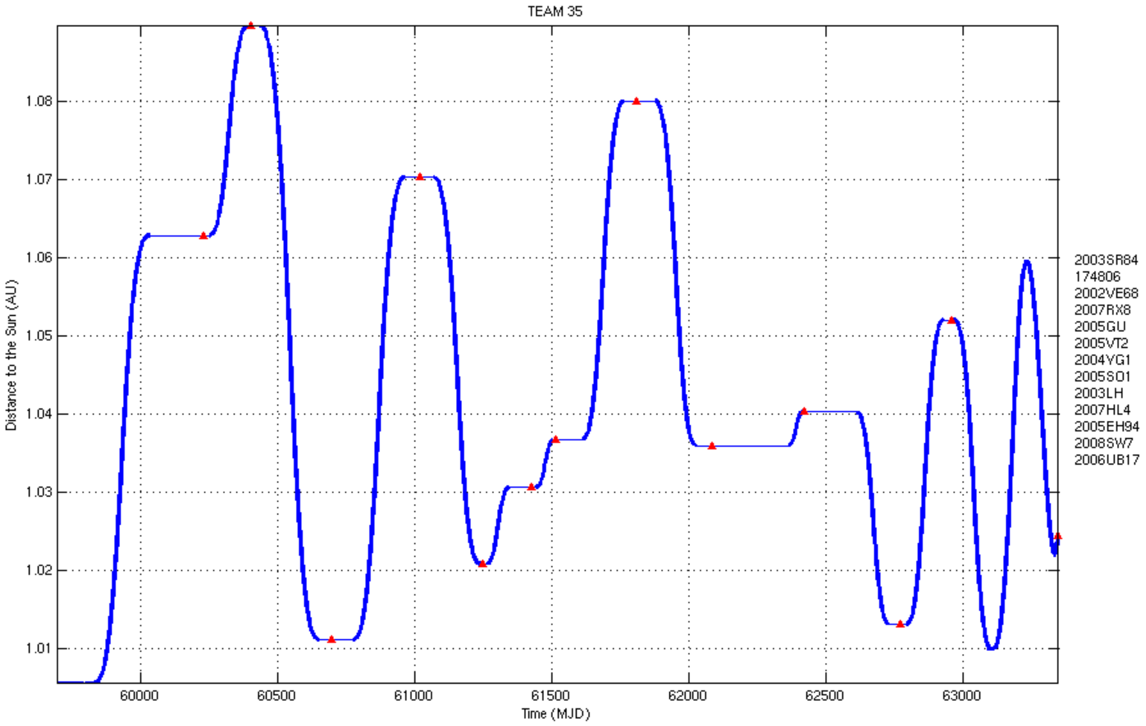
Team 9
Beijing University of Aeronautics and Astronautics (China)

In a first step, the position of the ascending and descending nodes of each asteroid is computed and the corresponding times are deduced. Then the GTOC4 problem is seen as a Lambert's problem between the ascending or descending nodes of the different asteroids. A flyby sequence and a ΔV budget can be optimised by means of the Tent-map Chaotic Particle Swarm Optimisation (TCPSO). Finally, in order to satisfy the position and velocity constraints at flybys and rendezvous, the BFGS optimisation method is used to correct the errors due to the differences between the impulsive problem and the relevant low thrust one.



Team 35
Texas A&M University (USA)

Assuming first that the spacecraft is launched directly in the plane of the asteroid with which it will rendezvous, only 535 asteroids are inclined such that the spacecraft can be launched directly onto the asteroid's plane given the restriction on the initial v_∞. For each of these 535 rendezvous asteroids, the Earth passes through the asteroid's plane 22 times over the 11 year launch window, yielding 11770 distinct launch dates and rendezvous scenarios. Then, by down-selecting to the impact points that fall within given bounds and taking into account phasing considerations, 600 points are selected. In a next step, starting from the above results, a path is built from the Earth to the rendezvous asteroid by constructing a graph in order to check the feasibility of the flybys and to find the maximum number of successive feasible flybys. Finally, for each sequence found above each leg is computed by considering a coast arc, a minimum time circular-to-circular transfer with constant thrust and a coast arc.



Team 37
Nanjing University of Aeronautics and Astronautics (China)

The solution method uses a dynamic programming method combined with a semi-analytical approximate trajectory algorithm. The dynamic programming process is based on three steps: the computation of a grid coordinate system from the departure to the arrival points of each trajectory leg, the definition of a moving corridor (i.e. domain of controllability) an finally a local optimisation of the selected trajectories.

Team 23
CHOPIN Team ISAS/JAXA, JAXA/JSPEC, The Graduate University for Advanced Studies, University of Tokyo, Kyushu University (Japan), Delft University of Technology (The Netherlands)

In a first phase candidates for the rendezvous asteroid are found by considering all the asteroids in the database and by choosing the asteroids with an orbit “similar” to that of the Earth. Combination of launch and arrival date are chosen from solving multi-revolution Lambert’s problems with 9, 10 or 11 revolutions and by considering the smallest v_∞ at the final asteroid while satisfying the constraint on the initial v_∞ at the Earth departure. Then, the final spacecraft positions at the selected final asteroids are integrated backward in time towards the departure time. Potential asteroid flybys are found by looking at the distance between the spacecraft and all the asteroids at each date. For a given sequence of asteroids, the exponential sinusoid Lambert’s problem is solved. This process is repeated until the Earth is reached with a total transfer time equal to around 9 years. In a last step a local optimisation is performed by using collocation and nonlinear programming in order to take into account the low thrust characteristics of the engine.

Team 18
Chinese Academy of Sciences (China)

The solution method is based on the following assumption: the flybys occur when the asteroids cross the ecliptic plane. Then, the first asteroid is chosen by computing the smallest distance from the Earth to all the asteroids crossing the ecliptic plane between 2015 and 2025. This approach is used again for selecting the other asteroids for flybys. The last rendezvous asteroid is chosen by considering asteroids with small inclinations with respect to the ecliptic plane. Then, burn-coast trajectories are considered between two flybys and a burn-coast-burn trajectory is considered for the last arc. Finally, with these assumptions concerning the thrust history, the continuous control problem is converted into a nonlinear programming problem that is solved thanks to an SQP method.