Problem Description for the 8th Global Trajectory Optimisation Competition

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1 Background

The Global Trajectory Optimisation Competition was inaugurated in 2005 by Dario Izzo of the Advanced Concepts Team, European Space Agency. GTOC2 through GTOC7 were organised by the winning teams of the preceding GTOC editions. Keeping this tradition, the Outer Planet Mission Analysis Group and Mission Design and Navigation Section of the Jet Propulsion Laboratory are pleased to organise the eighth edition of the competition, GTOC8. This document reveals the problem that is to be solved for GTOC8.

2 Introduction

The criteria for selecting a problem this year are similar to those used in the previous competitions:

- Global optimisation over a large design space with many local optima.
- Unusual objective function or constraints — no canned methods or existing software can likely fully solve the problem.
- Problem is easy enough to tackle in a 3-4 week timeframe for experienced mission designers or mathematicians, including exploration of new algorithms.
- Problem solutions can be easily verified.

The problem chosen this year, like that of previous years, involves low-thrust trajectory design. The multi-spacecraft aspect of GTOC7 is expanded to consider simple spacecraft formations. Gravity-assist flybys are permitted as in several of the previous competitions. The complexities of choosing suitable flyby or rendezvous asteroids from a large set, or choosing satellite gravity assist sequences, as in GTOC2–GTOC7, have been replaced by the complexities of managing a spacecraft formation and selecting suitable directions from a set of several hundred in which to orient it.

Sections 3-5 describe the problem in the customary language of mission design and astrodynamics. Section 6 provides a more mathematical description of the problem dynamics. Section 7 describes the solution submission requirements. Lastly, the appendix, Section 8, summarises the nomenclature and provides some background information.

3 Problem Summary

This year's problem is the high-resolution mapping of radio sources in the universe using space-based Very-Long-Baseline Interferometry (VLBI). This will be accomplished by three spacecraft flying around the Earth. The spacecraft trajectories are controlled by means of an initial impulse, low-thrust propulsion, and lunar gravity assists. The goal is to observe a number of radio sources by orienting the plane defined by the three

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spacecraft towards each source in turn. A performance index, which depends on the source direction and is loosely related to the efficacy of a triangular formation's resolving potential, must be maximised subject to a variety of constraints. Repeat observations of a source are rewarded extra if the observing triangles are of sufficiently different size. The dynamics are simplified: The Sun's gravity is excluded, Earth is modelled as a point mass, flybys of the Moon are to be modelled as "patched conics" (the Moon does not otherwise affect the trajectory), and the Moon is assumed to follow a conic orbit around Earth.

4 Spacecraft and Trajectory Parameters and Constraints

The three spacecraft are initially collocated in a 400-km altitude circular orbit around the Earth in the ecliptic plane:

$$\begin{aligned} t_0 &= 58849.0 \text{ MJD} \\ x(t_0) &= R_E + 400 \text{ km} \\ a(t_0) &= x(t_0) \\ e(t_0) &= 0 \\ i(t_0) &= 0 \text{ deg} \end{aligned}$$

That is to say, at the indicated epoch of 58849.0 MJD, the spacecraft all lie on the positive x-axis at a distance of $(R_E + 400 \text{ km})$ from the centre of the Earth, in a circular orbit of zero inclination. A single reference frame will be used throughout this problem statement (it happens to be the Earth Mean Ecliptic and Equinox of J2000 frame). Each spacecraft has two propulsion systems: A chemical propulsion system with a specific impulse (I_{sp}) of 450 s and an impulsive capability of up to 3 km/s; and a low-thrust system with a specific impulse of 5000 s and a thrust of up to 0.1 N. Each spacecraft can use its chemical system only once, and the chemical system must be used before the low-thrust system of that spacecraft is first used. The mission begins with the first use of propulsion by any of the spacecraft, *i.e.*, when the first impulse is performed or when the low thrust is first employed, whichever is sooner, on any spacecraft. The spacecraft are not required to perform their impulses at the same time as each other. The mission must begin between MJD 58849.0 and 58880.0 (which corresponds to 1–31 January 2020, inclusive). The mission must end within three years. The time at which the last observation is made marks the end of the mission.

Each spacecraft has an initial mass of 4000 kg and a minimum permissible final mass of 1890 kg. The initial mass is the mass before any propulsion system has been employed.

The direction of the initial impulse, and the direction of the low thrust are unconstrained. The spacecraft range to Earth must obey the following at all times:

$$6578.14 \text{ km} \leq r \leq 1000000.0 \text{ km}$$

For a patched-conic Moon flyby to occur, the spacecraft's geocentric trajectory must match the Moon's geocentric position within 1 km. Details on patched-conic modelling appear in a later section. The flyby altitudes at the Moon (*i.e.* the range to the Moon's centre at closest approach on the flyby minus the Moon radius) cannot be below 50 km:

$$h_{pM} = r_{pM} - R_M \geq 50 \text{ km}$$

Also, the hyperbolic excess velocity of the lunar flybys must be above 0.25 km/s:

$$v_\infty \geq 0.25 \text{ km/s}$$

5 Performance Index

To assess the interferometric resolving power of a given spacecraft formation, a simplified geometric measure is used and is coupled with a reward function for repeat observations as well as a reward function for observing radio sources at low declinations. The radio sources available for observation are a subset of those in the comprehensive catalogue maintained by Leonid Petrov.[2] The right ascension and declination of the selected

[2] `http://astrogeo.org/vlbi/solutions/rfc_2015b/`, accessed May 2015.

radio sources (420 in number) are listed in a separate file, `gtoc8_radiosources.txt`, in the coordinate frame of the present GTOC problem.

The three spacecraft will always form in space either a triangle wherein each spacecraft is at a vertex, or a line, in the degenerate case. An observation of a source occurs when the triangle normal (*i.e.*, the normal to the plane containing the triangle) points in the direction of a source. Either of the two diametrically opposed normal directions can be chosen. The position of the Earth and the Moon are immaterial during an observation. Observations occur instantaneously. Observations must be spaced at least 15 days apart. It is not, however, mandatory to consider an observation as "taken" just because the geometrical and timing constraints are met. Observations that are not taken do not, of course, contribute to the performance index.

The performance index to be maximised is thus:

$$J = \sum_{\substack{\text{all} \\ \text{observations}}} Ph\left(0.2 + \cos^2\delta\right)$$

In the above expression, h denotes the smallest of the three altitudes of the observing triangle and must satisfy

$$h \geq 10000.0 \text{ km}$$

An altitude of a triangle is the perpendicular distance from a vertex to the opposite side (or its extension). δ is the declination of the source being observed. P is a weighting factor for repeat observations that takes on the values of 1, 3, 6, or 0 according to the following rules, where "first", "second", "third", *etc.* are taken to denote chronological order:

- If an observation is the first observation taken of a source: $P = 1$.
- If an observation is the second observation taken of a previously observed source: $P = 3$ if $\frac{h_{\text{max}}}{h_{\text{min}}} \geq 3$, otherwise $P = 1$.
- If an observation is the third observation taken of a previously observed source: $P = 6$ if $\frac{h_{\text{max}}}{h_{\text{mid}}} \geq 3$ and $\frac{h_{\text{mid}}}{h_{\text{min}}} \geq 3$, else $P = 3$ if $\frac{h_{\text{max}}}{h_{\text{min}}} \geq 3$ and the second observation of the source had a weight of $P = 1$, otherwise $P = 1$.
- If an observation is the fourth or greater observation taken of a previously observed source: $P = 0$.

In these definitions, $h_{\text{min}}, h_{\text{max}}$ denote the minimum and maximum h, respectively, of the observing triangles involved in the repeat observations (regardless of the chronological order), and h_{mid} is the intermediate h value when three repeat observations are made. Examples of P values are given in Section 8.3.

Thus, J is effectively the weighted sum of a measure of the sizes of the observing triangles. J should be reported in units of km. The pointing tolerance that will be enforced during verification of the submitted solutions will be 0.1 deg.

In the event that solutions are evenly ranked in terms of J, the number of different sources observed (not the number of observations) will serve as a tie breaker; thereafter, if needed, the sum of the final masses of the three spacecraft will be considered. In both cases, the higher the value the better.

6 Dynamical models

6.1 Physical Constants and Orbit Parameters

The Moon is assumed to follow a Keplerian (conic) orbit around the Earth; the orbit elements are specified in Table 1. The spacecraft will also follow conic orbits around the Earth when they are not thrusting; lunar flybys during a non-thrusting period will cause an instantaneous jump to a new conic orbit by means of a velocity discontinuity as described in a later section. The gravitational parameters and radii of the Earth and Moon, along with other constants and conversions are listed in Table 2.

Table 1: **Keplerian orbit elements of the Moon at Epoch = 58849.0 MJD**

Orbit Element	Value
semimajor axis, a (km)	383500.0
eccentricity, e	0.04986
inclination, i (deg.)	5.2586
LAN, Ω (deg.)	98.0954
Arg. peri., ω (deg.)	69.3903
Mean anomaly, M_0 (deg.)	164.35025

Table 2: **Physical constants and conversions**

Gravitational parameter of Earth, μ (km^3/s^2)	398600.4329
Gravitational parameter of Moon, μ_M (km^3/s^2)	4902.8006
Earth radius, R_E, (km)	6378.14
Moon radius, R_M, (km)	1737.5
Standard acceleration due to gravity, g (m/s^2)	9.80665
Day, (s)	86400
Year, (days)	365.25

6.2 Dynamics and conversions between elements

The motion of the Moon around the Earth is governed by these equations:

$$\ddot{x} + \mu \frac{x}{r^3} = 0, \quad \ddot{y} + \mu \frac{y}{r^3} = 0, \quad \ddot{z} + \mu \frac{z}{r^3} = 0$$

where

$$r = \sqrt{x^2 + y^2 + z^2} = \frac{a(1-e^2)}{1 + e\cos\theta}$$

The motion of the spacecraft around the Earth is governed by the same formulae but with the addition of the x, y, z components of the thrust acceleration, an equation for the mass, as well as the ability to introduce discontinuities in the spacecraft velocity by means of an impulse from the chemical propulsion system or of a lunar flyby:

$$\ddot{x} + \mu \frac{x}{r^3} = \frac{T_x}{m}, \quad \ddot{y} + \mu \frac{y}{r^3} = \frac{T_y}{m}, \quad \ddot{z} + \mu \frac{z}{r^3} = \frac{T_z}{m}, \quad \dot{m} = -\frac{T}{I_{sp} g}$$

where

$$T = \sqrt{T_x^2 + T_y^2 + T_z^2} \leq 0.1\text{N}$$

The permitted discontinuities in the spacecraft velocity are described in Sections 6.3 and 6.4.

Conversion from orbit elements to cartesian quantities is as follows:

$$\begin{aligned}
x &= r[\cos(\theta+\omega)\cos\Omega - \sin(\theta+\omega)\cos i \sin\Omega] \\
y &= r[\cos(\theta+\omega)\sin\Omega + \sin(\theta+\omega)\cos i \cos\Omega] \\
z &= r[\sin(\theta+\omega)\sin i] \\
v_x &= v[-\sin(\theta+\omega-\gamma)\cos\Omega - \cos(\theta+\omega-\gamma)\cos i \sin\Omega] \\
v_y &= v[-\sin(\theta+\omega-\gamma)\sin\Omega + \cos(\theta+\omega-\gamma)\cos i \cos\Omega] \\
v_z &= v[\cos(\theta+\omega-\gamma)\sin i]
\end{aligned}$$

where the velocity v is

$$v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}},$$

the flight path angle is obtained from

$$\tan\gamma = \frac{e\sin\theta}{1 + e\cos\theta},$$

the true anomaly is related to the eccentric anomaly by

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2},$$

the eccentric anomaly is related to the mean anomaly by Kepler's equation,

$$M = E - e\sin E,$$

and the mean anomaly is related to time and the initial mean anomaly by

$$M - M_0 = \sqrt{\frac{\mu}{a^3}}(t - t_0).$$

Thus, based on the provided orbit elements for the Moon and Earth's gravitational parameter, the cartesian positions and velocities of the Moon may be computed as a function of time with only the minor nuisance of having to solve Kepler's equation for E by some iterative procedure. (That is, for the Moon and a non-thrusting spacecraft, the equations of motion do not need to be numerically integrated to find position and velocity at some given time.)

The orbit elements may also be computed from the cartesian state by inverting the equations.

6.3 The initial impulse from the chemical system

The chemical propulsion system provides an instantaneous change in the spacecraft velocity (an impulse) and mass. Specifically, suppose that the impulse occurs at a time t, and that t^-, t^+ denote the instants immediately preceding and succeeding the impulse. Then the spacecraft velocity and mass must satisfy:

$$\begin{aligned}
\Delta V &= \|\vec{v}(t^+) - \vec{v}(t^-)\| \;\leq\; 3\,\text{km/s} \\
m(t^+) &= m(t^-)\exp\left(-\frac{\Delta V}{g I_{sp}}\right)
\end{aligned}$$

where $\exp(p)$ denotes the base of the natural logarithm raised to the power p, g denotes the standard acceleration due to gravity, and I_{sp} is the chemical system's specific impulse.

6.4 Mathematical modelling of "patched-conic" flybys

Lunar flybys are modelled using the patched-conic approximation and neglecting the time spent inside the Moon's sphere of influence. The gravity assist (flyby) occurs at time t_G when the spacecraft geocentric position equals the moon's geocentric position to within 1 km; the spacecraft geocentric velocity undergoes a discontinuous change in such a way that the outgoing and incoming hyperbolic excess velocity relative to the Moon have the same magnitude and are separated by the turn angle δ_t. Specifically

$$\begin{aligned}
\vec{x}(t_{G-}) &= \vec{x}_M(t_{G-}) \\
\vec{x}(t_{G+}) &= \vec{x}_M(t_{G+}) \\
\vec{x}(t_{G+}) &= \vec{x}(t_{G-}) \\
\vec{v}_{\infty G-} &= \vec{v}(t_{G-}) - \vec{v}_M(t_{G-}) \\
\vec{v}_{\infty G+} &= \vec{v}(t_{G+}) - \vec{v}_M(t_{G+}) \\
|\vec{v}_{\infty G+}| &= |\vec{v}_{\infty G-}| = v_\infty \\
\vec{v}_{\infty G+}\cdot\vec{v}_{\infty G-} &= v_\infty^2\cos\delta_t \\
\sin(\delta_t/2) &= \frac{\mu_M/(R_M + h_{pM})}{v_\infty^2 + \mu_M/(R_M + h_{pM})}
\end{aligned}$$

subject to the timing and altitude constraints

$$t_{G+} = t_{G-}, \qquad h_{pM} \geq 50\text{km}, \qquad v_\infty \geq 0.25\text{km/s}$$

For computational purposes, the equality condition on the flyby position can be relaxed up to 1 km:

$$|\vec{x}(t_G) - \vec{x}_M(t_G)| \leq 1\text{km}$$

and similarly the tolerance on the velocity condition is 1 m/s:

$$|\vec{v}_{\infty G+}| - |\vec{v}_{\infty G-}| \leq 1\text{m/s}$$

This patched-conic method is the same as the one used in previous GTOC competitions.

6.5 Orienting the triangle normal

The normal, $\vec{n}$, of the observing triangle is given by

$$\vec{n} = \pm (\vec{r}_2 - \vec{r}_1) \times (\vec{r}_3 - \vec{r}_1)$$

where $\vec{r}_i$ is the position vector of the i^{th} spacecraft relative to the Earth, and a choice of sign is available. The degenerate case where the cross product is zero cannot be used to make an observation. The direction of a source is given by the vector $\vec{s}$:

$$\vec{s} = \cos\delta \cos\alpha \ \hat{x} + \cos\delta \sin\alpha \ \hat{y} + \sin\delta \ \hat{z}$$

where α is the right ascension of the source, and δ the declination.

An observation can be taken when the vectors $\vec{n}$ and $\vec{s}$ are aligned. The alignment tolerance during the solution verification process will be 0.1 deg.

7 Solution Format

Each team should return its best solution by email to Anastassios.E.Petropoulos@jpl.nasa.gov before 23 June 2015, 20:00 GMT. If any of the data files are too large to send by email, alternative arrangements will be made with the affected teams to submit the affected files via a JPL website; a 24-hour extension will be granted for those files (the other files will not be covered by the extension).

Five files must be returned: One summarising the solution and the methods used, a trajectory data file for each spacecraft, and an observation file.

1. **A Brief Description** of the methods used, a summary of the characteristics of the best trajectory found, including the value of the performance index J, and a visual representation of the trajectory. The file should preferably be in Portable Document Format (PDF) or PostScript (PS) format; Microsoft Word format should also be acceptable.

2. **A Trajectory File for each spacecraft**, which will be used to verify the solution returned, and must follow the format and units provided in the ASCII template file `gtoc8_traj_format.txt`. Time, state, mass and thrust must be reported at intervals that are described below, on a single line for each time point. The coordinate frame to be used is that used in the problem statement. The spacecraft number (1, 2, or 3) should be indicated in the header of each file. The required data are summarised here:

   - Time (MJD)
   - Spacecraft cartesian position (x, y, z, km) and velocity (v_x, v_y, v_z, km/s) relative to the Earth.
   - Spacecraft mass (kg)
   - Thrust vector, expressed in cartesian coordinates (N)

   The first time entry for each trajectory file must be for the mission start time. The reporting interval need not be at a constant time step. The three trajectory files need not have the same time steps as each other. Variable time steps are permitted to facilitate both the preparation and the verification of the trajectory file and will also allow for smaller file sizes. However, several constraints must be met:

- The angular separation of the position vectors listed on two successive lines must be between 0.1 deg and 1 deg during thrusting. This latitude is permitted so that rapid dynamics can be captured when needed, without over-reporting when the dynamics are smoother. It also ameliorates the potential difficulty of reporting at precise time intervals that are unrelated to the numerical integration process.
- If an impulse is applied, two lines must be listed with identical data for all quantities except velocity, mass, and possibly thrust; any thrust should be listed as zero for the first line and zero for the second line only if there is no thrust commencing at that time.
- When thrust turns on or off, two lines must be listed with identical data for all quantities except the thrust.
- When thrust is off, that is, during coast arcs, there must be no additional lines listed — the lines listed for the thrust on/off events are sufficient to fully describe the spacecraft state during coast arcs. Thus, a coast arc will always have exactly two lines to describe it — one for the start of the coast arc and one for the end. If a coast arc has an impulse on it, it must of course be separated into two coast arcs, one on either side of the impulse.

Verification of any impulse on the trajectory and the coast arcs will be done using standard orbit mechanics. Verification of the thrust arcs will be done by interpolating the reported thrust profile either using an Euler step or using higher order methods and using the interpolation to numerically propagate a reported state forward, comparing the resulting propagated state with the reported state. The forward propagation will typically be restarted after between one and three revolutions.

3. **An Observation File** which lists the observations made. It must follow the format and units provided in the ASCII template file `gtoc8_obs_format.txt`. The data to be reported are:
   - The time at which an observation is made (MJD)
   - The source being observed (Source ID)
   - The value of P for this observation (1, 3, or 6)
   - The value of h for this observation (km)
   - The cartesian coordinates of the first spacecraft (x, y, z, km)
   - The cartesian coordinates of the second spacecraft (x, y, z, km)
   - The cartesian coordinates of the third spacecraft (x, y, z, km)
   - The observing direction of the observing triangle ($\vec{n}/|\vec{n}|$) in cartesian coordinates.

8 Appendix

8.1 Nomenclature

Orbit elements and related quantities	
a	semimajor axis, km
e	eccentricity
i	inclination, rad
Ω	longitude of the ascending node (LAN), rad
ω	argument of periapsis, rad
M	mean anomaly, rad
θ	true anomaly, rad
E	eccentric anomaly, rad
r	radius from Earth, km
γ	the flight path angle, rad
r_{pM}	periapsis radius on a flyby with respect to the Moon, km
R_E	radius of Earth, km
R_M	radius of the Moon, km
h_{pM}	flyby altitude (at closest approach to Moon) $= r_{pM} - R_M$, km
δ_t	turn angle of the $\vec{v}_\infty$
μ	Gravitational parameter of Earth, km^3/s^2
μ_M	Gravitational parameter of the Moon, km^3/s^2
Cartesian position and velocity	
x, y, z	the cartesian position coordinates of an orbiting body with respect to Earth, km
$\vec{x}$, $\vec{r}$	vector of position coordinates, x, y, z
v_x, v_y, v_z	the cartesian velocity components of an orbiting body with respect to Earth expressed in an inertial reference frame, km/s
$\vec{v}$	vector of velocity components, v_x, v_y, v_z
$\vec{v}_\infty$	hyperbolic excess velocity vector, km/s
v_∞	hyperbolic excess velocity magnitude, km/s
Other quantities	
α	right ascension of a source (deg.)
δ	declination of a source (deg.)
P	repeat-observation weighting factor
h	the smallest of a triangle's three altitudes (km)
$\vec{n}$	a vector normal to the observing triangle
t	time, s
m	spacecraft mass, kg
I_{sp}	specific impulse, s
T	thrust of propulsion system, N
g	standard acceleration due to gravity at Earth's surface, m/s^2
Subscripts and superscripts	
$()_0$	value of quantity at some given instant
$()_G$	a quantity related to a gravity assist (flyby)
$()_{G-}$	a quantity immediately before a gravity assist (flyby)
$()_{G+}$	a quantity immediately after a gravity assist (flyby)
$()_E$	a quantity related to Earth
$()_M$	a quantity related to the Moon
$\dot{()}$	time derivative of quantity

8.2 Glossary

gravity assist A hyperbolic flyby of a [massive] body for purposes of achieving a desirable course change.

Modified Julian Date (MJD) Has units of days and is defined as MJD=(Julian_Date−2400000.5), where the Julian Date is simply the number of days past some defined point in the past.

8.3 Explanatory and background comments

8.3.1 Repeat observations: Example of P values

To illustrate the rules by which the value of P is determined, several examples are provided in Table 3 for observations of hypothetical sources along with the increment to J that each observation brings.

Table 3: **Example of P values and increments to J for hypothetical sources with $\delta = 0°$**

Observation Date (MJD)	Source	h (km)	P	Increment to J (km)
59000.0	S1	10,000.0	1	10,000.0×1.2
59050.0	S1	30,000.0	3	90,000.0×1.2
59100.0	S1	95,000.0	6	570,000.0×1.2
59200.0	S2	100,000.0	1	100,000.0×1.2
59300.0	S2	30,000.0	3	90,000.0×1.2
59400.0	S2	10,000.0	6	60,000.0×1.2
59500.0	S3	100,000.0	1	100,000.0×1.2
59550.0	S3	10,000.0	3	30,000.0×1.2
59600.0	S3	40,000.0	1	40,000.0×1.2
59025.0	S4	10,000.0	1	10,000.0×1.2
59075.0	S4	20,000.0	1	20,000.0×1.2
59125.0	S4	30,000.0	3	90,000.0×1.2
59225.0	S5	10,000.0	1	10,000.0×1.2
59275.0	S5	20,000.0	1	20,000.0×1.2
59325.0	S5	29,000.0	1	29,000.0×1.2

8.3.2 Patched conics

For a full understanding of the patched-conic method and its relation to real trajectories, the reader is referred for example to Richard Battin's textbook, "An Introduction to the Mathematics and Methods of Astrodynamics," AIAA, Reston, Virginia, 1999. Here, we say only a few words in an attempt to make the concept a little less foreign, and provide some context for the mathematics and terminology of Section 6.4.

For a real-world, lunar flyby trajectory, which a spacecraft could actually fly assuming perfect knowledge and perfect execution, there are three distinct conceptual parts: The trajectory before the flyby (where the spacecraft is predominantly affected by the Earth), an almost hyperbolic flyby (where the spacecraft is predominantly affected by the Moon and flies on an arc that is approximately part of a hyperbola with focus at the Moon's centre), and the trajectory after the flyby (where the spacecraft is again mostly affected by Earth). In the patched-conic approximation, the trajectory parts before and after the flyby are modelled as being continuous in position and passing through the centre of the Moon, without sensing the gravity of the Moon, but with a velocity discontinuity at the time when the spacecraft position matches the position of the Moon's centre. Given the patched-conic approximation for the pre- and post-flyby trajectories, the hyperbola which approximates the almost-hyperbolic part can be computed using the equations of Section 6.4. Specifically, what is termed the "flyby altitude" in the patched-conic method is the altitude computed based on this hyperbola; it has no relation to the fact that the pre- and post-flyby trajectories of the patched-conic approximation pass through the centre of the Moon. The flyby periapsis vector is similalry based on this hyperbola. There is, of course, also a timing error in the patched-conic approximation, because in the real world the spacecraft velocity will be noticeably altered by the Moon's gravity (unless the flyby altitude is very high), an effect which is not modelled in the approximation. In practice, the position, velocity, and timing errors of the patched-conic approximation are frequently small enough to allow the method to be used for preliminary design.

8.3.3 Consistency of units

Units should be consistent when using numerical values in the equations. For example, the equations of motion for a thrusting spacecraft have thrust appearing on the right, whose customary units are Newtons ($\mathrm{kg\,m\,s^{-2}}$). However, on the left, the customary length units are kilometres. Hence the thrust in Newtons should be divided by 1000, or the quantities involving length on the left should be converted to be expressed in metres.